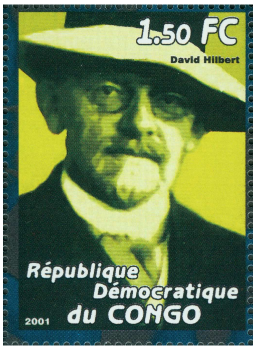
Hermann Minkowski (June 22, 1864 – January 12, 1909)

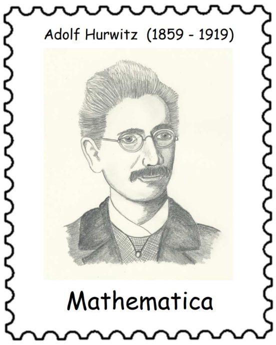
by Heinz Klaus Strick, Germany

"And EINSTEIN? He was always cutting my lectures – I would never have believed him capable of this," is what one of his former mathematics professors is supposed to have said on the occasion of the publication of EINSTEIN's *On the Electrodynamics of Moving Bodies*, which today is called the *Special theory of relativity*. In fact, the student ALBERT EINSTEIN was never much interested in abstract mathematics as it was taught at the Zurich Polytechnic (today ETH Zurich), something that he came later to regret. The professor of mathematics quoted above was none other than HERMANN MINKOWSKI, who was professor of mathematics in Zurich from 1896 to 1902.

HERMANN MINKOWSKI came from a Jewish merchant family in the Kaunas guberniya (then part of the Russian Empire, today in Lithuania). The family moved in 1872 to Königsberg, where the boy attended the Altstadt Gymnasium, skipping several grades and receiving his certificate of eligibility to enter university (Reifezeugnis) in March 1880, when he was only 16 years old.

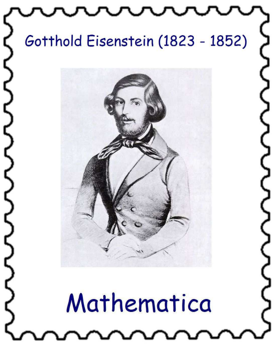
While at university, he became acquainted with fellow student DAVID HILBERT, and a lifelong friendship developed among the two students and the 25-year-old mathematics professor ADOLF HURWITZ, who had taken a position in Königsberg.

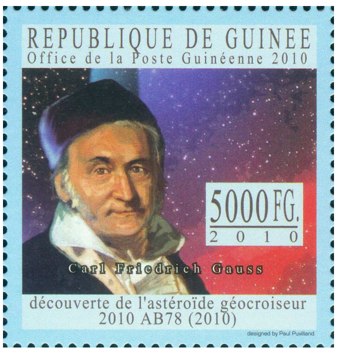




When in 1881, the French *Académie des Sciences* posed a problem on the number of ways in which an integer can be represented as the sum of five squares, the 17-year-old student HERMANN MINKOWSKI thereupon steeped himself so deeply in the proof of a conjecture of FERDINAND GOTTHOLD MAX EISENSTEIN (1823–1852) that he ended up developing a general theory that went far beyond the proposed topic. Although his work did not formally comply with the conditions of the competition (it was written in German), MINKOWSKI was awarded the prize by the *Académie*.

In his *Disquisitiones Arithmeticae*, CARL FRIEDRICH GAUSS had developed a comprehensive theory of binary quadratic forms $f(x,y)$ with $f(x,y) = ax^2 + bxy + cy^2$ ($a, b, c \in \mathbb{Z}$).





Minkowski generalized Gauss's work to the case of quadratic forms of more than two variables, and in 1885, he received his doctorate with a dissertation on this topic. In 1887, he was appointed *privatdozent* at the University of Bonn with a habilitation thesis on *Spatial Visualization and the Minima of Positive Definite Quadratic Forms*. In 1892, he was promoted to associate professor. There followed two years at the University of Königsberg (succeeding Hilbert) before he was finally called in 1896 to the Eidgenössische Polytechnic in Zurich, where his friend Hurwitz had been teaching since 1892.

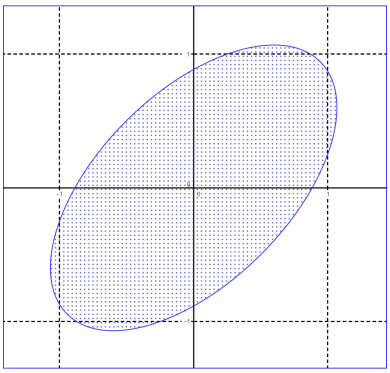
It was during his time in Zurich that his work on the geometry of numbers was published. It was Minkowski's contribution to a *Report on the Latest Developments in Number Theory* that he and Hilbert were putting together under the auspices of the *German Mathematical Society* (DMV). It was in this work that Minkowski developed the idea of an *n*-dimensional lattice. Such a lattice in $\mathbb{R}^n$ is generated by a set of basis vectors (for example, by the standard unit vectors). The set of integral linear combinations of these vectors determines the points of a lattice, and the *n*-dimensional parallelotope formed from neighbouring points forms a fundamental domain *M*.

Minkowski's lattice-point theorem:

A bounded convex subset T of $\mathbb{R}^n$ containing the origin whose volume is greater than 2^n times the volume of M must contain at least one point of the lattice in addition to the origin.

*) a set T is convex if $P, Q \in T$ implies that every point on the line PQ is also in T.

For example, if one considers the integral lattice of the usual rectangular Cartesian coordinate system, then every square of side length 1 formed by lattice points determines a fundamental domain of the lattice, with "volume" 1. For every subset T with the properties given above, we have then the following statement: If the area of T is greater than $2^2 \cdot 1 = 4$, then at least one lattice point in addition to the origin belongs to T. This implies, for example, that every ellipse that is symmetric about the origin (the equation of such an ellipse is $ax^2 + bxy + cy^2 = d$) and contains no lattice point other than the origin can have volume at most 4.

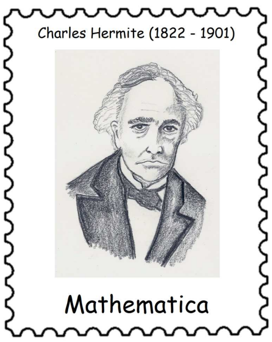


This theorem, which appears to have only geometrical significance, has astounding applications in number theory.

From the conditions on the coefficients of a quadratic term follows the existence of lattice points (whose coordinates are integer solutions), for example in the following theorems discovered by Leonhard Euler:

- Every prime number of the form $6k+1$ is the sum of a square and three times another square, and every prime number of the form $8k+1$ is the sum of a square and twice another square.

Minkowski's work *The Geometry of Numbers* contains a host of additional facets that have consequences, for example, in the theory of algebraic number fields, in the approximation of algebraic numbers by continued fractions, and in problems of optimal sphere-packing.



Charles Hermite was so enthralled by the book that he is said to have exclaimed, "*I believe that I have seen the promised land*", and he commissioned a translation of it so that he would be sure to understand every detail.

(drawings © Andreas Strick)

In 1902, MINKOWSKI moved to Göttingen, to take a chair that had been created especially for him. There he continued his fruitful collaboration with HILBERT. In the meantime, he developed a particular interest in problems of theoretical physics and wrote, among other works, an encyclopaedia entry on mathematical modelling of capillary action. In 1905, HILBERT organised a seminar to examine the consequences of the 1897 discovery of the electron by JOSEPH J. THOMSON and the related theories developed by HENDRIK ANTOON LORENTZ and HENRI POINCARÉ.

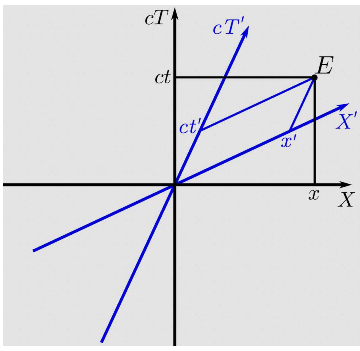
With the help of LORENTZ transformations, it is possible to bring into alignment the location in space and time of various observers. However, Lorentz 's theory presupposed the existence of an ether. In his theory of relativity, Einstein began with the constancy of the speed of light, which enabled him to explain all the relevant phenomena.

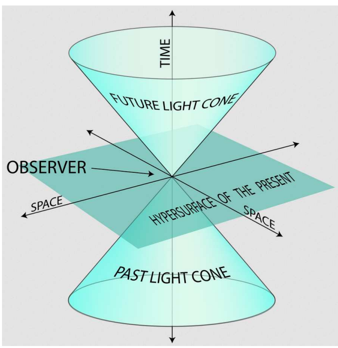
But it was MINKOWSKI who succeeded in making it possible to visualise EINSTEIN's theory. He began his address at the eightieth meeting of German natural scientists and physicians, in 1908, with the following words:

The ideas about space and time that I would like to develop here for you were grown in the soil of experimental physics. Therein lies their strength. The direction in which they lead is a radical one. From this time forth, space as space and time as time will sink into the shadows, and only a sort of union of the two will retain any independence.

He then described his geometrical model of a non-Euclidean four-dimensional space-time, in which points with the coordinates (t; x; y; z) are events (time point t and space coordinates x, y, z), while sequences of points are world lines.

The metric in this vector space is determined by the following scalar product of two event vectors: $(ct_1)(ct_2)+x_1x_2+y_1y_2+z_1z_2$, where c is the speed of light.





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For the sake of visualisation, he developed a diagram in which the space coordinates are represented by a single axis (left-hand diagram). With the help of this simplification, it is possible to understand seemingly paradoxical phenomena such as time dilation and length contraction. With the help of a three-dimensional representation of the light double cone (right-hand figure), he clarified what past and future mean from the point of view of an observer.

EINSTEIN was at first not particularly enthusiastic about MINKOWSKI's mathematical model – he said: *Since the mathematicians have taken over the theory of relativity, I myself no longer understand it*.

But he later conceded that without such an approach, the general theory of relativity would perhaps never have gotten itself out of nappies.

When MINKOWSKI died at the young age of 44 following a ruptured appendix, HILBERT gave a memorial eulogy that was full of gratitude for the friend who in the year 1900 had suggested to him the idea of offering in his address to the International Congress of Mathematicians a look into the future:

- *What are the problems that future mathematicians will be grappling with?*

– an address that to this day continues to play a role in the directions of mathematical research.

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https://www.spektrum.de/wissen/hermann-minkowski-1864-1909/1285967

Translated by David Kramer

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