## Thales Of Miletus (624-547 BC)

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Little is known about Thales of Miletus. What is known about him is that he came from a wealthy family from Miletus (Asia Minor, today Turkey) and was active as a philosopher, mathematician, astronomer, engineer and politician. Miletus was one of the leading city-states on the Aegean Sea at the time.


During his travels in the Mediterranean region, he acquired extensive astronomical knowledge, with the help of which he predicted a solar eclipse in 585 BC, which increased his reputation as a "wise man". Incidentally, the solar eclipse ended a war between Medes and Lydians, who still saw the wrath of the gods in the natural event.

As a philosopher, Thales of Miletus was important above all because he endeavoured to explain the world rationally, i.e. with the help of natural causes, rather than through myths. Even if, for example, his explanation of the regular flooding of the Nile turned out to be wrong ("winds from the Mediterranean dam up the Nile water"), unlike the Egyptians, he did not assume divine intervention, but sought a natural explanation.

For him, water was the origin of all (natural) things. He held the view that the earth, as a flat disc, floats on water like a ship and that the natural phenomenon of earthquakes could thus be explained (i.e. not caused by the god Poseidon). Thales recognised that solar eclipses were caused by the moon "stepping in front of the sun" and he made the claim that the moon is illuminated by the sun. He assumed that the stars were made of glowing earth.


Aristoteles reported that Thales had become rich because of his (natural) scientific knowledge. One year he had foreseen a good oil harvest, so he rented all the oil presses in Miletus and on the island of Chios in the winter and then rented them out at higher prices at harvest time.

Thales of Miletus was certainly not the discoverer of the mathematical theorem named after him (Theorem of THALES). The statement of the theorem was already known to the Egyptians and Babylonians and was applied by them in practice.

However, a new epoch of mathematics is connected with the person of Thales: Like other mathematicians before him, Thales also gave practical hints for the calculation of geometric quantities, but he was probably the first to try to give reasons for the methods. With him began a development of Greek mathematics that broke away from concrete measurements and led to abstract, idealised geometric objects (such as points, straight lines, circles, triangles, angles). The logical conclusions used must be correct independently of a concrete situation, i.e. they must also be valid independently of the drawings made and the concretely chosen angle sizes and side lengths there.

Thales formulated some theorems on geometry that seem "elementary", but which describe basic geometric insights:
(1) The diameter bisects the circle.
(2) Opposing angles of two intersecting straight lines are equal (vertex angle theorem).
(3) The sum of the interior angles in the triangle is $180^{\circ}$.
(4) The base angles in an isosceles triangle are equal.
(5) A triangle is defined by one side and the two adjacent angles.
(6) The peripheral angle in a semicircle is a right angle (Thales theorem).

Proclus, in the 5th century AD, i.e. 1000 years after Thales, reproduced his idea for the proof of theorem (1) in the following words:


Think of the diameter drawn and one half of the circle placed on the other. If it is not equal, it will come to lie either inside or outside. In both cases the conclusion will be that the shorter straight line is equal to the longer; for all the lines from the centre to the lines from the centre to the circular line are equal to each other. But this is impossible.

This is one of the first indirect proofs in the history of mathematics!
Theorem (2) is proved by EucLID as follows: Since

$$
\alpha_{1}+\alpha_{2}=180^{\circ} \text { and } \alpha_{2}+\alpha_{3}=180^{\circ} \text { thus } \alpha_{1}+\alpha_{2}=\alpha_{2}+\alpha_{3} \text {, i.e. } \alpha_{1}=\alpha_{3} .
$$

Theorem (6) also holds more comprehensively:


On the one hand, a right angle always arises at the circle line if one strikes a semicircle over a line, but on the other hand, the converse of the theorem also applies, which states that the centre of the circumcircle of a right triangle is also the centre of the hypotenuse of this triangle - or in other words: The geometric location of all points from which one sees a given line at a right angle is the (semi) circle over this line.
The proof of (6) uses the theorems (3) and (4). Namely, since:

$$
180^{\circ}=\alpha_{1}+\alpha_{4}+\left(\alpha_{3}+\alpha_{2}\right)=\alpha_{2}+\alpha_{3}+\left(\alpha_{3}+\alpha_{2}\right)=2 \cdot\left(\alpha_{2}+\alpha_{3}\right), \text { so it follows: } \alpha_{2}+\alpha_{3}=90^{\circ} .
$$

The proof of the converse can be done "dynamically" (see figure on the right): Consider the consequences regarding the sum $\alpha_{2}+\alpha_{3}$ if the point $C$ is not on the circle line, i.e. the triangles $A M C$ and $M B C$ are not isosceles.


The Theorem of Thales is a special case of a more general mathematical theorem:

The so-called Subtended Angle Theorem (circumferential angle theorem) states that all peripheral angles over any chord are equal.

The proof of the theorem is done by showing that each peripheral angle is half as large as the (one) central angle at the centre of the
 circle.

It is reported that Thales used geometric methods to determine the height of the pyramids in Egypt. He waited until the length of his own shadow was as long as the length of his own body (i.e. the sun's rays hit at an angle of $45^{\circ}$ ); he then applied this knowledge to the isosceles rightangled triangle on the pyramid.
According to another source, he is said to have stuck a rod vertically into the earth at the point where the shadow image of the pyramid's tip could be seen. From the ratio of the length of the shadow of the rod and the length of the rod and the length of the shadow of the pyramid, he was able to deduce the height of the pyramid (according to the ray theorem).


Thales is also said to have used various methods to determine the distance of inaccessible objects, e.g. the distance of a ship on the sea from a tower.

To do this, one pointed a sighting stick fixed to a vertical pole at the ship and then turned the pole around until one has a prominent object in sight on land. This then has the same distance from the tower as the ship (the tower is thus used as an axis of symmetry).


First published 2007 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg https://www.spektrum.de/wissen/thales-von-milet-624-547-v-chr/860557

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