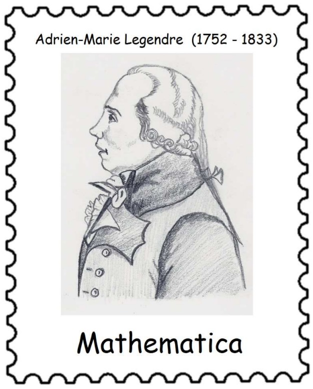
**ADRIEN-MARIE LEGENDRE** (September 18, 1752 – January 10, 1833)

by HEINZ KLAUS STRICK, Germany



(drawing: © Andreas Strick)

There exists a picture that was supposed to be of the mathematician ADRIEN-MARIE LEGENDRE, and if the French postal agency had issued a stamp on the occasion of LEGENDRE's 250th birthday, then it likely would have featured the portrait displayed above. However, it was recently discovered that the portrait is in fact not that of the mathematician, but of his contemporary the politician LOUIS LEGENDRE, a member of the group of *Montagnards* around DANTON, MARAT, and ROBESPIERRE.

It is no longer possible to unravel the thread by which this confusion arose. Perhaps it has something to do with ADRIEN-MARIE LEGENDRE's modesty, in that he placed little value on having himself immortalised in portraiture. Many years after his death, when a search was made for a picture of the great man, one was found with the inscription LEGENDRE, by which time the politician, who had died in 1797, had been long forgotten. Thus the only existing portrait appears to be a caricature from the year 1820 by JULIEN-LÉOPOLD BOILLY, who drew the members of the Institut de France.

(Source: English Wikipedia)

Little is known about LEGENDRE's childhood. He was raised in a well-to-do family, and he attended the *Collège Mazarin*, in Paris, where in the year 1770, he defended his dissertation in mathematics and physics.

From 1775 to 1780, he taught, together with PIERRE-SIMON LAPLACE, at the *École Militaire*, to which he had been given a recommendation by JEAN-BAPTISTE LE ROND D'ALEMBERT.

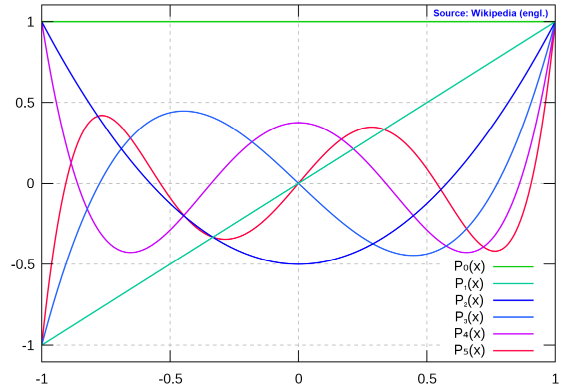

In 1782, he took part in a scientific competition sponsored by the *Prussian Academy of Sciences*. With his paper *Recherches sur la trajectoire des projectiles dans les milieux résistants* (*research on the trajectory of projectiles in resistant media*), he won the prize offered by the academy, thereby attracting the notice of JOSEPH-LOUIS LAGRANGE, who at the time was still serving in Berlin as director of the *Prussian Academy*.

In 1783, he presented his research on the gravitational attraction of ellipsoidal bodies to the *Académie des Sciences* in Paris, which led, following its enthusiastic appraisal by LAPLACE, to LEGENDRE being named a candidate for membership in the *Académie*. In the following year, he complemented his *Académie* paper with a paper entitled *Recherches sur la figure des planètes*.

In both papers, he introduced a class of polynomials that today are known as the LEGENDRE *polynomials* $P_n$.

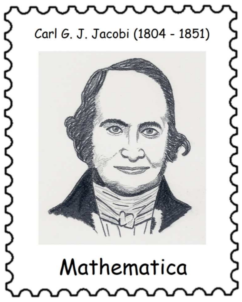
They have the following notable property:

$$\int_{-1}^{1} P_n(x) \cdot P_m(x)\,dx = 0 \text{ for } m \neq n,$$




so that these polynomials can be said to be *mutually orthogonal* with respect to the scalar product defined by the above integral.

In addition, LEGENDRE studied the integrals of functions that can be represented as square roots of polynomials of the third and fourth degree; since such polynomials arise in the determination of the arc length of ellipses, this entire class of functions has come (erroneously) to be called *elliptic functions* (*Mémoire sur les intégrations par d'arcs d'ellipse*, 1786). Over the course of the following decades, LEGENDRE continued to make progress in his research; for example, he was able to reduce such integrals to three normal forms.

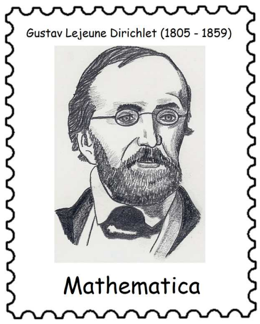



He worked for many years on tables for calculating the approximate values of these integral functions (which are not integrable in terms of elementary functions). As he was working on a final revision of his *Traité des fonctions elliptiques* in the years 1825 to 1828, he must have realized that his work of over thirty years as the unrivalled expert in the field had at last been made superfluous through the work of NIELS HENRIK ABEL and CARL GUSTAV JACOB JACOBI.


In 1785, LEGENDRE published a paper in the field of *number theory*. Among other things, it contained two important results, though his proofs had some gaps and indeed outright errors.

He proposed the following conjecture for arithmetic sequences:

If $a, m \in \mathbb{N}$ are relatively prime, then every arithmetic sequence $(a_n)_{n \in \mathbb{N}}$, with $a_n = a + m \cdot n$, contains infinitely many prime numbers.




A proof of this conjecture was finally given in 1837 by PETER GUSTAV LEJEUNE DIRICHLET, and today, it is referred to as DIRICHLET'*s theorem on primes in arithmetic progressions*, without any reference to the contribution made by LEGENDRE.

Elsewhere in the paper, LEGENDRE dealt with the *conjecture of* LEONHARD EULER known as the *law of quadratic reciprocity*:

If $p$ and $q$ are distinct prime numbers, then the congruences $x^2 \equiv p \bmod q$ and $x^2 \equiv q \bmod p$ are either both soluble or both insoluble, unless $p$ and $q$ both yield a remainder of 3 on division by 4 (that is, in mathematical terminology, both are congruent to 3 modulo 4), in which case exactly one of the two congruences in soluble.

In formulation the theorem, one uses today the notation introduced by LEGENDRE, namely the LEGENDRE *symbol* $\left(\frac{a}{p}\right)$ for an integer $a$ and prime number $p$. One writes $\left(\frac{a}{p}\right) = 1$ if $a$ is a quadratic residue modulo $p$, that is, if the congruence $x^2 \equiv a \bmod p$ is soluble in $\mathbb{Z}$.

The congruence $x^2 \equiv 2 \bmod 7$, for example, has the solutions 3 and 4. Therefore, one writes $\left(\frac{2}{7}\right) = 1$.

On the other hand, if a is not a quadratic residue modulo $p$, that is, if the congruence has no solution, then one expresses this fact by writing $\left(\frac{a}{p}\right) = -1$. For instance, the congruence $x^2 \equiv 2 \bmod 5$ has no solutions. Therefore, one writes $\left(\frac{2}{5}\right) = -1$. If $a$ is a multiple of $p$, then one has by definition $\left(\frac{a}{p}\right) = 0$. With this notation in hand, we may express the law of quadratic reciprocity for odd primes $p$ and $q$ in terms of the LEGENDRE symbol as follows:

$$\left(\frac{p}{q}\right) \cdot \left(\frac{q}{p}\right) = (-1)^{\frac{1}{4}\cdot(p-1)\cdot(q-1)}.$$

In 1787, LEGENDRE was given the assignment, as a member of the *Académie*, to bring the survey measurements of the observatories of Paris and Greenwich into alignment. In recognition of his services, he was inducted into the *Royal Society*. In the same year, he published a paper dealing with the errors and necessary adjustments of trigonometric measurements.

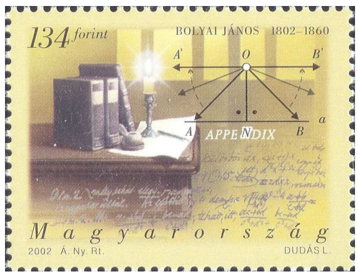
When in 1793, the French *National Convention* introduced the *metre* as the compulsory unit of linear measurement, defined as one ten-millionth of the distance of the *Paris Meridian* (the quarter of the Earth's circumference from the North Pole to the equator running along the meridian passing through the Paris Observatory), this value was based on data with whose coordination he had been tasked.

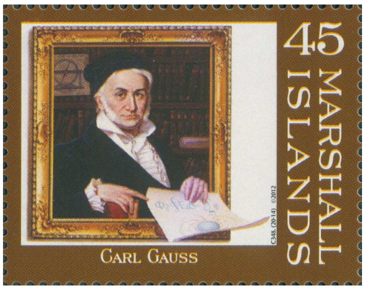
In the following years, LEGENDRE worked in a position of authority on the project of producing logarithmic tables in base 10 and associated tables of trigonometric functions. During the turmoil of the French Revolution, LEGENDRE, who was newly married, lost his entire inherited fortune, and when the *Académie des Sciences* was closed following a decision of the *National Convention*, he found himself temporarily destitute. After the reestablishment of the *Académie des Sciences* in 1795 as the *Institut National des Sciences et Arts*, LEGENDRE became one of the six representatives in the area of mathematics.



CONDORCET encouraged him to reorganize and simplify the *Elements* of EUCLID, and for over 100 years, French schoolchildren were taught geometry from his *Éléments de géométrie*; translations of the book were produced in other countries. In one of the last editions published under his editorship, he included simple proofs that $\pi$ and $\pi^2$ are both irrational numbers; without offering a proof, he conjectured that $\pi$ cannot be the solution of any algebraic equation, that is, that $\pi$ is not only irrational, but transcendental, a fact not proved until 1882, when a proof was given by FERDINAND VON LINDEMANN.


LEGENDRE worked for decades on the problem of the *parallel postulate*. In 1832, the year in which JÀNOS BOLYAI published his paper on non-Euclidean geometry, he stated his conviction that the theorem on the sum of angles in a triangle (an equivalent formulation of the parallel postulate) was a fundamental truth that could not be proved and that represented an example of incontrovertible mathematical truth.






In 1806, LEGENDRE published a book on computing the paths of comets, which he took to be parabolic. He then explained how one could determine the desired parameters of the comet's path through three observations over equal time intervals. In his calculations, he used the *method of least squares* in order to minimise the experimental error. When in 1809, CARL FRIEDRICH GAUSS published his derivation of the method, he claimed priority for himself, despite his knowledge of LEGENDRE's publication. LEGENDRE attempted in vain to enlist other mathematicians to acknowledge the fact that his publication was the earlier to appear.

In 1798, LEGENDRE published a summary of his research on the properties of numbers, *Essai sur la théorie des nombres*, the first book whose title included the term *number theory*. The then 21-year-old GAUSS noted the gaps in LEGENDRE's proof of the *law of quadratic reciprocity*.

When GAUSS himself presented a rigorous proof of the theorem in his 1801 *Disquisitiones Arithmeticae*, he claimed the theorem as his own, without mentioning LEGENDRE and his contribution. Over the course of his life, GAUSS produced an additional five distinct proofs of the theorem, but until his death, LEGENDRE complained of the incredible lack of respect from a man who himself had to his credit so many accomplishments that he had no need of appropriating the discoveries of others as his own.

When in 1808, LEGENDRE reissued his *Théorie des nombres*, he had no problem with publishing GAUSS's proof in full and fully giving him credit. This new edition also contained an estimate of the number $\pi(n)$ of prime numbers less than a given natural number $n$: $\pi(n) \approx \frac{n}{\log(n) - 1.08366}$.

Again, GAUSS claimed that he had found this asymptotic formula before LEGENDRE, and again he refused to accept that both discoveries had occurred in parallel and independently.

LEGENDRE's conjecture that between every two squares of natural numbers $n^2$ and $(n+1)^2$ there is at least one prime has to this day been neither proved nor refuted.

When in 1825, DIRICHLET, an unknown German student, presented to the Académie a proof of Fermat's *last theorem* for the case $n = 5$, LEGENDRE filled in the gap in the presented proof. Showing great friendship, he encouraged the young talent, just as he enthusiastically praised the advances made by ABEL and JACOBI.

LEGENDRE doubtless belongs among the most important French mathematicians, and indeed, one finds his name written in letters of gold among the 72 names of famous Frenchman appearing in a frieze on the EIFFEL *tower*, in Paris.

Yet in the history of mathematics he occupies a sort of tragic subsidiary role. In number theory, many of his contributions were surpassed by the young GAUSS, while in the case of elliptic integrals, his results were made obsolete by the considerably farther-reaching theories of ABEL and JACOBI.

LEGENDRE spent his final years in his house in Auteuil, ill and impoverished, beginning in 1824, when as a consequence of his refusal to vote for the government's candidate at the Institut National, he was deprived of his pension.

However his situation improved somewhat on the change in government in 1828, and in 1831, LEGENDRE was made an officer of the *Légion d'Honneur*.

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https://www.spektrum.de/wissen/adrien-marie-legendre-grossmeister-im-schatten-noch-groesserer-genies/1210734

Translated by David Kramer

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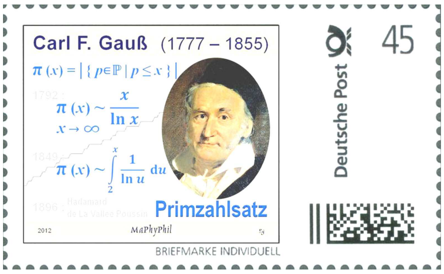
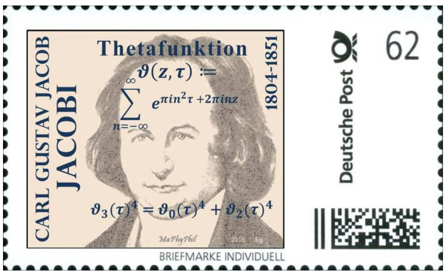
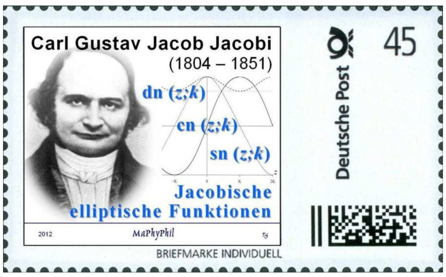
Here an important hint for philatelists who also like individual (not officially issued) stamps:



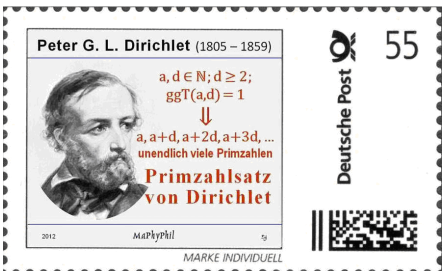










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