

Biographical Encyclopedia of Astronomers

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Lagrange, Joseph Louis

Born Turin, (Italy), 25 January 1736

Died Paris, France, 10 April 1813

Joseph Lagrange made numerous significant contributions to mathematics and to celestial mechanics (Lagrangian points), especially through his invention of the calculus of variations. A particular combination of the variables in a dynamical system is called the Lagrangian. He was the son of Giuseppe Francesco Lodovico Lagrangia and Teresa Grosso. Lagrange initially found mathematics uninteresting but became attracted to it due to the works of Edmond Halley. He lived in Turin until 1766, when he moved to Berlin to work at the Academy of Sciences. He stayed there until 1787, when he moved to Paris, where he remained until his death

Lagrange's mathematical career began in 1754 with a paper submitted to Leonhard Euler at the Berlin Academy on the calculus of variations. Euler passed copies of Lagrange's work to Pierre de Maupertuis, who offered Lagrange a position at the Academy of Sciences at Berlin. Lagrange declined, though he was made an associate foreign member. Meanwhile, with others, Lagrange co-founded the Royal Academy of Sciences in Turin in 1757

Lagrange applied his considerable talents to many of the unsolved problems in celestial mechanics. For the prize question of 1764, the Paris Academy of Sciences asked for an explanation of why the Moon's rotational and orbital periods coincide, thereby always showing the same face to us. The prize committee awarded the prize to Lagrange, the first of five times he would win a prize competition, a record surpassed only by Euler. Lagrange's winning solution included the first complete mathematical account of the phenomenon of libration.

The 1766 prize question concerned the explanation of the various inequalities in the motions of the four Galilean satellites of Jupiter. If two bodies were alone in the Universe, they would orbit each other along an unchanging elliptical orbit, but the real case is more complex; the gravitational perturbations of the Sun, planets, and satellites mean that orbital parameters constantly vary under changing gravitational influences. These variations or inequalities fall into two categories: periodic, which forever oscillate about a mean value, and secular, which increase without bound. Lagrange's submission won the prize, establishing him as one of the foremost mathematicians in Europe. Shortly afterward, Jean d'Alembert offered Euler's position at Berlin to Lagrange when Euler left for the Russian Academy of Sciences. Thus, in October 1766, Lagrange arrived in Berlin.

Jupiter and its satellites are an example of an n -body problem. The general method of approach is to solve a corresponding two-body problem and then take into account the perturbations caused by the addition of bodies. The motion of the Moon is even more difficult, for the Sun's perturbing effect on the Moon's motion about the Earth is far greater than any other perturbations in the Solar System. In 1770, the Academy, dissatisfied with existing tables of the Moon's position, offered a prize for a new theory of the Moon and thus a new method of

creating more accurate tables of the Moon's position, which in turn could be used to solve the longitude problem. None of the entries received by the Academy were deemed satisfactory, so in 1772 the Academy posed the question again, this time with a double prize of 5,000 florins. Both Lagrange and Euler submitted winning papers and shared the double prize

In the first section of this paper, Lagrange made one of his most celebrated contributions to celestial mechanics. He found five points in a two-body system where a third body, assumed not to gravitationally influence the other two, could be placed so as to remain in a stable position relative to the other two. These five points are known today as the Lagrangian points. Closely related is the inner Lagrangian Surface, for instance, in a binary star system, on which material can flow freely from one star to the other (also called a Roche lobe).

Despite their success, neither Euler nor Lagrange could explain a phenomenon noted by Halley. Observations from the time of Ptolemy could not be reconciled with contemporary observations unless one assumed that the Moon's mean motion was undergoing a steady acceleration. In March 1772, d'Alembert suggested to Lagrange that the lunar acceleration might be the subject of the next prize question. Lagrange's analysis showed that the perturbations caused by the planets were periodic in nature; hence, any variation they caused was also periodic, so they could not produce a secular variation. He concluded that because the secular acceleration could not be accounted for mathematically, it had to be an observational error. Lagrange won the prize, though it should be pointed out that he was not entirely correct. A later analysis by Pierre de Laplace showed that very slow (but periodic) changes in the eccentricity of the Earth's orbit were the source of very long-period (hence, apparently secular) variations in the Moon's mean motion.

The next prize question posed by the academy, for 1776, concerned the perturbation of the orbits of the comets. Lagrange began to compose an entry, but shortly afterward withdrew from the contest. The academy found none of the entries worthy of an award, so once again, they repeated the question with a double prize in 1780. This time, Lagrange won the double prize himself.

When Lagrange withdrew from the 1776 contest, he found himself involved in a far more important question: an examination of the orbits of the planets. An elliptical orbit has five main parameters: the inclination of the orbit to the plane of the ecliptic, the eccentricity, the semimajor axis, the position of the nodes, and the position of the aphelion. Gravitational perturbations vary these five parameters. Of the five, two (the position of the nodes and the aphelion positions) could be varied any amount without seriously affecting the constitution of the Solar System, but this was not true of the eccentricity, inclination, or semimajor axis; if these varied too greatly, the orderly march of the planets around the Sun would be disrupted. Hence, the question arose of the long-term stability of the Solar System

In 1774, Lagrange had shown that the inclinations of planetary orbits undergo only periodic variations. Lagrange's paper, sent to the Paris Academy, was given to Laplace to referee. Laplace applied Lagrange's method and showed that the eccentricities likewise underwent only periodic variations. Moreover, Laplace managed to rush his own work into print *before*

Lagrange's (though giving credit to Lagrange for having developed the method he used). Not surprisingly, Lagrange submitted no more papers to the Academy.

Lagrange's crowning result thus appeared in the 1776 *Memoirs* for the Berlin Academy of Sciences. Lagrange showed that the semi-major axes of the planetary orbits, like the inclinations and eccentricities, underwent only periodic variations, answering positively the question of the stability of the Solar System

By 1787, Lagrange's position at Berlin was more of a burden than a joy. He had married his cousin, Vittori Conti, in September 1767, but they had no children. She died in 1783 after some years of poor health. Lagrange's health, too, was far from perfect. Frederick the Great died in 1786, and after his death, the prestige and position of Prussia began a long decline. Lagrange accepted an offer from the Paris Academy to become a member, particularly since the offer specifically excluded teaching as one of his responsibilities. Lagrange left Berlin on 18 May 1787. In 1792, Lagrange married again, this time to Renée-François-Adélaïde le Monnier (daughter of Pierre le Monnier). The marriage contract was signed on 3 June by the royal family, one of the last official acts of the doomed Bourbons

Lagrange himself managed to survive the Revolution, though at times his fate was in doubt. In September 1793, at the height of the Reign of Terror, an order was issued for the arrest of all enemy aliens and the seizure of their property. A special dispensation was arranged for Lagrange at the behest of chemist Antoine Lavoisier, who would himself be executed by the radicals a few months later. Many schools were closed down during the first years of the Revolution, and in an attempt to restore higher learning, the *École Polytechnique* and the *École Normale* were established; Lagrange taught at both. The *École Normale* closed down after just three months of instruction, but the *École Polytechnique* survived the revolutions and beyond. Napoleon, who himself was well aware of the prestige associated with top scientists, showered honors on Lagrange and others: Lagrange was made a Senator, then a Grand Officer in the Legion of Honor, and then a Count of the Empire Finally, on 3 April 1813, Napoleon inducted Lagrange as a Grand Croix of the Order of the Reunion. A week later Lagrange died.

Jeff Suzuki

Alternate name

Lagrangia, Giuseppe Lodovico

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