An Interview with Beno Eckmann

Conducted by Martin Raussen (Aalborg, Denmark) and Alain Valette (Neuchâtel, Switzerland)
in Zurich on 10 January 2007.

Education

Professor Eckmann, you were born on 31 March 1917 in Bern, Switzerland, and you are approaching your 90th birthday now. Could you please tell us a bit about your school education, in particular who and what aroused your interest in mathematics?
I went to school in Bern. I will mainly talk about high school, which is called "gymnasium". I did the classical gymnasium – that means with Greek and Latin and languages. Everything was very good. Except mathematics; mathematics was very weak! I don't regret that I studied Greek and Latin. And I still know Latin well.

I really don't know why I decided to study mathematics. It is not that I no longer remember. I just don't know! I was thinking about German languages or other languages, or something else – all kind of things! All of a sudden, I said: "I want to study mathematics" – here in Zurich, at the ETH (Swiss Federal Institute of Technology). Someone told me: "Don't study mathematics. It's a very old science. Everything is known. There's nothing to get interested in." Nevertheless, I went to Zurich and studied maths!

Beno Eckmann during the interview (photos: Indira Lara Chatterji)

How old were you when you started?
Eighteen years.

What was your student time like in Zurich and who were your most important teachers and supervisors?
In the first year we had *Michel Plancherel* (1885–1967), *Ferdinand Gonseth* (1890–1975) and as a supervising assistant *Eduard Stiefel* (1909–1978). Plancherel was very old-fashioned, extremely old-fashioned; but in fact he was not bad! Since I was not really properly prepared, linear algebra and analysis were quite difficult for me. However, everything we learned was a revelation and I realised that mathematics was indeed something I had expected in my dreams.

The second and later years brought even more interesting teachers: *Heinz Hopf* (1894–1971), *George Polya* (1887–1985) and *Wolfgang Pauli* (1900–1958). As we understood, Hopf was working in a new field: algebraic topology, a higher type of geometry. I decided early that later on, I would try to work with him. Hopf was very modern, he taught in the style of van der Waerden's "Moderne Algebra" or later Bourbaki. In fact, at a very early stage, I started to read *B.L. van der Waerdens'* book, "Modern Algebra" (later called "Algebra"). Mathematical objects were sets provided with additional structure fulfilling certain axioms. This was exactly what made the definitions of Hopf very clear and transparent (groups, spaces, etc.).

Polya was a very good teacher. But he was always far too slow in the beginning, and in the end the courses were too difficult. Moreover, his definitions were often not that clear. His books with Szegö were very interesting. One could learn a lot from the problems when he followed them chapter by chapter.

As for Pauli, he gave a course in theoretical physics. Even though I was not really involved in physics, I realised that in his thinking all types of mathematics were involved – we had the possibility to get acquainted with many highly interesting aspects.

How many students were you altogether at the time?
Six students. We were twelve in the whole group: six mathematicians and six physicists. We were practically always together; there was not much difference between us except that the physicists had to go to the laboratories more often.

You graduated from the ETH at the dawn of World War II.
Yes, I got the diploma in 1939; this corresponds to a Masters Thesis today. I did my diploma thesis in topology under Hopf's supervision. He was really nice and a good man.

Please tell us about your doctoral thesis work!
After the diploma, I became an assistant to Professor *Walter Saxer* (1896–1974). There were not many assistants at the time because there were not many engineering students who needed assistants. Saxer was an analyst, not worldwide renowned but a good professor; he needed

Beno Eckmann with interviewers Alain Valette (left) and Martin Raussen (right) on top of the ETH building

assistants because he became rector at the ETH. As his assistant I replaced him at times and taught the problem sessions for him. That was a very good training.

Simultaneously I could start working on my PhD with Hopf. He asked: "Do you want to work on something else?" But I started immediately on the theme he gave me, which was homotopy groups. Nobody else had worked on homotopy groups, except *Witold Hurewicz* (1904–1956) in his famous and absolutely wonderful notes.

Career

Having finished your thesis, you were appointed to Lausanne.
Indeed, right after the PhD, in 1942. While I was in Lausanne, I remained lecturing at the ETH. At that time, I concentrated on combinatorial problems; I do not know why! At Lausanne, I became acquainted with *Georges de Rham* (1903–1990). He lived in Lausanne and he was a professor at both Lausanne and Geneva. My main mathematical contacts at the time were with him and with Hopf.

It was wartime in Europe and you had to serve military service at that time. What did you have to do?
Of course, I went to the Army, serving in the mountain artillery. We had to stay in the mountains, normally in summer, for two weeks at a time. Then I could go back for two weeks to give all my lectures in Lausanne, and so on.

But there was no communication with abroad during the war?
Very little. There was some before France was completely occupied. There was the "free zone" in the south. *Charles Ehresmann* (1905–1979) was in the free zone. He came to Switzerland; we had vacations together.

How did this situation change after the war?
In 1947, I went to the Institute for Advanced Studies (IAS) in Princeton for an academic year. I had to get myself to learn English at first, since I was supposed to give lectures in English, like everybody! Very few of my colleagues had a good knowledge of English; it was not part of the school curriculum everywhere. It was particularly important for my mathematical development that I had the opportunity to meet people like *Solomon Lefschetz* (1884–1972) and *Norman Steenrod* (1910–1971) at the IAS.

One year after that year in Princeton, in 1948, I was appointed at ETH. Soon after, Robert Oppenheimer (1904–1967) became the director of IAS. He invited me to spend another year at IAS, from 1951 to 1952. Again, I met interesting people, among them *Raoul Bott* (1926–2005) who was then a beginner with fascinating ideas. I had the possibility to discuss many different topics with *Albert Einstein* (1879–1955); he was happy to talk German and to remember his old experiences from Switzerland.

You travelled a lot to the United States and to other countries.
I went regularly to the US. Not for the full academic year but for summer vacation or shorter periods. MSRI at Berkeley was established and I went there when it was still very young and talked a great deal to *Shiing–Chen Chern* (1911–2004). He explained that they planned to have a specific topic for every year and they would invite people for that year. But it never worked that way: people would come for some period and then they would perhaps come two years later, and so on.

Scientific work

Under the influence of Heinz Hopf, you started to work in homotopy theory at a time when algebraic topology was hardly established. Please give us some reminiscences on the development. You must be one of the few left who can witness that algebraic topology has not always been associated with commutative diagrams, exact sequences, spectral sequences and so on.
Yes, indeed – exact sequences, commutative diagrams. When I wrote my first paper they did not exist at all. Not even a map was denoted as we do it today, with an arrow from its domain to its codomain. It's unbelievable! It was much more difficult to express things and to compute the exactness of a sequence. At each stage you had to show explicitly what you needed. And then, as soon as maps were denoted just by two letters with an arrow in-between and with this a suitable notation for exact sequences and diagrams, everything became simple and clear, and you could use them for clear statements and easy proofs. So many things that we used a lot of energy on in the past are almost obvious today!

And then you had to bring in homological algebra...
You see, if a topological space X is acyclic and has fundamental group G, then it follows from Hurewicz theory that the homology of X depends on G alone. Thus we were looking for an algebraic description of the homology depending only on G. It makes use of the group algebra of G, and thus the homology of a group algebra was introduced. These were problems that many people were

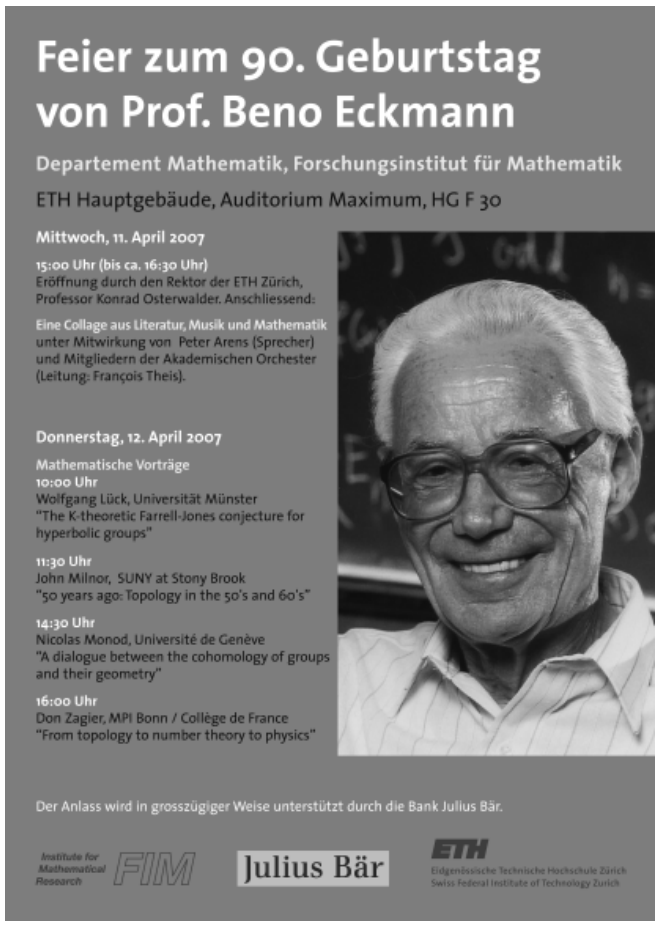
dealing with but it seems not to be widely known that Heinz Hopf was the first person to construct a free resolution over a group ring. People don't know that; they talk about *Eilenberg-McLane*[1] but it was Heinz Hopf who invented free resolutions. He also phrased precisely what it means that two free resolutions are equivalent. I carried this line of thought further on.

Let us talk about your many other contributions to mathematics. Apart from algebraic topology, your name is associated with results in differential geometry, group theory and more recently L^2-invariants, at the boundary between topology and analysis. How would you describe the common thread through your work?
Topology, in the spirit of Hopf, was always to be applied to geometry. That was the idea. It was not just something abstract. One of the geometries was differential geometry, manifolds. So, at an early stage I went through complex manifolds, Kähler[2] manifolds, with my student *Heinrich Guggenheimer*. We even created the name Kähler manifold!

Ah, that was your invention!
Indeed. The reason was that we used the operator of Kähler on differential forms. Of course we used the book by *William Hodge* (1903–1975). As I explained, the topology of the classifying space of a group depends only on the group. So at the same time, we had to develop the formalism to work directly with the homology of groups, and then the homology of algebras because groups lead to group algebras; so we went to algebras. Then I went on to dualize every map, considering a map in the other direction as well. From this point of view, one obtains new theorems. Pursuing this direction further on, I got interested in groups by themselves: in applying geometry to groups, topology to groups. And this then led to Poincaré duality, Poincaré duality groups and duality groups. It is a much more general setting that I developed in collaboration with *Robert Bieri*. Together with many other people and after a long development I could prove that a Poincaré duality group of cohomological dimension 2 is the group of a Riemann surface. That was actually a conjecture of *Jean-Pierre Serre*. "You have to prove it!" he had always insisted.

In my earlier papers in topology, I had used cell complexes, chains (which are linear combinations of cells) and harmonic chains (which are cycles and cocycles at the same time). There seemed to be something hidden, which is typical for operator analysis. It was Jean-Pierre Serre who always insisted: "There must be something much deeper!" I did not know what it was for a long time.

Finally L^2–theory came up, with idealized chains. Now you can have harmonic chains inside that space of L^2-chains. And then you get so many things from earlier considerations that guided me through L^2-theory: topology with Hilbert spaces instead of vector spaces, operators – these were the things I worked on, until I more or less stopped recently. Well, maybe not quite … I can still read, for example I read what *Wolfgang Lück* has done. You see, when Lück was very young, I got a paper from him where he proved something I had just announced also having proved.



There will be a meeting[3] next April for your 90th birthday. Clearly you are still active. What is the driving force that pushes you to continue doing mathematics?
The same force that was there at the beginning: because I like it. I like it and I find it fascinating. I try to follow a little bit of what the young people are doing, to understand a little bit of what directions they go and how they use the old stuff.

Coming back to your own contributions: is there one single result that you view as your most important?
One single result - that is difficult! But if I would single out something, it is probably what I did in the beginning. It is so elementary today that it belongs to the first semester of topology: homotopy groups, the exact sequence of a fibration, calculating the homotopy groups for the orthogonal groups and so on. There was nothing like that before! And then I used the same homotopy methods to prove that on a sphere of dimension $4k+1$, you can only have one tangent unit field, not two that are linearly independent. It was the beginning of the whole theory of

[1] Samuel Eilenberg (1913–1998), Saunders Mac Lane (1909–2005).
[2] Erich Kähler (1906-2000).
[3] Held 11–12 April 2007 at the ETH Zurich

vector fields on spheres, which was developed by others later on. It was very difficult in higher dimensions until the famous papers by *John Milnor* and *Michel Kervaire* (1927-2007). Milnor was not my student but I took him from Princeton to Zurich, when he was a student. Kervaire was my student but he wrote his thesis with Hopf.

At this stage, I should at least mention various geometric and algebraic techniques introduced in algebraic topology during my most active years, like spectral sequences, cohomology operations, general homology theories and so on. But in this conversation, I think we should concentrate on comparatively elementary aspects.

Another direction was Eckmann-Hilton duality, which went on for many years. There was even a section in Mathematical Reviews under that name, with many contributions.

Still another important area is Poincaré duality for groups, invented by Robert Bieri and myself. They behave like manifolds: homology, cohomology, you see, in complementary dimensions, but with another dualizing module. Many groups that are interesting in algebraic geometry, group theory or other areas are such duality groups. I had a draft of a paper about these topics with me in Princeton and Jean-Pierre Serre was there at the same time. He could not come to my lecture but the day after he wrote to me that he wanted to publish it in the Inventiones! It was not ready to be published yet – two months were still necessary.

I don't know what is very important, what is less important. What you do is always interesting; it is more difficult to judge importance! And then, I had so many students! I gave many ideas and interests to my PhD students and they then published work that I could not have thought about myself.

Students

You mentioned your many PhD students; indeed according to the Mathematics Genealogy Project, you had 60 PhD students and more than 600 descendants. How did you manage?

That is a good question! I don't know! The first of the students was an assistant who wanted to write a PhD with me. I told him to write down, in one or two weeks, what he really wanted to do, an abstract. I told him to read a little bit and after a long time, maybe half a year or even more, he should come and tell me what he really wanted to do. And once he had his topic, we would see each other, for one or two hours, and discuss things in detail, and start to write down first results. Then I had the next student, and the next, and the next … more and more.

Is there a particular reason why you attracted so many students?

I don't know! I mean, it's probably because they liked my style. In fact, I gave many lectures. At that time we gave more lectures than today. To teach, you must make it very clear in your mind what you want to lecture about, how to present it and what to say first, and then you head towards a result, a theorem.

My lecturing style is very old-fashioned, and probably young people do not agree with me; that's normal! When lecturing, I always used blackboard and chalk, developing the ideas gradually further and further, saying exactly where I wanted to go. Sometimes I had to lecture with overhead projectors. But then I wrote maybe four or five lines and I would cover all the rest, except the one line that I would have written on the blackboard. So it's really old-fashioned but I know there are many mathematicians who still organise their lectures in that way. My students seemed to like it because they followed my courses. I did not allow any script, mimeographed notes or anything. I said: "You have to think here and I go with you step by step." When the course is finished, you must get the book and read it, and you will find other similar things.

Today many students just use the mimeographed notes. They have their colour pens and they underline this and that; I don't think it's the same. But it works as well! Today also, my colleagues find good students and they have good PhDs. It's just different!

Collaboration

Among your many collaborators, Peter Hilton clearly plays a privileged role. Can you say some words about the way you did your joint work?

I met *Peter Hilton* when he was a graduate student with *Henry Whitehead* (1904–1960) at Oxford. I went in 1947 from the IAS to Oxford to meet Whitehead. Peter was very shy then and he asked me whether he could contact me at Zurich later on. I agreed and so he did. In 1955 he came to Zurich and he stayed here for the whole year. I could guide him a little bit and explain many things to him about homological algebra, and then he got more and more into that idea of dualizing lots of our mathematics. And this became Eckmann-Hilton duality, which was quite well followed for a while: in geometry, topology and algebra. When he left Zurich, we continued of course by correspondence. Sometimes I went to England, or he came to Zurich and that was alright. After a long time, he changed direction and I always had the wish to do more concrete mathematics, more geometry, more group theory; so we took different routes. Our minds were a bit different and that was alright: we remained very good friends but we did not collaborate after that.

In your long career, you met quite a number of famous mathematicians. Is there anybody whom you would like to mention in terms of influence, or friendship?

I already mentioned Peter Hilton and Robert Bieri. Then there is *Guido Mislin*; we have joint papers on Chern classes of group representations. This work again combines topology with group theory and with number theory because the Chern class gives really interesting limitations related to Bernoulli numbers and so on; it's an interesting topic!

These were collaborators with whom I wrote joint papers. Georges de Rham was very important for me in Lausanne, and also afterwards; I went to see him from time to time. But then I got of course a lot of very lucky influence very early on from *Henri Cartan*[4] , who is already more than one hundred years old, and later on from Jean-Pierre Serre. Actually Jean-Pierre Serre is younger than I am; he has followed my first papers very carefully and that was really an interesting advantage. I would always have wonderful contact with him; he asked important questions. Unfortunately, I could not follow him any longer when he went into number theory.

Forschungsinstitut für Mathematik

I would like to ask you about FIM, the Forschungsinstitut für Mathematik, which you founded at the ETH, and of which you were the first director for 20 years. What was the prime idea for creating this institute? How did it develop? Are you satisfied with its present activities?

Indeed, I founded it because I thought it was necessary to have an organization to welcome visitors and to do everything so that they can work here together with faculty members. The institute was not to have permanent members, except for the director who had to be one of the faculty members.

Something like that did not exist previously. The Institute for Advanced Study was separated from Princeton University and was not linked to it. It was essential for me that our institute was to be linked with the department here so that every member of the department could invite visitors to work with or to learn from. And the institute should care for these visitors in every respect. That was an idea that people found strange at the time and many did not agree. I went with this idea to the ETH president *Hans Pallmann*. I argued that we needed such an institute because otherwise our professors do not have enough interaction with the world outside. He said: "We do not have the money, but you get it! Just start right away!" Soon afterwards, I could invite *K. Chandrasekharan* and *Lipman Bers* (1914–1993). Many others followed.

I remained the director for twenty years. At the end we had a huge number of visitors. My successor was *Jürgen Moser* (1928–1999). He had a different style but he worked towards the same objectives. He was followed by *Alain-Sol Sznitman* and the present director is *Marc Burger*. I think it will continue in the same spirit, although many new features have been added, for example the Nachdiplomvorlesungen (post-diploma lectures): we invite people to the institute to give very high level graduate courses[5].

We have two or three such courses every semester. Nowadays, they organise workshops as well. All this changed the size of the institute, of course; it has grown. But the institute still takes care of apartments for visitors and for their offices within the ETH.

The director Marc Burger has an excellent knowledge of mathematics and of mathematicians all over the world, so he attracts good visitors. Moreover, with all our later appointments of high level mathematicians to the department, people expressed interest in the institute during negotiations: "Can I invite people to work with my PhD-students?" It is quite important and I am very pleased.

Nowadays, almost every university has such an institute but at that time, in 1964, there was not a single one – nowhere!

Publishing mathematics

Can we talk about your involvement in the publishing of mathematics? For many years, you have been an editor of the series Grundlehren der Mathematischen Wissenschaften and also of Lecture Notes in Mathematics.

I was asked to join the managing board of the *Grundlehren* because they needed people. *Wolfgang Schmidt* who was there wanted to retire and van der Waerden said that he no longer wanted to do that much.

At what time did you join Grundlehren?

That was in 1966. Every volume was refereed before being accepted and this was heavy work.

Konrad Springer, 4th generation of Springer, was a biologist who studied in Zurich and I talked with him about the publishing of lecture notes. The institute published lots of lecture notes. Who could be a commercial publisher of such notes? Springer-Verlag, of course! I talked to him and said: "That is something I have wished for a long time, so if you help me…" He tried to convince his father and they finally liked the idea. They made photocopies of the typescript and published it! You could send the typescript directly to Springer who provided the copies. It was immediately a great success. Since I could not take both series myself, I asked *Albrecht Dold* to take over the Lecture Notes; the first volume is under him. But he did not want to do the work alone. He argued that I knew all the Springer people and convinced me that we should both be editors.

Over the years it became easier, with computers and the internet; it certainly takes less time. Now they receive a computer-processed manuscript, only one or two referees need to accept it and it runs very quickly. The contact with Springer was always very interesting; we had long discussions over many years.

IMU

You were also very much involved in national and international mathematical societies. You have been the President of the Swiss Mathematical Society for a two year period…

That was almost compulsory; I had to do that…

[4] Born in 1904.

[5] In 1999, one of the interviewers, A.V., gave such a Nachdiplomvorlesung on the Baum-Connes conjecture. This led to a very stimulating interaction with Profs Eckmann and Mislin.

…and secretary of the IMU…
Well, that was not compulsory. Heinz Hopf was IMU's president and he asked me to become the secretary of the international union. I said: "Yes, if I can have a helping secretary, because I do not have a secretary here!" This is how I got a secretary to do the typing and mailing for me. It was a very interesting period, 1956–1961.

What were the important issues at the time, during the cold war?
Two important goals were achieved:
Many countries (some of them very large) that had not adhered to the IMU became members. One can imagine that many difficulties had to be overcome, difficulties of personal, political and financial character. This was heavy but gratifying work for the secretary.

The International Congress of Mathematicians had to become a task of the IMU. The last congress organised solely by a single country was the Congress in Edinburgh in 1958, organised by the UK. With the increasing number of research areas and of participants, this became too heavy a burden for a national mathematical society. The local organisation is of course still taken care of by the organising country but the scientific plans are made by the union.

Private Interests

What are your other main private interests – apart from mathematics?
Through my entire mathematical life I was always able to find time for other activities (sometimes combined with mathematical work): I spent interesting periods with my wife and my big family, on weekends, during vacations, with music and art, and with school and student problems. Love and happiness are important inside and outside mathematics.

Thank you very much for this interesting conversation!

A Survey of ICMI Activities

Maria G. (Mariolina) Bartolini Bussi (Modena, Italy)

Maria G. (Mariolina) Bartolini Bussi is a member of the editorial board of this newsletter and serves as a member of the executive committee of the International Commission on Mathematical Instruction from 01 January 2007 until 31 December 2009. In this column, she will periodically provide news from the ICMI.

The information below is taken from the official website of the eleventh International Congress on Mathematical Education (ICME11), which is to be held in Monterrey, Mexico, 6–13 July 2008. The interested reader is welcome to visit the website (http://icme11.org/), where the second announcement will appear over the next few weeks. Below is a summary of the session types given. Most activities **(topic study groups, discussion groups, workshops, sharing experiences groups, a poster exhibition and round tables)** welcome contributors. Each activity will have its own deadline, but not later than 20 January 2008.

The organizers expect to gather between 3000 and 4000 professionals from 100 countries in the mathematics education area, including researchers, educators and teachers.

The International Congress on Mathematical Education (ICME) aims to:

- Show what is happening in mathematics education worldwide, in terms of research as well as teaching practices.
- Inform about the problems of mathematics education around the world.
- Learn and benefit from recent advances in mathematics as a discipline.

ICME consists of several different *session types.*

Plenary Activities
Lectures or panels on themes of current actuality and relevance to the practice of the international community of mathematics educators.

National Presentations
It is customary to select a small number of countries so that the international mathematics community may gain a closer knowledge on the state and trends of mathematics education in those countries.

National representatives of those countries are asked to make the presentations.