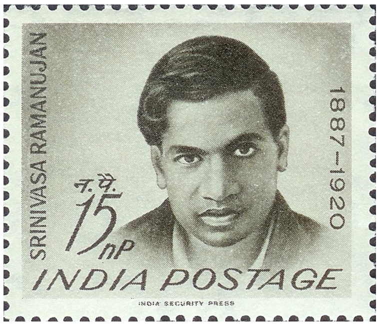
Srinivasa Ramanujan (December 22, 1887–April 26, 1920)

by Heinz Klaus Strick, Germany



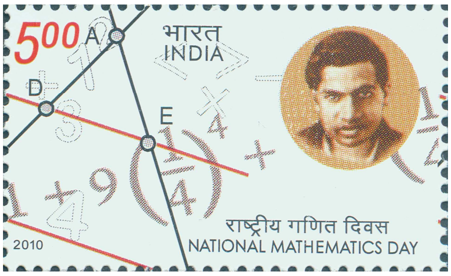
Dear Sir,

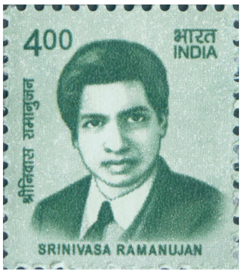
I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age. I have had no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as “startling”…

I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for the trouble I give you.

I remain, Dear Sir, Yours truly,

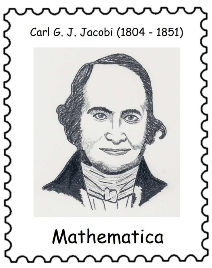
S. Ramanujan

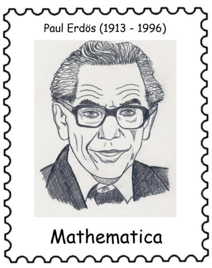




This letter was addressed to the famous mathematician Godfrey Harold Hardy (1877–1947), professor at *Trinity College* of the University of Cambridge, where the great Isaac Newton had taught. When Ramanujan’s letter arrived, containing some remarkable-looking formulas, Hardy consulted his colleague John Endensor Littlewood (1885–1977). “Some,” Hardy later wrote in his reminiscences, “I could prove (after harder work than I had expected) while others fairly blew me away. I had never seen the like! Only a mathematician of the highest class could have written them. They had to be true, for if they were not, no one would have the imagination to invent them.”

And in an interview with Paul Erdos, when asked what he believed to be his most significant contribution to mathematics, he replied without hesitation, “the discovery of Ramanujan.” At another time, he said, “He can be compared only with Euler or Jacobi.”





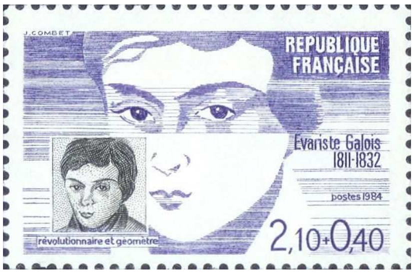
(Drawings: Andreas Strick ©)

Srinivasa Ramanujan grew up in southern India, about 240 km south of Madras. His father worked as a clerk in a sari shop, leaving early in the morning for work and returning late in the evening. His mother was responsible for raising the boy, giving him the religious education of a Brahman.

After attending primary school, he transferred to a secondary school, where he excelled in all subjects and received numerous school prizes and awards. In 1903, at the age of 16, he became acquainted with a book that had been published in 1886, *A Synopsis of Elementary Results in Pure and Applied Mathematics*, by G. S. Carr. This book had a decisive influence on his life, especially in how he approached mathematics. The book was a compendium of several thousand mathematical formulas and theorems, most given with neither motivation nor proof.

To Ramanujan, this appeared to be the way that mathematics was done: formulas and theorems were "discovered" (for example on the basis of numerical examples), and formal derivation was superfluous.

Ramanujan showed an exceptional interest in mathematical problems at a very early age. As a twelve-year-old, he became interested in questions about geometric and arithmetic series; at the age of 15, he learned about the solution method for cubic equations and discovered on his own the method for solving equations of degree 4. He searched in vain for a corresponding solution method for equations of the fifth degree, unaware that eighty years earlier, Galois and Abel had proved that there can exist no such method.



His work at school continued to be first rate, and he received a scholarship to attend Government College, in Kumbakonam. It was through such colleges that the British colonial regime sought to prepare native students for positions in the colonial administration. Therefore, the curriculum was a general one, and there was no possibility of specializing in a single subject. Ramanujan, who on account of his intensive involvement with mathematics had lost interest in all other subjects, suffered under this stricture. He failed some of his examinations and thereby lost his scholarship. Moreover, his attempt to graduate from a school in Madras that would allow him to attend university was unsuccessful.

In the meantime, Ramanujan was compiling a notebook (he would eventually fill four such books) in which he recorded his discoveries, arranged by topic. One of his first entries was the equation

$$\sqrt{1+2\cdot\sqrt{1+3\cdot\sqrt{1+\ldots}}}=3$$

The equations that he wrote down had no meaning in and of themselves, as he later said, unless they perhaps expressed an idea of God's. Likewise, continued fractions, that is, fractions whose denominators contain fractions whose denominators contain fractions whose denominators … exerted on him a particular fascination. For example, he discovered the relationships

$$\cfrac{1}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cfrac{e^{-8\pi}}{\ldots}}}}} = \left(\sqrt{\frac{5+\sqrt{5}}{2}} - \frac{\sqrt{5}-1}{2}\right)\cdot e^{\frac{2\pi}{5}} \quad \text{and}$$

$$1+\frac{1}{1\cdot 3}+\frac{1}{1\cdot 3\cdot 5}+\frac{1}{1\cdot 3\cdot 5\cdot 7}+\ldots+\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{1+\ldots}}}}} = \sqrt{\frac{\pi\cdot e}{2}}$$

In 1911, his first scholarly article, “Some Properties of Bernoulli’s Numbers,” appeared in the Journal of the *Indian Mathematical Society*. (These are numbers discovered by Jacob Bernoulli in his series development of the tangent function.) Ramanujan’s reputation as a genius began to spread among the mathematicians in and around Madras. In vain, the founder of the *Indian Mathematical Society* (IMS), Ramachandra Rao, sought to obtain a stipend for him. And letters of recommendation by Indian mathematicians to professors at English universities also failed to achieve their objective. The impoverished Ramanujan lived on the charity of his friends. When he became ill, a necessary operation was performed only because the surgeon agreed to perform it without remuneration.

In an effort to support himself financially, he applied for a position as bookkeeper at the Madras harbour authority. Thanks to support from members of the IMS, he received the appointment.

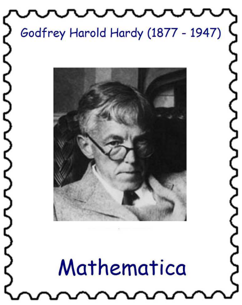
Ramanujan’s dissatisfaction with his circumstances grew, and he finally took the initiative himself and wrote to three professors of mathematics at English universities; only Godfrey Harold Hardy replied. (“I am already a half-starving man,” he later wrote to Hardy. “To preserve my brains I want food, and this is my first consideration.”) Among the approximately one hundred equations that Ramanujan included in his letter can be found the following:

$$1-5\cdot\left(\frac{1}{2}\right)^3+9\cdot\left(\frac{1\cdot 3}{2\cdot 4}\right)^3-13\cdot\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^3+\ldots=\frac{2}{\pi} \quad \text{and}$$

$$1+9\cdot\left(\frac{1}{4}\right)^4+17\cdot\left(\frac{1\cdot 5}{4\cdot 8}\right)^4+25\cdot\left(\frac{1\cdot 5\cdot 9}{4\cdot 8\cdot 12}\right)^4+\ldots=\frac{\sqrt{8}}{\sqrt{\pi}\cdot\Gamma^2\left(\frac{3}{4}\right)}$$

in which appears the gamma function discovered in 1730 by Leonhard Euler, defined by

$$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}\,dt\,.$$



HARDY succeeded in obtaining for RAMANUJAN a stipend for a two-year residence at *Trinity College*, Cambridge; but RAMANUJAN had grave doubts about leaving his native country. Among other things, he feared that as an orthodox Brahman, he would lose membership in his caste. It was only under the influence of Indian mathematics professors that he finally began the four-week sea voyage, reaching his destination in April 1914, only a few months before the outbreak of the First World War.

With great care, HARDY and LITTLEWOOD taught RAMANUJAN the fundamentals of "modern" mathematics so that the brilliant ideas of the Indian mathematician could be expressed in a way that met current scientific standards. Numerous papers appeared in research journals, primarily joint work with HARDY. Seven of these papers, on highly composite numbers, formed the basis of RAMANUJAN's dissertation in 1916.

Natural numbers that are not prime are called composite numbers. If one considers the composite numbers that have more factors than any smaller natural number, then one obtains the following sequence: 4 (three divisors), 6 (four divisors), 12 (six divisors), 24 (eight divisors), 36 (nine divisors), 48 (ten divisors), 60 (twelve divisors), ….

RAMANUJAN discovered that in all of these highly composite numbers, if one writes them as a product of prime powers, the sequence of exponents of the prime numbers $2, 3, 5, \ldots$ for each such number is monotone decreasing (nonincreasing):

$6 = 2^1 \cdot 3^1$ (the sequence of exponents is $(1 \geq 1)$); $12 = 2^2 \cdot 3^1$ $(2 \geq 1)$; $24 = 2^3 \cdot 3^1$ $(3 \geq 1)$;

$36 = 2^2 \cdot 3^2$ $(2 \geq 2)$; $48 = 2^4 \cdot 3^1$ $(4 \geq 1)$, $60 = 2^2 \cdot 3^1 \cdot 5^1$ $(2 \geq 1 \geq 1)$, ….

One of the classical problems of *number theory* was to determine the number of ways that a natural number n can be decomposed as a sum of positive integers (the partition into a "sum" of one number, that is the number n itself, is also considered such a decomposition).

For example, the numbers $p(n)$ of such partitions for $n = 3, 4, 5$ are given by

$p(3) = 3$, since $3 = 2+1 = 1+1+1$;

$p(4) = 5$, since $4 = 3+1 = 2+2 = 2+1+1 = 1+1+1+1$;

$p(5) = 7$, since $5 = 4+1 = 3+2 = 3+1+1 = 2+2+1 = 2+1+1+1 = 1+1+1+1+1$.

The number $p(n)$ increases rapidly with n. For example, $p(50) = 204226$ und $p(100) = 190569292$.
LEONHARD EULER discovered a recurrence formula for the calculation of $p(n)$.

RAMANUJAN and HARDY discovered the asymptotic formula $p(n) \approx \dfrac{1}{4n\sqrt{3}} e^{\pi\sqrt{2n/3}}$.

RAMANUJAN was also interested in prime numbers ($p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$, …), and he discovered "simple" relationships for certain infinite products:

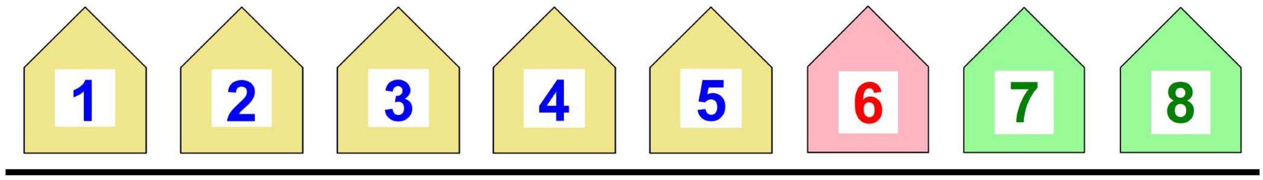
$$\prod_{k=1}^{\infty} \frac{p_k^2+1}{p_k^2-1} = \frac{5}{3} \cdot \frac{10}{8} \cdot \frac{26}{24} \cdot \ldots = \frac{5}{2} \quad \text{and} \quad \prod_{k=1}^{\infty} (1+\frac{1}{p_k^2}) = (1+\frac{1}{4}) \cdot (1+\frac{1}{9}) \cdot (1+\frac{1}{25}) \cdot \ldots = \frac{15}{\pi^2}.$$

RAMANUJAN's discoveries in the area of number theory show that he possessed a certain "insight" into the nature of numbers that was closed to others.

Two episodes in his life might serve to elucidate this assertion. Once, HARDY visited RAMANUJAN in hospital and observed, just to make conversation, that the taxi in which he had arrived had the number 1729, which was surely a number without any particular interest.

Ramanujan immediately countered that 1729 is a very interesting number indeed. In fact, 1729 is the smallest natural number that can be expressed in two different ways as the sum of two positive cubes: $1729 = 12^3 + 1^3 = 10^3 + 9^3$.

Another time, Ramanujan is said to have solved the following puzzle instantaneously: The consecutively numbered houses on the road of a one-road village all lie on one side of the road. Someone in the village lives in a house with a house number such that the sum of the house numbers on one side of the house is the same as the sum of the house numbers on the other side. How many houses are in the village? What is the number of the house in question? (Ramanujan solved the problem with the aid of continued fractions.)



Having been born and raised in a tropical climate, Ramanujan could not adapt to the cold and rainy climate of England. He regularly became ill during the winter months and was then incapable of scientific work. He also had great difficulties in adhering to the severe prescriptions of his religion, which required a vegetarian diet. During the war years, it was impossible to return home, and once, in a fit of depression, he attempted to throw himself in front of a subway train. In 1917, Ramanujan became seriously ill. He saw this as part of his predetermined fate, and it was only after he had received a number of honours that the will to live again grew strong within him, and he resumed his scientific labours. In 1918, he was made a member of the *Cambridge Philosophical Society* as well as a fellow of *Trinity College*. A few weeks later, he became a member of the *Royal Society* of London.

Following the end of the war, Ramanujan returned to his homeland, but he died a half year later, possibly from tuberculosis, but it may also have been from dysentery, for there was an epidemic of the disease in Madras at the time.

For a long time, it was thought that some of Ramanujan's papers had disappeared. But they were discovered in 1976 in a box of effects stored in the library at *Trinity College*, Cambridge. This "lost notebook" consists of 87 loose and unordered sheets of paper, comprising more than 600 formulas, without proof.

In a series of four volumes published between 2005 and 2012, George Andrews and Bruce Berndt have provided statements, proofs, and discussions of all the claims made by Ramanujan in his lost notebook and all his other manuscripts and letters published with the lost notebook.

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www.spektrum.de/wissen/srinivasa-ramanujan-1887-1920/1030269

Translated by David Kramer

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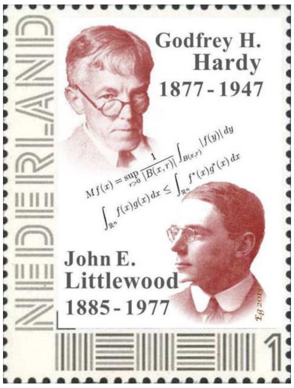

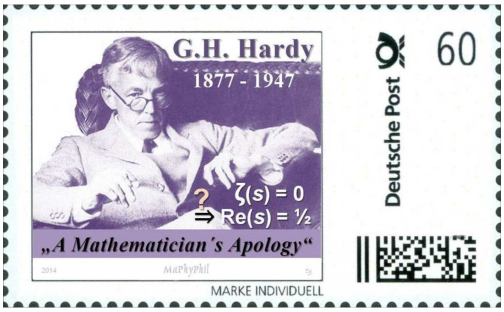

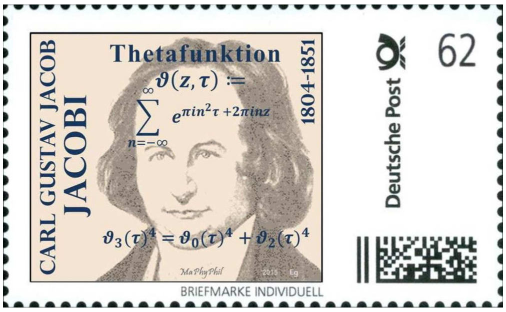

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