Akhiezer, Naum Il’ich | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
6-8 minutes

(*b*. Cherikov. Belorussia, 6 March 1901; *d*. Kharkov. U.S.S.R. 3 June 19801)

mathematics.

Akhiezer graduated horn the Kiev Institute of People’s Education in 1923. then continued his studies for the candidate degree at the i University of Kiev under D. A. Grave, who was well known in the field of algebra. In his candidate’s thesis, on his aerodynamic research, he applied methods of complex analysis to aerodynamic problems. He was the first to obtain a formula for the conformal mapping of a double-connected polygonal domain onto a ring.

Akhiezer was very much involved in problems of approximation theory. In 1928 he solved a difficult problem: Among all the polynomials of degree n with three fixed senior coefficients, find the least deviation from the zero polynomial in a given interval of the real axis. Akhiezer showed that the solution of this problem can be obtained with the help of Schottky’s functions. This result gave impetus to further development of the classical theory of the least deviation from the zero polynomials of P. L. Chebyshev, N. E. Zolotarev, and V. A. Markov. Akhiezer later showed that the problem of the least deviation from the zero polynomial in the case where k senior coefficients are fixed can be reduced to the problem of finding some domain that is the complex plane with k segmental sections along the real axis, and to the construction of the Green function of this domain.

In 1933 Akhiezer moved to Kharkov, where he was head of the complex analysis department and president of the Kharkov Mathematical Society for many years. In 1934 he was elected corresponding member of the Ukrainian Academy of Sciences. At that time Akhiezer and M. G. Krein began to investigate the L-moments problem: Find a density ϱ (t)($-\infty \leq a \leq t \leq b \leq \infty$)of a mass distribution with given moments that satisfies an additional condition $0 \leq \varrho(t) \leq$ L. Their researches in this field were summarized in *Some Questions in the Theory of Moments* (1933). They also found the precise value of a constant in the theorem of Dunham Jackson concerning the approximation of a periodic function by trigonometric polynomials (this value was obtained independently by Jean Favard)

During World War II, Akhiezer was at the Alma-Ata Mining Institute (1941–1943) and the Moscow Power Engineering Institute (1943–1947). He returned to Kharkov in 1947, the year in which his book on approximation theory was published. In it he combined new ideas of functional analysis with classical methods and presented his own results. In 1949 Akhiezer was rewarded by the Chebyshev Prize for this book.

Further investigations were inspired by S. N. Bernstein, one of the founders of approximation theory, who in 1924 had formulated the following problem: Let $\Phi(x)(-\infty\ x < \infty)$ be a function such that and lim Let us introduce the space C_{Φ} of continuous functions *f* (t) such that him and let be the norm in C_{Φ}. Find the necessary and sufficient conditions on such that polynomials will form a dense set of C_{Φ}. In their joint work on this problem, Akhiezer and K. I. Babenko studied an important class M_{Φ} polynomials $P(x)$ satisfying the inequality P(x)| $\leq \Phi$(x) and introduced the expression

In 1953 Akhiezer and Bernstein found that the condition $j_{\psi}= \infty$, where $\psi=(1 + x^2)^{1/2}\phi$ X, is a sufficient and necessary condition for the completeness of the set of polynomials in C_{ϕ}.This result established the complete solution of Bernstein’s problem. In another group of works Akhiezer studied the following problem: Among all the entire functions of a finite degree with given values or derivatives at a finite set of given points in the complex plane, find the least deviation from the zero entire function. Akhiezer also obtained a generalization of Bernstein’s inequality for the derivative of an entire function of a finite degree. Later he and B. Ia. Levin extended these results to important classes of many-valued functions. In 1961 another important book. *The Classical Moment Problem*, was published. At the same time Akhiezer worked on problems connected with “continual analogues” of the classical problem of moments, He also further developed a work by Mark Kac on the Fredholm determinants of the Wiener-Hopf equation with a hermitian kernel.

Akhiezer investigated the orthogonal polynomials with respect to a weight fund ion on a set of arcs of a circle or on a set of intervals of the real axis; he then applied these methods to inverse problems in spectral analysis. Let

be a Sturm-Liouville operator, where q(x) is a real continuous function and h is a real constant. Let us assume that the spectrum of L has g gaps. Akhiezer constructed and studied a hyperelliptical Riemannian surface of genus g, associated with the operator L: the corresponding Bloch function E(λ:x) on this surface is single-valued with respect to λ. Theset $\{P_1(x), \ldots, P_g(x)\}$ of its zeros plays an important role: B. A. Dubrovin later showed that the potential q(x) can be expressed explicitly with the help of the functions $\{P_1(x), \ldots, P_g(x)\}$ These results are presented in the appendix to the third edition of *Theory of Operators in Hilbert Space*. Using the above results. S. P. Novikov, B. A. Dubrovin, and more recently V. A. Marchenko obtained solutions of remarkable classes of nonlinear partial differential equations of the Korteweg-de Vries type in an explicit and effective form.

BIBLIOGRAPHY

I. Original Works. Akhiezer wrote 130 papers and 8 books. The latter include *Theory of Approximation*, Charles J. Hyman. trans. (New York, 1956); *Some Questions in the Theory of Moments*, W. Fleming and D. Prill, trans. (Providence, R.I, 1962) written with M. G. Krein; The Classical Moment Problem, N. Kemmer. trans. (New York, 1965): and *Theory of Operators in Hibert Space*, 2 vols., E. R. Dawson, trans, and W. N. Everitt, ed. (Boston, 1981; 3rd ed., 1987), written with I. M. Glassman.

II. Secondary Literature. M. Berezanskii. A. N Kolmogorov. M. G.Krein, B. la. Levin. B. M. Levitan. and V. A. Marchenko. “Naum ll’ich Akhiezer (k semidesiatiletiiu so dnia rozhdeniia” (… to the seventieth birthday). in *Uspekhi matematicheskikh nauk*. **26** , no. 6 (1971), 257–261: M. G. Krein and B. la. Levin. “*Naum ll’ich Akhiezer* (k shestidesiatiletiiu so dnia ruzhdeniia)” ([… to the sixtieth birthday]), *ibid*., **16** , no. 4 (1961), 223–232; and, V. A. Marchenko, *Nonlinear Equations and Operator Algebra*, V. I. Rublinetskii, trans, (Dordrecht and Boston. 1988).

Moshe LivŜiĈ