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Eratosthenes of Cyrene

Born Cyrene (near Darnah, Libya), circa 274 BCE

Died Alexandria, (Egypt), circa 194 BCE

Eratosthenes, Greek scholar, scientist, and mathematician, is chiefly remembered for devising and performing the first measurement of the circumference of the Earth, and for inventing the algorithm known as the sieve of Eratosthenes

According to the Suda, Eratosthenes was born in the 126th Olympiad (276/273 BCE), but this is difficult to reconcile with Strabo's assertion that he studied in Athens with Zeno the Stoic, who died in 262/261 BCE. Around 246 BCE, Eratosthenes moved to Alexandria

where he succeeded Apollonius as chief librarian. We are told he lived to be 80. According to the Suda, the next chief librarian, Aristophanes of Byzantium, was also his pupil

There are very few remains of Eratosthenes' epic poem *Hermes* and of his elegy *Erigone*. His 12 books on ancient Attic comedy have been lost. The extant book *Katasterismos* (Star Arrangements), which explains the mythological origin of the names of the constellations, is presumably an ancient abridgment of the work he wrote on the subject. According to R. Pfeiffer, Eratosthenes was the founder of critical chronology. In his lost *Chronographi* (Chronographies), he gave a full chronological survey of Greek history from the fall of Troy to the death of Alexander, based on the lists of Spartan kings and of Olympian victors. His precise reconstruction of the latter list, *Olumpionikai* (Olympian Victories), is also lost.

Eratosthenes' contributions to mathematics included research on the duplication of the cube and the famous sieve. The "sieve of Eratosthenes" was, until the recent invention of advanced computer programs, the only algorithm available for finding prime numbers To find all primes smaller than a given integer N , we write down the first N positive integers in order. We then begin a sequence of operations, in each of which we cross out one or more integers without deleting them. In the first operation, we cross out 1, which is not a prime. The first uncrossed integer is then the first prime, namely two; we leave it untouched and cross out every second integer from then on. After the second operation, the first uncrossed integer is the second prime, namely three; we leave it untouched and cross out every third integer from then on. (Some integers, like six and 12, will then be crossed out more than once.) And so on... After the n th operation, the first uncrossed integer is the n th prime, which we denote by $p(n)$. We leave it untouched and cross out every $p(n)$ th integer from then on. The procedure stops as soon as the first uncrossed integer is greater than the square root of N (e.g., after the 12th operation, if $N = 1,000$). At that stage, every uncrossed item in the list is a prime number $\leq N$.

Eratosthenes' method for measuring the circumference of the Earth is reported by Cleomedes. It rests on two idealizing assumptions: (1) The Earth is a perfect sphere and (2) the Sun is so far away that light coming from it reaches the surface of the Earth along parallel lines.

Moreover, Eratosthenes incorrectly assumed (3) that Alexandria and Syene (today's Aswan) lie on the same meridian. On the summer solstice, a pole planted vertically on the ground at Syene throws no shadow at noon. At Alexandria, at that same noon, a pole of the same height h similarly planted on the ground casts a shadow of length l . From the ratio $h:l$, Eratosthenes could figure out the size of the angle a made by the vertical pole and the direction from which solar light fell on it in Alexandria. By assumptions (2) and (3), this direction is parallel to the direction of the solar light falling on Syene at that moment; hence, by assumption (1), angle a is equal to the difference in latitude between Syene and Alexandria.

If Syene and Alexandria both lie on the same great circle of a sphere of circumference equal to K , if A is the length of arc between them, and if the angle a subtended by this arc is expressed in degrees, then, evidently,

$$K = 360\Delta/a.$$

According to Cleomedes, Eratosthenes's calculations yielded $K = 250,000$ stadia. The accuracy of Eratosthenes's estimate depends, of course, on the actual length of one stadion. In classical Greece, it measured exactly 600 ft. The length of a foot varied from one city-state to another, but not by much, and Tannery suggests one stadion = 185 ± 5 m. Then, $K = 46,250$ km, a fair estimate of the Earth's circumference. However, Pliny says that Eratosthenes counted 40 stadia per schoenus, an Egyptian unit that we know was equal to 630 km. Using this equivalence, we get $K = 39,375$ km, a figure eerily close to the actual length of a terrestrial meridian (39,942 km).

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