Ibn Al-Haytham, Abū | Encyclopedia.com

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called **al-Baṣrī** (of Baṣra, Iraq), **al-Miṣrī** (of Egypt); also known as **Alhazen** , the Latinized form of his first name, **al-Ḥasan** (*b*. 965; *d*. Cairo, *ca*. 1040)

optics, astronomy, mathematics.

About Ibn al-Haytham's life we have several, not always consistent, reports, most of which come from the thirteenth century. Ibn al-Qifṭī (*d*. 1248) gives a detailed account of how he went from Iraq to Fāṭimid Egypt during the reign of al-Ḥākim (996–1021), the caliph who patronized the great astronomer Ibn Yūnus (*d*. 1009) and who founded in Cairo a library, the Dār al-ʿIlm, whose fame almost equalled that of its precursor at Baghdad (the Bayt al-Ḥikma, which flourished under al-Maʾmūn [813–833]). Impressed by a claim of Ibn al-Haytham that he would be able to build a construction on the Nile which would regulate the flow of its waters, the caliph persuaded the already famous mathematician to come to Egypt and, to show his esteem, went out to meet him on his arrival at a village outside Cairo called al-Khandaq.

Ibn al-Haytham, according to Ibn al-Qifṭī, soon went at the head of an engineering mission to the southern border of Egypt where, he had assumed, the Nile entered the country from a high ground. But even before reaching his destination he began to lose heart about his project. The excellently designed and perfectly constructed ancient buildings which he saw on the banks of the river convinced him that if his plan had been at all possible it would have been already put into effect by the creators of those impressive structures. His misgivings were proved right when he found that the place called *al-Janādil* (the cataracts), south of Aswan, did not accord with what he had expected. Ashamed and dejected he admitted his failure to al-Ḥākim, who then put him in charge of some government office. Ibn al-Haytham at first accepted this post out of fear, but realizing his insecure position under the capricious and murderous al-Ḥākim he pretended to be mentally deranged and, as a result, was confined to his house until the caliph's death. Whereupon Ibn al-Hāytham revealed his sanity, took up residence near the Azhar Mosque, and, having been given back his previously sequestered property, spent the rest of his life writing, copying scientific texts, and teaching.

To this account Ibn al-Qifṭī appends a report which he obtained from his friend Yūsuf al-Fāsī (*d*. 1227), a Jewish physician from North Africa who settled in Aleppo after a short stay in Cairo where he worked with Maimonides.[1] Yūsuf al-Fāsī had "heard" that in the latter part of his life Ibn al-Haytham earned his living from the proceeds (amounting to 150 Egyptian dinars) of copying annually the *Elements of Euclid, the Almagest*, and the *Mutawassiṭāt*,[2] and that he continued to do so until he died "in [*fī ḥudūd*][3] the year 430 [a.d. 1038–1039] or shortly thereafter [*aw baʿdaha bi-qalīl*]." These words are immediately followed by a statement, of which the author must be presumed to be Ibn al-Qifṭī, to the effect that he possessed a volume on geometry in Ibn al-Haytham's hand, written in 432 (a.d. 1040–1041).

An earlier account of Ibn al-Haytham's visit to Egypt is given by ʿAlī ibn Zayd al-BayhaqĪ (*d*. 1169–1170).[4] According to him the mathematician had only a brief and unsuccessful meeting with al-Ḥākim outside an inn in Cairo. The caliph, sitting on a donkey with silver-plated harness, examined a treatise composed by Ibn al-Haytham on his Nile project, while the author, being short of stature, stood on a bench (*dukkān*) in front of him. The caliph condemned the project as impractical and expensive, ordered the bench to be demolished, and rode away. Afraid for his life, Ibn al-Haytham immediately fled the country under cover of darkness, going to Syria, where he later secured the patronage of a well-to-do governor. But this account, vivid though it is, must be discarded as being unsupported by other evidence. For example, we are told by Ṣāʿid al-Andalusī (*d*. 1070) that a contemporary of his, a judge named ʿAbd al-Rahman ibn ʿĪsā, met Ibn al-Haytham in Egypt in 430 a.h., that is, a short time before the latter died.

Ibn Abī Uṣaybiʿa (*d*. 1270) gives the name of Ibn al-Haytham as Muḥammad (rather than al-Ḥasan) ibn al-Ḥasan; and he joins Ibn al-Qifṭī's story (which he quotes in full with the omission of the last statement about Ibn al-Haytham's autograph of 432) to a report which he heard from ʿAlam al-Dīn Qaysar ibn Abi ʿl-Qāsim ibn Musāfir, an Egyptian mathematician who resided in Syria and died at Damascus in 649 a.h./a.d. 1251.[5] According to this report, Ibn al-Haytham at first occupied the office of minister at Basra and its environs, but to satisfy his strong desire to devote himself entirely to science and learning he feigned madness until he was relieved from his duties. Only then did he go to Egypt, where he spent the rest of his life at the Azhar Mosque, living on what he earned from copying Euclid and the *Almagest* once every year. We may add that the title of one of his writings (no. II 13, see below) appears to imply that he was at Baghdad in 1027, six years after al-Ḥākim died.[6]

It is unfortunate that the autobiography of Ibn al-Haytham, which Ibn Abī Uṣaybiʿa quotes from an autograph, throws no light on these different reports. Written at the end of 417 a.h./a.d. 1027, when the author was sixty-three lunar years old, and clearly modeled after Galen's *De libris propriis*,[7] it lists the works written by Ibn al-Haytham up to that date but speaks in only general terms about his intellectual development.

As cited by Ibn Abī Uṣaybiʿa, Ibn al-Haytham, reflecting in his youth on the conflicting but firmly held beliefs of the various religious sects, was led to put them all in doubt and became convinced that truth was one. When in later years he was ready to grasp intellectual matters, he decided to turn his back on the common people and devote himself to seeking knowledge of the truth as the worthiest possession that could be obtained in this world and the surest way to gain favor with God—a decision which, using Galen's expressions in *De methodo medendi*,[8] he attributed to his "good fortune, or a divine inspiration, or a kind of madness." Frustrated in his intensive inquiries into the religious sciences, he finally emerged with the conviction that truth was to be had only in "doctrines whose matter was sensible and whose form was rational." Such doctrines Ibn al-Haytham found exemplified in the writings of Aristotle (of which he here gave a conspectus) and in the philosophical sciences of mathematics, physics, and metaphysics. As evidence of his having stood by his decision, he provided a list of his writings to 10 February 1027, containing twenty-five titles on mathematical subjects (list I*a*) and forty-five titles on questions of physics and metaphysics (list I*b*).

Ibn Abī Uṣaybiʿa gives two more lists of Ibn al-Haytham's work, which we shall designate as II and III. List II, which he found attached to list I and in the author's hand, contains twenty-one titles of works composed between 10 February 1027 and 25 July 1028. Ibn Abā Uṣaybiʿa does not say whether he also copied list III from an autograph but simply describes it as a catalogue (*fihrist*) which he found of Ibn al-Haytham's works to the end of 429 a.h./2 October 1038. Nor does he specify the *terminus a quo* of this catalogue. However that may be, two things are remarkable about this last list: consisting of ninety-two titles, it includes all sixty-nine titles ascribed to Ibn al-Haytham by Ibn al-Qifṭī, with two exceptions; and in it are to be found all of Ibn al-Haytham's extant works (not fewer than fifty-five), again with only a very few exceptions. It may also be noted that the order of works in list III almost always agrees with the chronological order of their composition, whenever the latter can be independently determined from internal cross references. Thus, III 2 was written before III 53; III 3 before III 36, III 49, III 60, III 77, and III 80; III 20 before III 21; III 25 before III 31; III 26 before III 38 and III 68; III 42 before III 74; III 53 before III 54; III 61 before III 63; III 63 before III 64; and III 66 before III 77. Item III 17, however, was written before III 16 (see bibliography).

Among the subjects on which Ibn al-Haytham wrote are logic, ethics, politics, poetry, music, and theology (*kalām*); but neither his writings on these subjects nor the summaries he made of Aristotle and Galen have survived. His extant works belong to the fields in which he was reputed to have made his most important contributions: optics, astronomy, and mathematics.

Optics: Doctrine of Light . Ibn al-Haytham's theory of light and vision is neither identical with nor directly descendant from any one of the theories known to have previously existed in antiquity or in Islam. It is obvious that it combines elements of earlier theories—owing perhaps more to Ptolemy than to any other writer—but in it these elements are reexamined and rearranged in such a way as to produce something new. Ibn al-Haytham's writings on optics included a treatise written "in accordance with the method of Ptolemy" (III 27), whose *Optics* was available to him in an Arabic translation lacking the first book and the end of the fifth and last book, and a summary of Euclid and Ptolemy in which he "supplemented the matters of the first Book, missing from Ptolemy's work" (I*a* 5). These two works are now lost.

But in his major work, the *Optics or Kitāb al-Mānazir* (III 3),[9] in seven books, Ibn al-Haytham deliberately set out to dispel what appeared to him to be a prevailing confusion in the subject by "recommencing the inquiry into its principles and premises, starting the investigation by an induction of the things that exist and a review of the conditions of the objects of vision." Once the results of induction were established he was then to "ascend in the inquiry and reasonings, gradually and in order, criticizing premises and exercising caution in the drawing of conclusions," his aim in all this being "to employ justice, not to follow prejudice, and to take care in all that we judge and criticize that we seek the truth and not to be swayed by opinions" (Fatih MS 3212, fol. 4a r).

The book is in fact an earnest and assiduous exercise in the method outlined. Its arguments are either inductive, experimental, or mathematical, and it cites no authorities. Experiment (*iʿtibār*) in particular emerges in it as an explicit and identifiable methodological tool involving the manipulation of artificially constructed devices. (In the Latin translation of the *Optics* the word *iʿtibār* and its cognates *iʿtabara* and *muʿtabir* became *experimentum, experimentare*, and *experimentator*, respectively.) Perhaps as a result of its derivation from the astronomical procedure of testing past observations by comparing them with new ones, the method of *iʿtibār* often appears aimed at proof rather than discovery. It establishes beyond doubt that which is insecurely suggested by inadequate observations.

The *Optics* is not a philosophical dissertation on the nature of light, but an experimental and mathematical investigation of its properties, particularly insofar as these relate to vision. With regard to the question "What is light?," Ibn al-Haytham readily adopted the view ascribed by him to "the physicists" or natural philosophers (*al-ṭabīʿiyyūn*)—not, however, because that view was by itself sufficient, but because it constituted an element of the truth which had to be combined with other elements derived from "mathematicians" (*ta'līmiyyūn*) such as Euclid and Ptolemy. In the resulting synthesis (*tarkīb*) the approach of "the mathematicians" dominated the form of inquiry, while their doctrines were altered, indeed reversed, in the light of those of "the physicists." That "the physicists" were the natural philosophers working in the Aristotelian tradition is clear enough from comparing the view attributed to them by Ibn al-Haytham with expressions and doctrines that had been current in the works of peripatetics from Alexander to Avicenna.

Light, says Ibn al-Haytham, is a form (*ṣūra, εîαδοs*) essential (*dhātiyya*) in self-luminous bodies, accidental (*ʿaraḍiyya*) in bodies that derive their luminosity from outside sources. Transparency (*al-shafīf*) is an essential form in virtue of which transparent bodies, such as air or water, transmit light. An opaque body, such as a stone, has the power to "receive" or take on and make its own the light shining upon it and thereby to become itself a luminous source. This received light is called accidental because it belongs to the body only as long as the body is irradiated from outside. There are no perfectly transparent bodies. All transparent bodies possess a certain degree of opacity which causes light to be “received” or “fixed” in them as accidental light.

The light which radiates directly from a selfluminous source is called “primary” (*awwal*); that which emanates from accidental light is called “secondary” (*thānī*). Primary and secondary lights are emitted by their respective sources in exactly the same manner, that is, from every point on the source in all directions along straight lines. The only difference between these two kinds of light is one of intensity: accidental light is weaker than its primary source and the secondary light deriving from it is weaker still. All radiating lights become weaker the farther they travel. The distinction is made in transparent bodies between the accidentally fixed and the traversing light, and it is from the former that secondary light is emitted. Thus from every point of the sunlit air, or on the surface of an illuminated opaque object, a secondary light, fainter than the light coming to this point directly from the sun, radiates “in the form of a sphere,” rectilinearly in all directions. (The picture is interesting since it later appears in the doctrine of the multiplication of species and it is at the basis of Huygens’ principle.)

Two other modes of propagation are the reflection of light from smooth bodies and its refraction when passing from one transparent body into another. Unlike an opaque body, a smooth surface does not behave, when illuminated, like a self-luminous object; rather than “receive” the impinging light it sends it back in a determinate direction. In *Optics*, book I, chapter 3, numerous experiments involving the use of various devices—sighting tubes, strings, dark chambers—are adduced to support all of the above statements, and in particular to establish the property of rectilinear propagation for all four kinds of radiation: primary, secondary, reflected, and refracted.

Colors are asserted to be as real as light and distinct from it; they exist as forms of the colored objects. A self-luminous body either possesses the form of color or something of “the same sort as color.” Like light, colors radiate their forms upon surrounding bodies and this radiation originates from every point on the colored object and extends in all directions. It is possible that colors should be capable of extending themselves into the surrounding air in the absence of light; but experiments show that they are always found in the company of light, mingled with it, and they are never visible without it. Whatever rules apply to light also apply to colors.

Some time after writing the *Optics*, Ibn al-Haytham remarked in the *Discourse* (III 60) that natural philosophers, in contrast to mathematicians, had failed to supply a definite concept of ray. In book IV of the *Optics* he had in fact tried to remedy the defect by introducing the concept of a physical ray. The underlying idea is that for a body to be able to carry the form of light it must be of certain minimal magnitude. Imagine, then, that a transparent body through which light travels is made progressively thinner by a process of division. (The operation is essentially the same as that of narrowing an aperture through which the light passes.)

Ibn al-Haytham considered that a limit would be reached after which further division would cause the light to vanish. At this limit there would pass through the thin body a light of finite breadth which he calls the smallest or least light (*aṣghar al-ṣaghīr min al-ḍaw’*), a single ray whose only direction of propagation is the straight line extending through its length. A wider volume of light should not, however, be regarded as an aggregate of such minimal parts (*aḍwāʾ diqāq mutaḍāmma*), but a continuous and coherent whole in which propagation takes place along all the straight lines, both parallel and intersecting, that can be imagined within its width. It follows that an aperture will either be wide enough to allow only rectilinear propagation, or too small to let any light pass through; there is no room for the diffraction of light. The result of the new concept is thus an uncompromising formulation of the ray theory of light. (Compare Newton’s concept of “least Light or part of Light” which accords with his interpretation of diffraction as a kind of refraction.)[10]

Theory of Vision . As employed by Ibn al-Haytham the language of forms serves merely to express the view that light and color are real properties of physical bodies. He sometimes conducted his discussion without even using the term “form” (as in the greater part of book I, chapter 3) and his experimental arguments would lose nothing of their import if that term were to be removed from them. And yet it was the term “form” that had been closely associated with the intromission theory of vision maintained in the Peripatetic tradition, whereas mathematical opticians had formulated their geometrically represented explanations in terms of “visual rays” issuing from the eye. Ibn al-Haytham adopted the intromission hypothesis as the more reasonable one and took over with it the vocabulary of forms. To this he added, as we saw, a new concept of ray that satisfied the mathematical condition of rectilinearity but was consistent with the physics of forms. His theory of vision (to be described presently) may thus be seen as one chief illustration of the program he outlined in the *Optics* (III 3), in the treatise *On the Halo and the Rainbow* (III 8), and in the *Discourse on Light* (III 60): optical inquiry must “combine” the physical and the mathematical sciences.

In chapter 5 of book I of the *Optics* Ibn al-Haytham described the construction of the eye on the basis of what had been generally accepted in the tradition of medical and anatomical writings derived from Galen’s works. But he adapted the geometry of this construction to suit his own explanation of vision. In particular he assumed both surfaces of the cornea opposite the pupil to be parallel to the anterior surface of the crystalline humor, all these surfaces being spherical and having the center of the eye as common center. He placed the center of the eye behind the posterior surface of the crystalline humor. The latter surface may be plane or spherical, so that the line passing through the middle of the pupil and the center of the eye would be perpendicular to it. (See Figure I.[11]) The theory of vision is itself expounded in chapters 2, 4, 6, and 8.

Observations (such as the feeling of pain in the eye when gazing on an intense light, or the lingering impression in the eye of a strongly illuminated object) show that it is a property of light to make an effect on the eye, and a property of sight to be affected by light; visual sensation is therefore appropriately explained solely in terms of light coming to the eye from the object. As maintained by natural philosophers this effect is produced by the forms of the light and color in the visible object. But as an explanation of vision this statement in terms of forms is, by itself, "null and void" (*tantaqiḍ wa-tabṭul, destruitur*).[12]

The problem Ibn al-Haytham posed for himself was to determine what further conditions are needed in order to bring the form of an external object intact into the eye where it makes its visual effect. His solution assumed the crystalline humor to be the organ in which visual sensation first occurs—an assumption which had been current since Galen. The solution also employs the experimentally supported principle which considers the shining object as a collection of points individually radiating their light and color (or the forms of their light and color) rectilinearly in all directions.[13] In consequence of this principle any point on a visible object may be regarded as the origin of a cone of radiation with a base at the portion of the surface of the eye opposite the pupil. Since this holds for all points of the object, there will be spread over the whole of that portion the forms of the light and color of every one of these points.

Further confusion will result after the majority of these forms have been refracted upon their passage through the cornea. Ibn al-Haytham considered that for veridical perception to be possible it must be assumed that vision of any given object point can occur only through a given point on the surface of the eye, and he defined the latter point as that at which the perpendicular from the object point meets the cornea. It follows from the geometry of the eye that forms coming from all points on the object along perpendiculars to the surface of the eye will pass unrefracted through the pupil into the albugineous humor and again strike the anterior surface of the crystalline at right angles. There will then be produced on the crystalline humor a total form whose points will correspond, one-to-one, with all the points on the object, and it is this "distinct" and erect form which the crystalline humor will sense. Because the effective perpendiculars are precisely those that make up the outward extension of the cone having the center of the eye as vertex and the pupil as base (the so-called "radial cone," *makhrūṭ al-shuʿāʿ*), what we have in the end is the geometry of the Euclidean visual-ray theory.

But now the "mathematicians'" rays are strictly mathematical, that is, they are no more than abstract lines along which the light travels toward the eye—which is enough to save the geometrical optics of the ancients. As for the hypothesis that something actually goes out of the eye, it is clearly declared to be "futile and superfluous"—"Exitus ergo radiorum est superfluus et otiosus."[14] It would be absurd, says Ibn al-Haytham, to suppose that a material effluence flowing out of the eye would be capable of filling the visible heavens almost as soon as we lift our eyelids. If such effluence or visual rays are not corporeal, then they would not be capable of sensation and their function would merely be to serve as vehicles for bringing back something else from the object which itself would produce vision in the eye. But since this function is already fulfilled by the transparent medium through which light and color (or their forms) extend, visual rays are no longer of any use. (In the presence of this decisive argument it is curious that the editor of the Latin translation should misinterpret Ibn al-Haytham's remarks about preserving the geometrical property of the mathematicians' rays as an argument in support of the "Platonic" theory of *συναύγεια*, combining the intromission and extramission hypotheses.)[15]

Ibn al-Haytham managed to introduce the form of the visible object into the eye—an achievement which had apparently defeated his predecessors. But it should be noted that the "distinct form" he succeeded in realizing inside the eye is apparent only to the sensitive faculty; it is not a visibly articulate image such as that produced by a pinhole camera. In one place he ascribed the privileged role of the perpendicular rays to their superior strength. But there is another dominant idea. As a transparent body the crystalline humor allows non-perpendicular rays to be refracted into it from all points on its surface; as a sensitive body, however, it is especially concerned with those rays that go through it without suffering refraction. Veridical vision is thus due in the first place to the selective or directional sensitivity of the crystalline humor.

The vitreous humor, whose transparency differs from that of the crystalline, has still another property, namely that of preserving the integrity of the form handed down to it at its common face with the crystalline, where refraction of the effective rays takes place away from the axis of symmetry. The sensitive body (visual spirit), issuing along independent and parallel lines from the brain into the optic nerve, finally receives the form from the vitreous body and channels it back along the same lines to the front of the brain where the process of vision is completed. In the optic chiasma, where corresponding lines of the optic nerves join together, the form from the one eye coincides with that from the other, and from there the two forms proceed to the brain as one.

In book VII Ibn al-Haytham introduced what may be considered a generalization of the theory of vision already set out in book I. The form of his inquiry is the same as before: the determination of the conditions that must be assumed in order to accommodate the results of certain indubitable experiments. The experiments described here at first appear to speak against the earlier theory. A small object placed in the radial cone close to one eye, while the other is shut, does not hide an object point lying behind it on the common line drawn from the center of the eye. This means that the object point must in this case be seen by means of a ray falling obliquely, and therefore refracted, at the surface of the eye. Again, a small object placed outside the radial cone, as when a needle is held close to the corner of one eye, can be seen while the other eye is shut. Since no perpendicular can be drawn from the object in this position to any point in the area cut off from the eye-surface by the radial cone, the object must be seen by refraction.

Briefly stated (and divorced from its rather problematic, though interesting, arguments), the final doctrine intended to take all of these observations into account is that vision of objects within the radial cone is effected both by direct and refracted rays, whereas objects outside the cone are seen only by refraction. Ibn al-Haytham here maintains that sensation of refracted as well as direct forms or rays takes place in the crystalline humor, although (in accordance with the earlier theory) he states that the "sensitive faculty" apprehends them all along perpendiculars drawn from the center of the eye to the objects seen. It is this general doctrine, that whatever we see is seen by refraction,[16] whether or not it is also seen by direct rays, that, according to Ibn al-Haytham, had not been grasped or explained by any writer on optics, ancient or modern.

The main part of Ibn al-Haytham's general theory of light and vision is contained in book I of the *Optics*. In book II he expounded an elaborate theory of cognition, with visual perception as the basis, which was referred to and made use of by fourteenthcentury philosophers including, for example, Ockham,[17] and which has yet to receive sufficient attention from historians of philosophy. Book III deals with binocular vision and with the errors of vision and of recognition. Reflection is the subject of book IV, and here Ibn al-Haytham gave experimental proof of the specular reflection of accidental as well as essential light, a complete formulation of the laws of reflection, and a description of the construction and use of a copper instrument for measuring reflections from plane, spherical, cylindrical, and conical mirrors, whether convex or concave. He gave much attention to the problem of finding the incident ray, given the reflected ray (from any kind of mirror) to a given position of the eye. This is characteristic of the whole of the *Optics*—an eye is always given with respect to which the problems are to be formulated. The investigation of reflection—with special reference to the location of images—is continued in book V where the well-known "problem of Alhazen" is discussed, while book VI deals with the errors of vision due to reflection.

Book VII, which concludes the *Optics*, is devoted to the theory of refraction. Ibn al-Haytham gave considerable space to a detailed description of an improved version of Ptolemy's instrument for measuring refractions, and illustrated its use for the study of air–water, air–glass, and water–glass refractions at plane and spherical surfaces. Rather than report any numerical measurements, as in Ptolemy's tables, he stated the results of his experiments in eight rules which mainly govern the relation between the angle of incidence i (made by the incident ray and the normal to the surface) and the angle of deviation d (*zāwiyat al-inʿiṭāf, angulus refractionis*) contained between the refracted ray and the prolongation of the incident ray into the refracting medium. (This concentration on d rather than the angle of refraction r—which being equal to $i - d$ he called the remaining angle, *al-bātqiya*—was also a feature of Kepler's researches.)

His rules may be expressed as follows. Let d_1, d_2 and r_1, r_2 correspond to i_1, i_2 respectively, and let $i_2 > i_1$. It is asserted that

(1) $d_2 > d_1$;

(2) $d_2 - d_1 < i_2 - i_1$;

(3)

(4) $r_2 > r_1$;

(5) In rare-to-dense refraction. $d < ½\, i$;

(6) In dense-to-rare refraction, $d < ½\, (i + d)$ [$d < ½\, r$];

(7) A denser refractive medium deflects the light more toward the normal; and

(8) A rarer refractive medium deflects the light more away from the normal.

It is to be noted that (2) holds only for rare-to-dense refraction, and (5) and (6) are true only under certain conditions which, however, were implicit in the experiments, as Naẓīf has shown.[18] Concluding that "these are all the ways in which light is refracted into transparent bodies," Ibn al-Haytham does not give the impression that he was seeking a law which he failed to discover; but his "explanation" of refraction certainly forms part of the history of the formulation of the refraction law. The explanation is based on the idea that light is a movement which admits of variable speed (being less in denser bodies) and of analogy with the mechanical behavior of bodies. The analogy had already been suggested in antiquity, but Ibn al-Haytham's elaborate application of the parallelogram method, regarding the incident and refracted movements as consisting of two perpendicular components which can be considered separately, introduced a new element of sophistication. His approach attracted the attention of such later mathematicians as Witelo, Kepler, and Descartes, all of whom employed it, the last in his successful deduction of the sine law.

Minor Optical Works . The extant writings of Ibn al-Haytham include a number of optical works other than the *Optics*, of which some are important, showing Ibn al-Haytham's mathematical and experimental ability at its best, although in scope they fall far short of the *Optics*. The following is a brief description of these works.

The Light of the Moon (III 6). Ibn al-Haytham showed here that if the moon behaved like a mirror, the light it receives from the sun would be reflected to a given point on the earth from a smaller part of its surface than is actually observed. He accordingly argued that the moon sends out its borrowed light in the same manner as a self-luminous source, that is, from every point on its surface in all directions. This is confirmed through the use of an astronomical diopter having a slit of variable length through which various parts of the moon could be viewed from an opposite hole in a screen parallel to the slit. The treatise is a beautiful combination of mathematical deduction and experimental technique. The experiments do not, however, lead to the discovery of a new property, but only serve to prove that the mode of emission from the moon is of the same kind as the already known mode of emission from selfluminous objects. Here, as in the *Optics*, the role of experiment is in contrast to its role in the work of, say, Grimaldi or Newton.

The Halo and the Rainbow (III 8). The subject is not treated in the *Optics*. In this treatise Ibn al-Haytham's explanation of the bow fails, being conceived of solely in terms of reflection from a concave spherical surface formed by the "thick and moist air" or cloud. The treatise did, however, become one of the starting points of Kamāl al-Dīn's more successful researches.

On Spherical Burning Mirrors (III 18). In contrast to the eye-centered researches of the *Optics* the only elements of the problems posed in this treatise (and in III 19) are the luminous source, the mirror, and the point or points in which the rays are assembled. Ibn al-Haytham showed that rays parallel to the axis of the mirror are reflected to a given point on the axis from only one circle on the mirror; his remarks imply a recognition of spherical aberration along the axis.

On Paraboloidal Burning Mirrors (III 19). This refers to Archimedes and Anthemius "and others" as having adopted a combination of spherical mirrors whose reflected rays meet in one point. Drawing ably on the methods of Apollonius, Ibn al-Haytham set out to provide a proof of a fact which, he said, the ancients had recognized but not demonstrated: that rays are reflected to one point from the whole of the concave surface of a paraboloid of revolution.

The Formation of Shadows (III 36). That there were many writings on shadows available to Ibn al-Haytham is clear from his reference here to *aṣḥāb al-aẓlāl* (the authors on shadows). Indeed, a long treatise on shadows by his contemporary al-Bīrunī is extant. Ibn al-Haytham defines darkness as the total absence of light, and shadow as the absence of some light and the presence of another. He made the distinction between umbra and penumbra—calling them *ẓultma* (darkness) or *ẓill maḥḍ* (pure shadow), and ẓill (shadow), respectively.

The Light of the Stars (III 48). This argues that all stars and planets, with the sole exception of the moon, are self-luminous.

Discourse on Light (III 60). Composed after the *Optics*, this treatise outlines the general doctrine of light. Some of its statements have been used in the account given above.

The Burning Sphere (III 77). In this work, written after the *Optics*, Ibn al-Haytham continued his investigations of refraction, but, as in III 18 and III 19, without reference to a seeing eye. He studied the path of parallel rays through a glass sphere, tried to determine the focal length of such a sphere, and pointed out spherical aberration. The treatise was carefully studied by Kamāl al-Din, who utilized it in his account of the path of rays from the sun inside individual rain drops.

The Shape of the Eclipse (III 80). This treatise is of special interest because of what it reveals about Ibn al-Haytham's knowledge of the important subject of the *camera obscura*. The exact Arabic equivalent of that Latin phrase, *al-bayt al-muzlim*, occurs in book I, chapter 3 of the *Optics;*[19] and indeed dark chambers are frequently used in this book for the study of such various properties of light as its rectilinear propagation and the fact that shining bodies radiate their light and color on neighboring objects. But such images as those produced by a pinhole camera are totally absent from the *Optics*. The nearest that Ibn al-Haytham gets to such an image is the passage in which he describes the patches of light cast on the inside wall of a "dark place" by candle flames set up at various points opposite a small aperture that leads into the dark place; the order of the images on the inside wall is the reverse of the order of the candles outside.

The experiment was designed to show that the light from one candle is not mingled with the light from another as a result of their meeting at the aperture, and in general that lights and colors are not affected by crossing one another. Although this passage occurs in book I in the context of the theory of vision,[20] the eye does not in Ibn al-Haytham's explanation act as a pinhole camera and it is expressly denied the role of a lens camera. In the present treatise, however, he approached the question, already posed in the pseudo-Aristotelian *Problemata*, of why the image of a crescent moon, cast through a small circular aperture, appears circular, whereas the same aperture will cast a crescent-shaped image of the partially eclipsed sun. Although his answer is not wholly satisfactory, and although he failed to solve the general problem of the pinhole camera, his attempted explanation of the image of a solar crescent clearly shows that he possessed the principles of the working of the camera. He formulated the condition for obtaining a distinct image of an object through a circular aperture as that when

where m_a, m_s are the diameters of the aperture and of the object respectively, and d_a, d_s the distances of the screen from the aperture and from the object respectively.

Ibn al-Haytham's construction of the crescentshaped image of the partially eclipsed sun can be clearly understood by reference to Figure 2. (Because Ibn al-Haytham's own diagram shows the crescents but not the circles, the figure shown is that constructed by Naẓīf.) It represents the special case in which the two ratios just mentioned are equal. It is assumed that the line joining the centers of the two arcs forming the solar crescent is parallel to the planes of the aperture and the screen, and further that the line joining the center of the sun and the center of the circular hole is perpendicular to the plane of the latter and to the plane of the screen.

The crescents *p q, r*, are inverted images produced

by three double conical solids of light whose vertices are three different points on the aperture, and whose bases are, on the one side, the shining solar crescent, and, on the other, the inverted image. These solids are each limited by two conical surfaces of which one is convex and the other concave; and in every double solid the convex surface on one side of the aperture corresponds to the concave surface on the other. The middle crescent image *q* is produced by such a double solid having its vertex at the aperture-center; *p* and *r* have their vertices at the extremities of a diameter of the aperture. The circular images are each produced by a single cone whose vertex is a single point on the shining crescent; as many such circles are produced as there are points on the crescent sun.

The center of each circle is therefore the point at which the axis of the cone, passing through the center of the aperture, intersects the screen. It is clear that the centers of all circles will be points on crescent *q*, and that their radii, as well as those of the arcs forming crescents *p, q, r*, will all be equal. The resultant image will therefore be bounded from above by a convex curve of which the upper part is the tangential arc of a circle whose center is the midpoint *K* of the convex arc of crescent *p*, and whose radius is twice the radius of that arc. Although circles of light will occur below arc *GTH*; they will be relatively few.

The sensible overall effect will be, according to Ibn al-Haytham, a crescent-shaped image bordered on the lower side by a sensibly dark cavity. He showed by a numerical example that the cavity will increase or decrease in size according as the ratio $m_a{:}m_s$ is less or greater than $d_a{:}d_s$ It is certain that the treatise *On the Shape of the Eclipse* was composed after the Optics, to which it refers. It is not impossible that, at the time of writing the Optics, Ibn al-Haytham was acquainted with the remarkable explanation revealed in the later work, but of this we have no evidence.

Transmission and Influence of the Optics . of all the optical treatises of Ibn al-Haytham that have been mentioned, only the *Optics* (III 3) and the treatise *On Paraboloidal Burning Mirrors* (III 9) are known to have been translated into Latin in the Middle Ages, the latter probably by Gerard of Cremona.[21] It is remarkable that in the Islamic world the *Optics* practically disappeared from view soon after its appearance in the eleventh century until, in the beginning of the fourteenth century, the Persian scholar Kamāl al-Dīn composed his great critical commentary on it, the *Tanaīh al-Manāzir,* at the suggestion of his teacher Qutb al-Dīn al-Shīrazī.

By this time the *Optics* had embarked on a new career in the West where it was already widely and avidly studied in a Latin translation of the late twelfth or early thirteenth century, entitled *Perspectiva* or *De aspectibus*. of the manuscript copies that have been located (no fewer than nineteen[22]), the earliest are from the thirteenth century; but where and by whom the *Optics* was translated remains unknown. The Latin text was published by Frederick Risner at Basel in 1572 in a volume entitled Opticae thesaurus, which included Witelo's *Perspectiva*. In both Risner's edition and the Latin manuscripts examined by the present writer (see bibliography) the Latin text wants the first three chapters of book I of the Arabic text (133 pages, containing about 130 words per page, in MS Fatih 3212).

The Latin *Perspectiva* shows the drawbacks as well as the advantages of the literal translation which in general it is. Often, however, it only paraphrases the Arabic, sometimes inadequately or even misleadingly, and at times it omits whole passages. But an exhaustive and critical study of the extant manuscripts is needed before a full and accurate evaluation of the translation can be made. In any case there is no doubt that through this Latin medium a good deal of the substance of Ibn al-Haytham's doctrine was successfully conveyed to medieval, Renaissance, and seventeenth-century philosophers in the West. Roger Bacon's *Perspectiva* is full of references to "Alhazen," or auctor perspectivae, whose influence on him cannot be overemphasized. Pecham's *Perspectiva communis* was composed as a compendium of the *Optics* of Ibn al-Haytham.[23] That Witelo's *Opticae libri decem* also depends heavily on *Alhazeni libri septem* has been noted repeatedly by scholars; the cross-references provided by Risner in his edition of the two texts have served as a sufficient indication of that. But Witelo's precise debt to Ibn al-Haytham, as distinguished from his own contribution, has yet to be determined.

The influence of Ibn al-Haytham's *Optics* was not channelled exclusively through the works of these thirteenth-century writers. There is clear evidence that the book was directly studied by philosophers of the fourteenth century[24] and an Italian translation made at that time was used by Lorenzo Ghiberti.[25] Risner's Latin edition made it available to such mathematicians as Kepler, Snell, Beeckman, Fermat, Harriot, and Descartes, all of whom except the last directly referred to Alhazen. It was, in fact, in the sixteenth and seventeenth centuries that the mathematical character of the *Optics* was widely and effectively appreciated.

Astronomy . No fewer than twenty of Ibn al-Haytham's extant works are devoted to astronomical questions. The few of these that have been studied by modern scholars do not appear to justify al-Bayhaqī's description of Ibn al-Haytham as "the second Ptolemy." (The description would be apt, however, if al-Bayhaqī had optics in mind.) Many of these works are short tracts that deal with minor or limited, although by no means trivial, theoretical or practical problems (sundials, determination of the direction of prayer, parallax, and height of stars), and none of them seems to have achieved results comparable to those of, say, Ibn Yūnus, al-Ṭūsī, or Ibn al-Shāṭir. Nevertheless, some of Ibn al-Haytham's contributions in this field are both interesting and historically important, as has sometimes been recognized.

As a writer on astronomy Ibn al-Haytham has been mainly known as the author of a treatise *On the Configuration of the World* (III = I*b* 10). The treatise must have been an early work: it speaks of "the ray that goes out of our eyes" and describes the moon as a "polished body" which "reflects" the light of the sun-two doctrines which are refuted in the Optics (III 3) and in *The Light of the Moon* (III 6), respectively. The treatise was widely known in the Islamic world,[26] and it is the only astronomical work of Ibn al-Haytham to have been transmitted to the West in the Middle Ages. A Spanish translation was made by Abraham Hebraeus for Alfonso X of Castile (*d*. 1284), and this translation was turned into Latin (under the title *Liber de mundo et coelo*) by an unknown person.

Jacob ben Maḥir (Prophatius Judaeus, *d*. *ca*. 1304) translated the Arabic text of the *Configuration* into Hebrew, a task which was suggested to him as a corrective to the *Elements of Astronomy* of al-Farghānī, whose treatment of the subject "did not accord with the nature of existing things," as the unknown person who made the suggestion said.[27] The physician Salomo ibn Pater made another Hebrew translation in 1322. A second Latin version was later made from Jacob's Hebrew by Abraham de Balmes for Cardinal Grimani (both of whom died in 1523). In the fourteenth century Ibn al-Haytham's treatise was cited by Levy ben Gerson. Its influence on early Renaissance astronomers and in particular on Peurbach's *Theoricae novae planetarum* has recently been pointed out.[28]

The declared aim of the *Configuration* was to perform a task which, in Ibn al-Haytham's view, had not been fulfilled either by the popularly descriptive or the technically mathematical works on astronomy. The existing descriptive accounts were only superficially in agreement with the details established by demonstrations and observations. A purely mathematical work like the *Almagest*, on the other hand, explained the laws (*qawānīn*) of celestial motions in terms of imaginary points moving on imaginary circles. It was necessary to provide an account that was faithful to mathematical theory while at the same time showing how the motions were brought about by the physical bodies in which the abstract points and circles must be assumed to exist. Such an account would be "more truly descriptive of the existing state of affairs and more obvious to the understanding."[29]

Ibn al-Haytham's aim here was not therefore to question any part of the theory of the *Almagest* but, following a tradition which goes back to Aristotle and which had been given authority among astronomers by one of Ptolemy's own works, the *Planetary Hypotheses*, to discover the physical reality underlying the abstract theory. The description had to satisfy certain principles already accepted in that tradition: a celestial body can have only circular, uniform, and permanent movement; a natural body cannot by itself have more than one natural movement; the body of the heavens is impassable; the void does not exist. Ibn al-Haytham's procedure was then, for every simple motion assumed in the *Almagest*, to assign a single spherical body to which this motion permanently belongs, and to show how the various bodies may continue to move without in any way impeding one another or creating gaps as they moved.

The heavens were accordingly conceived of as consisting of a series of concentric spherical shells (called spheres) which touched and rotated within one another. Inside the thickness of each shell representing the sphere of a planet other concentric and eccentric shells and whole spheres corresponded to concentric and eccentric circles and epicycles respectively. All shells and spheres rotated in their own places about their own centers, and their movements combined to produce the apparent motion of the planet assumed to be embedded in the epicyclic sphere at its equator. In his careful description of all movements involved Ibn al-Haytham provided, in fact, a full, clear, and untechnical account of Ptolemaic planetary theory—which alone may explain the popularity of his treatise.

A brief look at Ibn al-Haytham's other works will give us an idea of how seriously he took the program he inherited and of its significance for the later history of Islamic astronomy. Perhaps at some time after writing the *Configuration of the World* (III 1 = I*b* 10) Ibn al-Haytham composed a treatise (III 61) on what he called the movement of *ilifāf*, that is the movement or rather change in the obliquities (singular, *mayl*) of epicycles responsible for the latitudinal variations of the five planets (*Almagest*, XIII.2). This treatise is not known to have survived. But we have Ibn al-Haytham's reply to a criticism of it by an unnamed scholar. From this reply, the tract called *Solution of the Difficulties* [*shukūk*] *Concerning the Movement of Iltifāf* (III 63), we learn that in the earlier treatise he proposed a physical arrangement designed to produce the oscillations of epicycles required by the mathematical theory. The same subject is discussed, among other topics, in the work entitled *Al-Shukūk ʿalā Baṭlamyūs* (*Dubitationes in Ptolemaeum*) (III 64). More than any other of Ibn al-Haytham's writings, this work (almost certainly composed after the reply just mentioned) reveals the far-reaching consequences of the physical program to which he was committed.

The *Dubitationes* is a critique of three of Ptolemy's works: the *Almagest*, the *Planetary Hypotheses* and the *Optics*. As far as the first two works are concerned, the criticism is mainly aimed against the purely abstract character of the *Almagest* (this exclusiveness being in Ibn al-Haytham's view a violation of the principles accepted by Ptolemy himself) and against the fact that the *Planetary Hypotheses* had left out many of the motions demanded in the *Almagest* (a proof that Ptolemy had failed to discover the true arrangement of the heavenly bodies).

Ibn al-Haytham's objection to the "fifth motion" of the moon, described in *Almagest* V.5, is particularly instructive, being nothing short of a *reductio ad absurdum* by "showing" that such a motion would be physically impossible. Ptolemy had assumed that as the moon's epicycle moves on its eccentric deferent, the diameter through the epicycle's apogee (when the epicycle-center is at the deferent's apogee) rotates in such a way as to be always directed to a point on the apse-line (called the opposite point, *nuqṭat al-muḥādhāt*), such that the ecliptic-center lies halfway between that point and the deferent-center. The assumption implied that the epicycle's diameter alternately rotates in opposite senses as the epicycle itself completes one revolution on its deferent. But, Ibn al-Haytham argued, such a movement would have to be produced either by a single sphere which would alternately turn in opposite senses, or by two spheres of which one would be idle while the other turned in the appropriate sense. "As it is not possible to assume a body of this description, it is impossible that the diameter of the epicycle should be directed towards the given point."[30] Whatever one thinks of the argument, the problem it raised was later fruitfully explored by Naṣīr al-Dīn al-Ṭūsī in the *Tadhkira*.[31]

Perhaps most important historically was Ibn al-Haytham's objection against the theory of the five planets, and in particular against the device introduced by Ptolemy which later came to be known as the equant. Ptolemy supposed that the point from which the planet's epicycle would appear to move uniformly is neither the center of the eccentric deferent nor that of the ecliptic, but another point (the equant) on the line of apsides as far removed from the deferent-center as the latter is from the eclipticcenter. This entailed, as Ibn al-Haytham pointed out, that the motion of the epicycle-center, as measured on the circumference of its deferent, was not uniform, and consequently that the deferent sphere carrying the epicycle was not moving uniformly—in contradiction to the assumed principle of uniformity.

Although the equant had succeeded in bringing Ptolemy's planetary theory closer to observations, the validity of this criticism remained as long as the principle of uniform circular motion was adhered to. To say that the equant functioned merely as an abstract calculatory device designed for the sake of saving the phenomena was an answer which satisfied none of Ptolemy's critics, down to and including Copernicus. Nor was Ptolemy himself unaware of the objectionable character of such devices. In the *Dubitationes*, Ibn al-Haytham points to a passage in *Almagest* IX. 2 where Ptolemy asks to be excused for having employed procedures which, he admitted, were against the rules (*παρὰ τὸν λόγον, khārij ʿan al-giyās*), as, for example, when for convenience's sake he made use merely of circles described in the planetary spheres, or when he laid down principles whose foundation was not evident. For, Ptolemy said, "when something is laid down without proof and is found to be in accord with the phenomena, then it cannot have been discovered without a method of science [*sabīl min al-ʿilm*], even though the manner in which it has been attained would be difficult to describe."[32]

Ibn al-Haytham agreed that it was indeed appropriate to argue from unproved assumptions, but not when they violated the admitted principles. His final conclusion was that there existed a true configuration of the heavens which Ptolemy had failed to discover.

It has been customary to contrast the "physical" approach of Ibn al-Haytham with the "abstract" approach of mathematical astronomers. The contrast is misleading if it is taken to imply the existence of two groups of researchers with different concerns. The "mathematical" researches of the school of Marāgha (among them al-Ṭūsī and al-Shīrazī) were motivated by the same kind of considerations as those revealed in Ibn al-Haytham's *Dubitationes*.[33] Al-Ṭūsī, for instance, was as much worried about the moon's "fifth movement" and about the equant as was Ibn al-Haytham, and for the same reasons.[34]*His Tadhkira* states clearly that astronomical science is based on physical as well as mathematical premises. From a reference in it to Ibn al-Haytham,[35] made in the course of expounding alterations based on what is now known as the "Ṭūsiī couple," it is clear that al-Ṭūsī recognized the validity of Ibn al-Haytham's physical program, although not the particular solutions offered by his predecessor.

The longest of the astronomical works of Ibn al-Haytham that have come down to us is a commentary on the *Almagest*. The incomplete text in the unique Istanbul manuscript which has recently been discovered occupies 244 pages of about 230 words each (see bibliography, additional works, no. 3). The manuscript, copied in 655 a.h./a.d. 1257, bears no title but twice states the author's name as Muhammad ibn al-Ḥasan ibn al-Haytham, the name found by Ibn Abī Uṣaybiʿa in Ibn al-Haytham's own bibliographies, that is lists I and II. No title in list III seems to correspond to this work, but there are candidates in the other lists. The first title in Ibn al-Qifṭī's list is *Tahdhīb al-Majisṭī or Expurgation of the Almagest*. Number 19 in list II is described as "A book which works out the practical part of the *Almagest*." And number 3 in list la begins as follows: "A commentary and summary of the *Almagest*, with demonstrations, in which I worked out only a few of the matters requiring computation...." The last title is highly appropriate to the work that has survived.

Most commentators on the *Almagest*, Ibn al-Haytham says in the introduction, were more interested in proposing alternative techniques of computation than in clarifying obscure points for the beginner. As an example he mentions al-Nayrīzī who "crammed his book with a multiplicity of computational methods, thereby seeking to aggrandize it." Ibn al-Haytham sought rather to explain basic matters relating to the construction of Ptolemy's own tables, and he meant his commentary to be read in conjunction with the *Almagest*, whose terminology and order of topics it followed. The book was therefore to comprise thirteen parts, but, for brevity's sake and also because the *Almagest* was "well known and available," Ibn al-Haytham would not follow the commentators' customary practice of reproducing Ptolemy's own text. Unfortunately the manuscript breaks off before the end of the fifth part, shortly after the discussion of Ptolemy's theories for the sun and the moon. In the course of additions designed to complete, clarify, or improve Ptolemy's arguments, Ibn al-Haytham referred to earlier Islamic writers on astronomy, including Thābit ibn Qurra (on the "secant figure"), Banū Mūsā (on the sphere), and Ibrāhīm ibn Sinān (on gnomon shadows). All diagrams have been provided and are clearly drawn in the manuscript but the copyist has not filled in the tables.

Mathematics . Ibn al-Haytham's fame as a mathematician has rested on his treatment of the problem known since the seventeenth century as "Alhazen's problem." The problem, as viewed by him, can be expressed as follows: from any two points opposite a reflecting surface—which may be plane, spherical, cylindrical, or the surface of a cone, whether convex or concave—to find the point (or points) on the surface at which the light from one of the two points will be reflected to the other. Ptolemy, in his *Optics*, had shown that for convex spherical mirrors there exists a unique point of reflection. He also considered certain cases relating to concave spherical mirrors, including those in which the two given points coincide with the center of the specular sphere; the two points lie on the diameter of the sphere and at equal or unequal distances from its center; and the two points are on a chord of the sphere and at equal distances from the center. He further cited some cases in which reflection is impossible.[36]

In book V of his *Optics*, Ibn al-Haytham set out to solve the problem for all cases of spherical, cylindrical, and conical surfaces, convex and concave. Although he was not successful in every particular, his performance, which showed him to be in full command of the higher mathematics of the Greeks, has rightly won the admiration of later mathematicians and historians. Certain difficulties have faced students of this problem in the work of Ibn al-Haytham. In the Fatih manuscript, and in the Aya Sofya manuscript which is copied from it, the text of book V of the *Optics* suffers from many scribal errors, and in neither of these manuscripts are the lengthy demonstrations supplied with illustrative diagrams.[37] Such diagrams exist in Kamāl al-Dīn's commentary and in Risner's edition of the medieval Latin translation, but neither the diagrams nor the texts of these two editions are free from mistakes. One cannot, therefore, be too grateful to M. Naẓīf for his clear and thorough analysis of this problem, to which he devotes four chapters of his masterly book on Ibn al-Haytham.

Ibn al-Haytham bases his solution of the general problem on six geometrical lemmas (*muqaddamāt*) which he proves separately: (1) from a given point *A* on a circle *ABG*, to draw a line that cuts the circumference in *H* and the diameter *BG* in a point *D* whose distance from *H* equals a given line; (2) from the given point *A* to draw a line that cuts the diameter *BG* in a point E and the circumference in a point *D* such that *ED* equals the given line; (3) from a given point *D* on the side *BG* of a right-angled triangle having the angle *B* right, to draw a line *DTK* that cuts *AG* in *T* (and the extension of *BA* in *K*), such that *KT*:*TG* equals a given ratio; (4) from two points *E,D* outside a given circle *AB*, to draw two lines *EA* and *DA*, where *A* is a point on the circumference, such that the tangent at A equally divides the angle *EAD*; (5) from a point outside a circle having *AB* as diameter and *G* as center, to draw a line that cuts the circumference at *D* and the diameter at *Z* such that *DZ* equals *ZG*; and (6) from a given point *D* on the side *GB* of a right-angled triangle having the angle *B* right, to draw a line that meets the hypotenuse *AG* at *K* and the extension of *AB* on the side of *B* at *T*, such that *TK:KG* equals a given ratio.[38]

Obviously lemmas (1) and (2) are special cases of one and the same problem, and (3) and (6) are similarly related. In his exposition of Ibn al-Haytham's arguments Naẓīf combines each of these two pairs in one construction. It will be useful to reproduce here his construction for (1) and (2) and to follow him in explaining Ibn al-Haytham's procedure by referring to this construction. It happens that (1) and (2) contain characteristic features of the proposed solution of the geometrical problem involved. In Figure 3, *A* is a given point on the circumference of the small circle with diameter *BG*. It is required to draw a straight line from *A* that cuts the circle at *D* and the diameter, or its extension, at *E*, such that *DE* equals the given segment *z*.

From *G* draw the line *GH* parallel to *AB*; let it cut the circle at *H*; join *BH*. Let the extensions of *AG, AB* respectively represent the coordinate axes *x*. *y* whose origin thus coincides with *A*. Draw the hyperbola passing through *H* with, *x, y* as asymptotes. Then, with *H* as center, draw the circle with radius

(*HS* being the side of a rectangle whose other side is *z*, and whose area equals BG^2). The circle will, in the general case, cut the two branches of the hyperbola at four points, such as *S*. *T, U, V*. Join *H* with all four points, and from *A* draw the lines parallel to *HS, HT, HU*, and *HV*. Each of these parallels will cut the circle circumscribing the triangle *ABG* at a point, such as *D*, and the diameter, or its extension, at another point, such as *E*. It is proved that each of these lines satisfies the stated condition.

As distinguished from the above demonstration, Ibn al-Haytham proceeded by considering three cases one after the other: (a) the required line is tangential to the circle, that is, *A* and *D* coincide; (b) *D* is on the arc *AG*; (c) *D* is on the are *AB*. Despite the generality of the enunciation of lemma (1), he does not consider the case in which the line cuts the extension

of *BG* on the side of *B*. Similarly in dealing with lemma (2) he separately examines three possibilities in respect of the relation of the circle *HS* to the "opposite branch" of the hyperbola: (a) the circle cuts that branch at two points; (b) the circle is tangential to it at one point; (c) the circle falls short of it. For finding the shortest line between *H* and the "opposite branch" of the hyperbola he refers to Apollonius' *Conics*, V. 34. Ibn al-Haytham did not, of course, speak of a coordinate system of perpendicular axes whose origin he took to be the same as the given point *A*. He did, however, consider a rectangle similar to *ABHG*, and described the sides of it corresponding to *AB*, *AG* as asymptotic to the hyperbola he drew through a point corresponding to *H* in Figure 3. For drawing this hyperbola he referred to *Conics*, II. 4.

Applying the six geometrical lemmas for finding the points of reflection for the various kinds of surface, Ibn al-Haytham again proceeded by examining particular cases in succession. Naẓīf shows that the various cases comprised by lemma 4 constitute a general solution of the problem in respect of spherical surfaces, concave as well as convex. With regard to cylindrical mirrors, Ibn al-Haytham considered the cases in which (a) the two given points are in the plane perpendicular to the axis; and (c) the general case in which the intersection of the plane containing the two points with the cylinder is neither a straight line nor a circle, but an ellipse. He described six different cases in an attempt to show that reflection from convex conical surfaces can take place from only one point, which he determined. For concave conical mirrors, he showed that reflection can be from any number of points up to but not exceeding four. And he argued for the same number of points for concave cylindrical mirrors.

Apart from the mathematical sections of the *Optics*, some twenty of the writings of Ibn al-Haytham which deal exclusively with mathematical topics have come down to us. Most of these writings are short and they vary considerably in importance. About a quarter of them have been printed in the original Arabic and about half of them are available in European translations or paraphrases. Some of the more important among these works fall into groups and will be described as such.

List III includes three works (III 39, III 55, and III 56) which are described as solutions of diffiāculties arising in three different parts of Euclid's *Elements*. There are no manuscripts exactly answering to these descriptions. There exist, on the other hand, several manuscripts of a large work entitled *Solution of the Difficulties in Euclid's Elements*, which does not appear in list III. It therefore seems likely that III 39, III 55, and III 56 are parts of the larger work which is listed below in the bibliography as additional work no. 1.

The object of the *Solution* was to put into effect a rather ambitious program. Unlike earlier works by other writers, it proposed to deal with all or most of the difficulties occasioned by Euclid's book, and not with just a few of them; it examined particular cases and offered alternative constructions for many problems; it revealed the "remote mathematical causes" (*al-ʿilal al-taʿlīmiyya al-baʿīda*) of the theoretical propositions (*al-ashkāl al-ʿilmiyya*)—something which "none of the ancients or the moderns had previously mentioned"; and, finally, it replaced Euclid's indirect proofs with direct ones. In this book Ibn al-Haytham referred to an earlier *Commentary on the Premises of Euclid's Elements* (III 2), and said that he meant the two works to form together a complete commentary on the whole of the *Elements*. This earlier work, restricted to the definitions, axioms, and postulates of the *Elements*, is extant both in the Arabic original and in a Hebrew translation made in 1270 by Moses ibn Tibbon. Ibn al-Haytham's interesting treatment of Euclid's theory of parallels well illustrates his approach in these two "commentaries."

In III 2 Ibn al-Haytham ascribed to Euclid the "axiom" that "two straight lines do not enclose a space [*saṭḥ*]" (his own opinion is that the statement should be counted among the "postulates"). Concerning Euclid's definition of parallel lines as nonsecant lines he remarked that the "existence" of such lines should be proved and, for this purpose, introduced the following "more evident" postulate: if a straight line so moves that the one end always touches a second straight line, and throughout this motion remains perpendicular to the second and in the same plane with it, then the other end of the moving line will describe a straight line which is parallel to the second. Ibn al-Haytham thus replaced parallelism in Euclid's sense by the property of equidistance, a procedure which had originated with the Greeks and which had characterized many Islamic attempts to prove Euclid's postulate 5.

Like Thabit ibn Qurra before him Ibn al-Haytham based his proof on the concept of motion—which procedure al-Khayyāmī and, later, al-Ṭūsī found objectionable as being foreign to geometry. The crucial step in the deduction of Euclid's postulate is the demonstration of Saccheri's "hypothesis of the right angle" by reference to a "Saccheri quadrilateral." Let *AG*, *BD* be drawn at right angles to *AB* (Figure 4): it is to be proved that perpendiculars to *BD* from points on *AG* are equal to *AB*, and, consequently, perpendicular to *AG*. From any point *G* draw *GD* perpendicular to *BD*; produce *GA* to such that *AE* equals *AG*; draw –*ET* perpendicular to *DB* produced; and join *BG*, *BE*. Considering, first, triangles *ABG*, *ABE*, then triangles *BDG*, *BTE*, it is seen that *GD* equals *ET*. Let *GD* now move along *DBT*, the angle *GDT* being always right. Then, when *D* coincides with *B*, *G* will either coincide with *A*, or fall below it on *AB*, or above it (occupying the position of *H* in the figure) on *BA* produced, according as *GD* is assumed equal to, less than, or greater than *AB*. When *D* reaches *T*, *GD* will exactly coincide with *ET*. During this motion, *G* will have described a straight line which, on the hypothesis that *DG* is not equal to *AB*, would enclose an area, such as *GHEA*, with another straight line, *GAE*—which is impossible. Finally, by considering in turn triangles *BDG*, *BDA*, and *AKB*, *GKD*, it is clear that *DGA* = *BAG* = a right angle. The Euclidean postulate follows as a necessary consequence.

In the larger commentary, Ibn al-Haytham reformulated

postulate 5, stating that two intersecting straight lines cannot both be parallel to a third ("Playfair's axiom"), and referring to the proof set forth in the earlier, and shorter, work. It is to be noted that al-Ṭūsī's criticism (in his own work on the theory of parallels, *Al-Risāla al-Shāfiya*) of Ibn al-Haytham's attempt was based on the remarks in this larger commentary, not on the earlier proof, which al-Ṭūsī said was not available to him.[39]

Ibn al-Haytham wrote two treatises on the quadrature of crescent-shaped figures (*al-ashkāl al-hilā-liyya*) or lunes. (Their titles have sometimes been misunderstood as referring to the moon.) The second, and fuller, treatise (III 21), although extant in several manuscripts, has not been studied. From the introduction we gather that it was composed quite some time after the first (III 20, now lost), although the two works appear consecutively in list III. The treatise comprises twenty-three propositions on lunes, of which some are generalizations of particular cases already proved in the earlier treatise, as the author tells us, while others are said to be entirely new. The subject was connected with that of squaring the circle: if plane figures bounded by two unequal circular arcs could be squared, why not the simpler figure of a circle? Ibn al-Haytham put forward such an argument in a short tract on the *Quadrature of the Circle* (III 30), which has been published. The object of the tract is to prove the "possibility" of squaring the circle without showing how to "find" or construct a square equal in area to a given circle.

To illustrate his point, Ibn al-Haytham proves a generalization of a theorem ascribed to Hippocrates of Chios. The proof is reproduced from his earlier work on lunes. In Figure 5, let *B* be any point on the semicircle with diameter *AG*; describe the smaller semicircles with *AB*, *BG* as diameters; it is shown that the lunes *AEBH*, *BZGT* are together equal in area to the right-angled triangle *ABG*. On the basis of Euclid XII.2, which states that circles are to one another as the squares on the diameters, it is easily proved that the semicircles on *AB*, *BG* are together equal to the semicircle on the hypotenuse AG. The equality of the lunes to the triangle *ABG* follows from subtracting the segments *AHB*, *BTG* from both sides of the equation. Hippocrates had considered the particular case in which the triangle *ABG* is isosceles.[40]

Two more works which are closely related are *On Analysis and Synthesis* (*Maqāla fi 'l-taḥlīl wa 'l-larkīb*, III 53) and *On the Known Things* (*Maqāla fi 'l-Maʿlū-māt*,. III 54). The subject matter of the latter work overlaps with that of Euclid's *Data*, which is called in Arabic *Kitāb al-Mu'ṭayāt*, (*Δεδομένα*). Ibn al-Haytham's use of *al-maʿlūmāt*, rather than *al-muʿṭayāt,*

has a precedent in the Arabic translation of Euclid's book itself, where *al-maʿlūm* (the known) is regularly employed to denote the given. *On Analysis* is a substantial work of about 24,000 words whose object is to explain the methods of analysis and synthesis, necessary for the discovery and proof of theorems and constructions, by illustrating their application to each of the four mathematical disciplines: arithmetic, geometry, astronomy, and music. It lays particular emphasis on the role of "scientific intuition" (*al-ḥads al-ṣināʿī*), when properties other than those expressly stated in the proposition to be proved have to be conjectured before the process of analysis can begin.

In describing the relationship of this treatise to the one on *The Known Things* Ibn al-Haytham made certain claims which should be quoted here. The art of analysis, he says, is not complete without the things that are said to be known.

Now the known things are of five kinds: the known in number, the known in magnitude, the known in ratio, the known in position, and the known in species [*al-maʿlū al-ṣūra*]. The book of Euclid called *Al-Muʿṭayāt* includes many of these known things which are the instruments of the art of analysis, and on which the larger part of analysis is based. But that book does not include other known things that are indispensable to the art of analysis... nor have we found them in any other book. In the examples of analysis we give in the present treatise we shall prove the known things used, whether or not we have found them in other works.... After we have completed this treatise we shall resume the subject in a separate treatise in which we shall show the essence of the known things that are used in mathematics and give an account of all their kinds and of all that relates to them.[41]

The treatise on known things, which is extant, divides in fact into two parts, of which the first (comprising twenty-four propositions) is said to be the invention of Ibn al-Haytham himself. In 1834 L. Sédillot published a paraphrase of the introduction to this work (a discussion of the concept of knowledge) together with a translation of the enunciations of the propositions constituting both parts. There is no study of the work on *Analysis and Synthesis*. The more important of the remaining mathematical works are all available in European translations.

NOTES

1. On al-Fāsī, see Ibn al-Qifṭī, Ta'rīkh, pp. 392–394.

2. *Al-Mutawassiṭāt*, or intermediate books, so called because they were studied after the *Elements* of Euclid and before the *Almagest*, They included, for instance, Euclid's *Data*. Theo-dosius *Spherics*, and the *Spherics* of Menelaus. See the explanation of Abu'l-Ḥasan al-Nasawī in al-Ṭūsī, *Majmūʿ al-Rasā'il*, II (Hyderabad, 1359 a.h. [1940]), *risāla* no. 3, p. 2. The existence of a copy of Apollonius' *Conics* in Ibn al-Haytham's hand (MS Aya Sofya 2762, 307 fob., dated Safar 415 a.h. [1024]) may be taken to confirm the story that he lived on selling copies of scientific texts, although the *Conics* is not one of the books mentioned in the story.

3. The expression "fi hudūd" could also mean "about" or "toward the end of."

4. On Bayhaqī"s dates see the article devoted to him in *Encyclopaedia of Islam*, 2nd ed,

5. On Qayṣar see A. I. Sabra, "Simplicius's Proof of Euclid's Parallels Postulate," in *Journal of the Warburg and Courtauld Institutes*, **32** (1969), 8.

6. The title is "[Ibn al-Haytham's] Answer to a Geometrical Question Addressed to Him in Baghdad [*su'ita 'anhā bi-Bagh-dād*] in the Months of the Year Four Hundred and Eighteen."

7. See Galen's *Opera omnia*, C. G. Kühn, ed., XIX (repr. Hildesheim, 1965), 8–61; and F. Rosenthal, "Die arabische Autobiographie," pp. 7–8. Galen's *De libris propriis* was translated into Arabic by Ḥunayn ibn Isḥāq in the ninth century.

8. See Galen's *Opera omnia, ed. cit.*, X (repr. Hildesheim, 1965), 457, II. 11–15.

9. In the context of Arabic optics manāẓir is the plural, not of *manẓar* (view, appearance) but of *manẓara*, that by means of which vision is effected, an instrument of vision. One evidence for this is Ḥunayn ibn Isḥāq's Arabic translation of Galen's *De usu partium*, where *manẓara* and *manāẓir* correspond to *ὄψls* and *ὄψεls*, respectively (see Escorial MS 850, fol. 29v). *Al-Manāẓir* had been used as the Arabic title of Euclid's (and Ptolemy's) 'Oπttkα.

10. Newton, *Opticks*, bk. 1, pt. 1, def. I. See A. 1, Sabra, *Theories of Light*, pp. 285–289, 310–311, n. 25.

11. The diagram of the eye in Risner's ed. of the Latin text is taken from Vesalius' *De corporis humani fabrica* (D. Lindberg, "Alhazen's Theory of Vision... ." p. 327, n. 30). The diagram in MS Fatih 3212 of *Kitāb al-Manāẓir* bk. 1, does not clearly and correctly represent Ibn al-Haytham's descriptions; it can be seen in S. Polyak, *The Retina* (Chicago, 1957), and in G. Nebbia, "Ibn al-Haytham nel millesimo anniversario della nascita," p. 204.

12. MS Fatih 3212, fol. 83r; *Opticae thesaurus, Alhazeni libri VII*, p. 7, sec. 14, 1. 26.

13. The importance of this principle and of its application by Ibn al-Haytham to the problem of vision has been rightly emphasized by Vasco Ronchi in his *Storia delta luce*, 2nd ed, (Bologna. 1952), pp. 33–47, trans. into English as *The Nature of Light* (London, 1970), pp. 40–57.

14. *Opticae thesaurus*. *Alhazeni libri VII*, p. 14, sec. 23, I. 20.

15. *Ibid*., p. 15, sec. 24: "Visio videtur fieri per συναύγειαν, id est receptos simul et emissos radios."

16. At least some of the Latin MSS have "reflexe." Risner's text, however, correctly reads "refracte" (*bi ʾl-inʿitāf*). See Vescovini, *Studi*, p. 93, n. 10,

17. Vescovini, *Studi*, p. 141.

18. See M. Naẓīf. *Al-Ḥasan ibn al-Haytham*, pp. 709–721. Unlike the Arabic, the Latin text in Risner's ed. expresses rule (5) as $d < i$ (*Alhazeni libri VII,*, p. 247, 11. 8–11). Both the Arabic MSS and Risner's ed. omit the words "less than" from rule (6), and consequently express this rule as $d = 1.2\ (i + d)$[!]. The correction has been made by Naẓīf and is supported by Kamāl al-Dīn's formulation of the rule on the basis of Ibn al-Haytham's autograph (*see Tanqīḥ*, I, 7; II, 134, II. 10–11).

19. The earliest occurrence of "al-bayt al-muzlim" is in a ninthcentury tract on burning mirrors by ʿUṭārid ibn Muḥammad al-Ḥāsib: Istanbul MS Laleli 2759, fols. 1–20. The tract is based on earlier Greek works including at least one by Anthemius of Tralles, and the term may therefore have been derived from them. See M. Schramm, "Ibn al-Haythams Stellung in der Geschichte der Wissenschaften," pp. 15–16.

20. Bk. 1, ch. 5, sec. 29, p. 17 in Risner's ed. of the Latin text. The discussion continues in the Arabic for the greater part of two pages to which nothing corresponds in Risner's text.

21. M. Clagett, "A Medieval Latin Translation of a Short Arabic Tract on the Hyperbola," in *Osiris*, **11** (1954). 361.

22. D. Lindberg, *Pecham and the Science of Optics* (Madison, Wis., 1970), p. 29, n. 69. (See bibliography, "Original Works," no. III 3.)

23. *Ibid*., p. 20.

24. Vescovini, *Studi*. pp. 137 ff.

25. See especially Vescovini, "Contributo per la storia della fortuna di Alhazen in Italia" (in the Bibliography).

26. W. Hartner, "The Mercury Horoscope...," esp. pp. 122–124.

27. M. Steinschneider, "Notice...," p. 723.

28. Hartner, *op. cit*. pp. 124, 127 If.

29. MS India Office, Loth 734, fol. 101r.

30. *Dubitationes* (III 64), *ed. cit*., p. 19.

31. W. Hartner, "Nasīr al-Dīn al-Ṭūsīs Lunar Theory," in *Physis*. **11** (1969), 287–304.

32. *Dubitationes* (III 64). *ed. cit*., p. 39; also p. 33. The English trans. is of Isḥāq's Arabic version quoted by Ibn al-Haytham. The Greek differs only slightly from the Arabic: "... οϋτε τά άναποδείκτως ύποτιθέμενα, έάν άπαξ σύμφωνα τοίς φαινομένοις καταλαμβάνται, χωρίς όδού τινος καί έπιστάσεως εύρήσθαι δύναται, κάν δυσέκθετος ήό τρόπος αύτών τής καταλήψεως (Ptolemy, *Syntaxis mathematica*, J. L. Heiberg, ed., II [Leipzig, 1903], 212, II. 11–14).

33. See E. S. Kennedy, “Late Medieval Planetary Theory,” in *Isis***57** (1966), 365–378. esp. 366–368.

34. I have consulted the British Museum MSS Add. 23, 394; Add. 23, 397; and Add. 7472 Rich, the last two being al-Nīsabūrī’s commentary, *Tawdīh*, on the *Tadhkira*.

35. The reference is very probably to the lost tract on the movement of *iltifāf* (III 61).

36. A. Lejeune, *Recherches sur la catoptrique grecque* (Brussels, 1957), pp. 71–74.

37. Diagrams are supplied in MS KÖprülü 952 (see bibliography, “Original Works,” under III 3). As far as I know, this MS has not been used in studies of the *Optics*.

38. *Opticae thesaurus. Alhazeni libri VII*, pp. 142–150.

39. See al-Ṭūsī *Rasā’il*. II (cited in note 2 above), *risālā* no. 8, pp. 5–7, See also A. I. Sabra, “Thābit ibn Qurra on Euclid’s Parallels Postulate,” in *Journal of the Warburg and Courtauld Institutes*, **31** (1968). 12–32: and A. P.juschkewitsch [Youschkevitch], *Geschichte der Mathematik im Mittelater* (Leipzig, 1964), pp. 277–288.

40. Sir Thomas Heath, *A History of Greek Mathematics*, I (Oxford, 1921), pp. 183 ff.

41. Chester Beatty MS 3652, fol. 71r-v.

BIBLIOGRAPHY

Ibn Abī Usaybiʿ’s lists la, lb, II, and III of Ibn al-Haytham’s works, described in the article, have been published, wholly or in part, more than once in European languages. They have recently been reproduced in a convenient form in Italian trans. by G. Nebbia in “Ibn al-Haytham nel millesimo anniversario della nascita,” in *Physis*. **9** (1967), 165–214. Since practically all of Ibn al-Haytham’s extant works are included in list III, they will be arranged here according to their numbers in that list. The same numbers can be used to refer to Nebbia’s article, where the reader will find a useful bibliography, and to M. Schramm’s book, *Ibn al-Haythams Weg zur Physik* (Wiesbaden, 1963), where many of the works listed are discussed. (The titles constituting what Nebbia calls list I*c* are in fact chapter headings of the last work in list I*b*.)

Arabic MSS of Ibn al-Haytham’s works are listed in H. Suter, “Die Mathematiker und Astronomen der Araber und ihre Werke,” in *Abhandlungen zur Geschichte der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, **10** (Leipzig, 1900), no. 204, 91–95; and “Nachträge und Berichtigungen zur ‘Mathematiker...,’” *ibid*., **14** (1902), esp. 169–170; H. P. J. Renaud, “Additions et corrections á Suter ‘Die Mathem. u. Astr. der Arab.,’” *Isis*, **18** (1932), esp. 204; C. Brockelmann, *Geschichte der arabischen Literatur*, I (Weimar, 1898), 469–470; 2nd ed. (Leiden, 1943), pp. 617–619; supp. I (Leiden, 1937), 851–854. The Istanbul MSS are more fully described in M. Krause, “Stambuler Handschriften islamischer Mathematiker,” in *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. B, Studien, **3** (1936), 437–532. P. Sbath, in *Al-Fihris: Catalogue de manuscrits arabes*, 3 pts. plus *Supplement* (Cairo, 1938–1940), pt. I, p. 86, cites MSS, belonging to a private collection in Aleppo, of the following works: III 6, III 8, III 48, III 60, III 65, III 67, III 68, and III 82. (I owe this reference to Robert E. Hall.)

In the list of extant original works that follows, reference will be made to Brockelmann and Krause by means of the abbreviations “Br.” and “Kr.,” followed by the numbers given to Ibn al-Haytham’s treatises in these two authors.

Other abbreviations used are the following:

Rasā’il: Majmū’ al-Rasā’il (Hyderabad, 1357 a.h. [1938]). A collection of eight treatises by Ibn al-Haytham to which a ninth, published at Hyderabad, 1366 a.h. (1947), has been added.

Tanqīḥ: Tanqīḥ nl-Manāzir..., 2 vols. (Hyderabad, 1347–1348 a.h. [1928–1930]). This is Kamāl al-Dīn al-Fārisīs “commentary” (“Tanqīḥ” means revision or correction) on Ibn al-Haytham’s *Kitāb al-Manāzir*. Vol. II has a sequel (*dhayl*) and an appendix (*mulhaq*) which contain Kamāl al-Dīn’s recensions (sing., *tahrīr*) of a number of Ibn al-Haytham’s other optical works.

I. Original Works.

III 1 (Br. 28). *M*(*aqāla*). *f*(*ī*). *Hayʾat al-ʿālam* (“On the Configuration of the World”). A MS that has recently come to light is Kastamonu 2298, 43 fols.; unlike the India Office MS it is incomplete. For Hebrew and Latin MSS see M. Steinschneider, “Notice sur un ouvrage astronomique inédit d’Ibn Haitham,” in *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, **14** (1881), 721–736, also published as *Extrait*... (Rome, 1883); “Supplément,” *ibid*., **16** (1883), 505–513; and *Die hebraeischen Uebersetzungen des Mittelalters und die Juden als Dolmetscher* (Berlin, 1893), II, 559–561; F. Carmody, *Arabic*

Astronomical and Astrological Sciences in Latin Translation (Berkeley, Cal., 1955), pp. 141–142; and Lynn Thorndike and Pearl Kibre, *Catalogue of Incipits of Mediaeval Scientific Writings in Latin* (Cambridge, Mass., 1963), cols. 894, 895, 1147 (the last being a Spanish trans. by Abraham Hebraeus).

The Arabic text has not been edited. A Latin version has been published from a MS of the 13th or early 14th century in Millás Vallicrosa, *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo* (Madrid, 1942), app. II, 285–312; see pp. 206–208. There is a German trans. by K. Kohl, "Uber den Aufbau der Welt nach Ibn al Haiṯam," in *Sitzungsberichte der Physikalischmedizinischen Sozietät in Erlangen*, **54–55** (1922–1923), 140–179.

III 2 (Br. 8, Kr. 14). *M.f. Sharh musādarāt Kitāb Uqlīdis ("Commentary on the Premises of Euclid's Elements")*. Composed before III 53 and before the larger commentary on the *Elements* (see below, "Additional Works," no. 1). MSS of Ibn Tibbon's Hebrew trans. are listed in M. Steinschneider, *Hebraeischen Uebersetzungen* (cited under III 1), II, 509–510. A partial Russian trans. of this work (using a Kazan MS not recorded in Brockelmann) has been published by B. A. Rozenfeld as "Kniga kommentariev k vvedeniyam knigi Evklida 'Nachala,'" in *Istoriko-matematicheskie issledovaniya*, **11** (1958), 743–762.

III 3 (Br. 34, Kr. 15). Kitāb al-Manāzir ("Optics"). All known Arabic MSS of this work are in Istanbul; see Krause. Köprülü MS 952 contains practically the whole of bks. IV, V, VI, and VII. The folios must be rearranged as follows: IV, 108r–133v; V, 2r–v, 74r–81v, 89r–107v, 134r–135v; VI, 3r–47v; VII, 1r–v, 48r–73v, 82r–88v. The reference in Brockelmann to a recension of this work in the Paris MS, ar. 2460 (Br. has 2640) is mistaken; the MS is a recension of Euclid's *Optics* which is attributed on the title page to Hasan ibn [Mūusā ibn] Shākir.

I have examined the following Latin MSS: Bruges 512, 113 fols., 13th c.; Cambridge University Library, Peterhouse MS 209 (= 11 · 10 · 63), III fols., 14th c.; Cambridge University Library, Trinity College MS 1311 (= 0 · 5 · 30), 165 fols., 13th c.; Edinburgh, Royal Observatory, Crawford Library MS 9 · 11 · 3 (20), 189 fols., dated 1269; Florence, Biblioteca Nazionale, Magliabechi CI.XX.52, 136 fols., incomplete, 15th c.; London, British Museum, Royal 12 G VII, 102 fols., 14th c.; British Museum, Sloane 306, 177 fols., 14th c.; Oxford, Corpus Christi 150, 114 fols., 13th c.; Vienna, Nationalbibliothek 2438, a fragment only from beginning of bk. I, ch. 1 (ch. 4 in Arabic text), fols. 144r-147r, 15th c. Other Latin MSS have been reported in F. Carmody, *Arabic Astronomical and Astrological Sciences*, cited under III 1, p. 140; L. Thorndike and P. Kibre, *Catalogue*, cited under III 1, cols. 774, 803, 1208; and G. F. Vescovini, *Studi sulla prospettiva medievale* (Turin, 1965), pp. 93–94, n. 10.

The only known copy of the fourteenth-century Italian trans. of the *Optics* is MS Vat. Lat. 4595, 182 fols. Like the Latin text it lacks chs. 1–3 of bk. I. It includes an Italian trans. of the *Liber de crepusculis* (see below), fols. 178r–182v.

The Latin text was published in the collective volume bearing the following title: *Opticae thesaurus. Alhazeni Arabis libri septem, nunc primum editi, eiusdem liber de crepusculis et nubium ascensionibus, item Vitellionis Thuringo-Poloni Libri X, omnes instaurati, figuris illustrati et aucti, adjectis etiam in Alhazenum commentariis a Federico Risnero* (Basel, 1572). Concerning the authorship of *De crepusculis*, see below, "Spurious Works."

Kamāl al-Din's commentary, the *Tanqīḥ* (cited above), does not reproduce the integral text of the *Optics*, as was at one time supposed. An ed. of the Arabic text of *Kitāb al-Manāzir* and English trans. are being prepared by the present writer.

III 4 (Br. 51). *M.f. Kayfiyyat al-arsād* ("On the Method of [Astronomical] Observations").

III 6 (Br. 27). *M.f. Daw᾿ al-qamar* ("On the Light of the Moon"). Composed before 7 Aug. 1031, the date on which a copy was completed by ʿAlī ibn Ridwān (Ibn al-Qifṭī, *Ta'rīkh*, p. 444). Published as no. 8 in *Rasā'il*. There is a German trans. by Karl Kohl, "Über das Light des Mondes. Eine Untersuchung von Ibn al-Haitham," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **56–57** (1924–1925), 305–398.

III 7 (Br. 22, Kr. 18). *M.* (or *Qawl*) *f. Samt al-qibla bi 'l-hisāb* ("Determination of the Direction of the Qibla by Calculation"). A German trans. is C. Schoy, "Abhandlung des Hasan ibn al-Ḥasan ibn al-Haiṯam (Alhazen) Über die Bestimmung der Richtung der Qibla," in *Zeitschrift der Deutschen morgenländischen Gesellschaft*, **75** (1921), 242–253.

III 8 (Br. 41. Kr. 19). *M.f. al-Hāla wa-qaws quzah* ("On the Halo and the Rainbow"). Completed in Rajab, 419 a.h. (a.d. 1028); see *Tanqīḥ*, II, p. 279. Recension by Kamāl al-Dīn in *Tanqīḥ*, II, 258–279. A shortened German trans. of this recension is E. Wiedemann, "Theorie des Regenbogens von Ibn al Haiṯam," in *Süzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **46** (1914), 39–56.

III 9 (Br. 42. Kr. 20). *M.f Mā yaʿrid min al-ikhtilaf fī irtifāʿ āt al-kawākib* ("On What Appears of the Differences in the Heights of the Stars").

III 10 = Ia10 (Br. 39, Kr. 16). *M.f. Hisāb al-muʿāmalāt* ("On Business Arithmetic").

III 11 (Br. 43, Kr. 21). *M.f. al-Rukhāma al-ufuqiyya* ("On the Horizontal Sundial"). This work refers to a treatise to be written later on "shadow instruments" (*ālātal-azlāl*); the reference may be to III 66.

III 14. *M.f. Marākiz al-athqāl* ("On Centers of Gravity"). This is not extant but has been abstracted by al-Khāzinī in *Mīzān al-ḥikma;* see the Hyderabad ed. (1359 a.h. [1940]), pp. 16–20.

III 15 (Br. 13 a, Kr. 22). *M.f. Usūl al-misāha* ("On the Principles of Measurement"). A summary of the results of an earlier work or works by Ibn al-Haytham on the subject. Published as no. 7 in *Rasāil*. German trans, by E. Wiedemann in "Kleinere Arbeiten von Ibn al Haiṯam," in *Sitzungsberichte der Physikalisch-medizinischen Sozietāt in Erlangen*, **41** (1009), 16–24.

III 16 (Br, 2, Kr. 23). *M.f. Misāhat al-kura* ("On the Measurement of the Sphere"). Later in composition than III 17; it may be one of the works referred to in III 15.

III 17 (Br. 14). *M.f. Misāhat al-mujassam al-mukāfī* ("On the Measurement of the Paraboloidal Solid"). See III 16. Refers to a work on the same subject by Thābit ibn Qurra and another by Wayjan ibn Rustam al-Qūhī. German trans, by H. Suter, "Die Abhandlung über die Ausmessung des Paraboloides von el-Hasan b. el-Hasan b. el-Haitham," in *Bibliotheca mathematica*, 3rd ser., **12** (1912), 289–332. See also H. Suter, "Die Abhandlungen Thābit b. Kurras and Abū Sahl al-Kûhîs über die Ausmessung der Paraboloide," in *Sitzungsberichte der Physikalisch-medizinischen Sozietāt in Erlangen*, **48–49** (1916–1917), 186–227.

III 18 (Br. 3.1, Kr. 10). *M.f. al-Marāya al-muhriqa bi 'l-dawā'ir* ("On Spherical Burning Mirrors"). Published as no. 4 in *Rasā'il*. German trans, by E. Wiedemann, "Ibn al Haiṯams Schrift über die sphärischen Hohlspiegel," in *Bibliotheca mathematica*, 3rd ser., **10** (1909–1910), 293–307. See also E. Wiedemann, "Zur Geschichte der Brennspiegel," in *Annalen der Physik und Chemie*, n.s. **39** (1890), 110–130, trans. into English by H. J. J. Winter and W. ʿArafāt, "A Discourse on the Concave Spherical Mirror by Ibn al-Haitham," in *Journal of the Royal Asiatic Society of Bengal*, 3rd ser., Science, **16** (1950), 1–16.

III 19 (Br. 33). *M.f. al-Marāya al-muhriqa bi 'l-qutūʿ* ("On Paraboloidal Burning Mirrors"). A hitherto unrecorded MS of the Arabic text is Florence, Biblioteca Medicea-Laurenziana, Or. 152, fols. 90v-97v. It was copied in the 13th century and bears no title or author's name. Here Ibn al-Haytham mentions an earlier treatise of his on how to construct all conic sections by mechanical means (*istikhrāj jamīʿ al-qutūʿ bi-tarīq al-āla*); see below, "Additional Works," no. 2. The Arabic text has been published as no. 3 in *Rasā'il*. A medieval Latin trans. as *Liber de speculis comburentibus*, probably made by Gerard of Cremona, has been published together with a German trans. from the Arabic by J. L. Heiberg and E. Wiedemann: "Ibn al Haiṯams Schrift über parabolische Hohlspiegel," in *Bibliotheca mathematica*, 3rd ser., **10** (1909–1910). 201–237. See also E. Wiedemann, "Über geometrische Instrumente bei den muslimischen Völkern." in *Zeitschrift für Vermessungswesen*, nos. 22–23 (1910). 1–8; and "Geschichte der Brennspiegel," cited under III 18. An English trans. is H. J. J. Winter and W. ʿArafāt, "Ibn al-Haitham on the Paraboloidal Focusing Mirror," in *Journal of the Royal Asiatic Society of Bengal*, 3rd ser., Science, **15** (1949). 25–40.

III 20. *Maqāla mukhtasara fi 'l-Ashkāl al-hilāliyya* ("A Short Treatise on Crescent-Shaped Figures"). Not extant; see III 21.

III 21 (Br. 1, Kr. 12). *Maqāla mustaqsāt fi 'l–Ashkāl al-hilāliyya* ("A Longer Treatise on Crescent-Shaped Figures"). Composed after III 20, which it is intended to supersede, and before III 30 (*q.v.*). It may also have been written before the work listed below as "Additional," no. 1 (*q.v.*).

III 22 ([?] Br. 6). *Maqāla mukhtasara fī Birkār al-dawā ir al-'izām* ("A Short Treatise on the Birkār of Great Circles"). See Wiedemann, "Geometrische Instrumente...," cited under III 19. See III 23. "*Birkār*" is Persian for compass. Ibn al-Haytham explains the theory and construction of an instrument suitable for accurately drawing very large circles.

III 23 ([?] Br. 6). *Maqāla mashrūha fī Birkār al-dawāir al-'izām* ("An Expanded Treatise on the Birkār of Great Circles"). See III 22.

III 25 (Br. 52). *M.f. al-Tanbīh 'alā mawādi' al-ghalat fī kayfiyyat al-rasd* ("On Errors in the Method of [Astronomical] Observations"). Earlier in composition than III 31.

III 26 (Br. 44, Kr. 24). *M.f. anna 'l-Kura awsaʿ al-ashkāl al-mujassama allatī ihāiatuhā muta-sāwiya, wa-anna 'l-dā'ira awsaʿ al-ashkāl al-musattaha allatī ihatatuhā mutasāwiya* ("That the Sphere Is the Largest of the Solid Figures Having Equal Perimeters, and That the Circle is the Largest of the Plane Figures Having Equal Perimeters"). Composed before III 38 and III 68. Refers to Archimedes' *On the Sphere and the Cylinder*.

III 28 (Br. 29). *Kitāb fī Tasīh al-a'māl al-nujūmiyya, maqālatān* ("A Book on the Corrections of Astrological Operations, Two Treatises").

III 30 (Br. 9. Kr, 2). *M.f. Tarbī’ al-dā’ira* (“On the Quadrature of the Circle”). Refers to the “book on lunes” (*kitābina fi ’l-hilāliyyat*), that is cither III 20 or III 21. There is an ed. of the Arabic text and German trans. by H. Suter, “Die Kreisquadratur des Ibn el-Haiṯam,” in *Zeitschrift für Mathematik und Physik*, Hist.-lit. Abt., **44** (1899), ‘33–47.

III 31 (Br. 45, Kr. 25). *M.f. Istikhrāj khatt nisf al-nahār ‘alā ghāyat al-tahqīq* (“Determination of the Meridian with the Greatest Precision”). Composed after III 25. Points out the relevance of the subject to astrology.

III 36 (Br. 31. Kr. 7) *M.f. Kayfiyyat al-azlāl* (“On the Formation of Shadows”). Composed before the “Commentary on the *Almagest*” (see below, “Additional Works,” no. 3) and after III 3. A recension by Kamāl al-Dīn al-Fārisī is in *Tanqīḥ*, II, 358–381. A German trans. by E. Wiedemann is “Über eine Schrift von Ibn al Haiṯam: Über die Beschaffenheit der Schatten,” in *Sitzungsberichte der Physikalisch-medizinischen Sozietäl in Erlangen*, **39** (1907),” 226–248.

III 38 (Br. 30, Kr. 26). *M.f. Hall shukūk fi ’l-maqāla l-ūlā min Kitāb al-Majistī yushakkiku fīhā baʿd ahl al-ʿilm* (“Solution of Difficulties in the First Book of the *Almagest* Which a Scholar Has Raised”). This work is to be distinguished from III 64. It was composed after III 26. The name of “the scholar” appears in MS Fatih 3439, fol. 150v to be Abu ‘l-Qāsim ibn [?]Maʿdān, who is otherwise unknown to me. The Title in the Fatih MS (*Hall Shukūk fī Kitāb al-Majistī yushakkiku fīhā baʿd ahl al-ʿilm*) does not limit the discussion to book I of the *Almagest*. The text in fact discusses, among other things, book V of Ptolemy’s *Optics*.

III 39 ([?] Kr. 27). *M.f. Hall shakk fī mujimamāt Kitāb Uqlīdis* (“Solution of a Difficulty in the Part of Euclid’s Book Dealing With Solid Figures”). This may be part of the work listed below as “Additional Works,” no. 1. But see Krause, no. 27, where reference is made to a work bearing a partially similar title and of uncertain authorship. (I have not examined MS Yeni Cami T 217, 2^0, 893 a.h., referred to by Krause.)

III 40 (Br. 3). *Qawl fī Qismat al-miqdārayn al-mukhtalifayn al-madhkūrayn fi ’l-shakl al-awwal min al-maqā ’l-‘ āshira min Kitāb Uqlīdis* (“On the Division of the Two Unequal Magnitudes Mentioned in Proposition I of Book X of Euclid’s Book”). The subject is closely connected with the so-called “axiom of Archimedes.”

III 41 (Br. 23). *Mas’ala fī lkhtilāf al-nazar* (“A Question Relating to Parallax”). MS India Office, Loth 734, fols. 120r–120v, specifies that lunar parallax is meant.

III 42 (Br. 17). *Qawl fī Istikhrāj muqaddamat dil‘ almusabba‘* (“On the Lemma [Used by Archimedes] for [Constructing] the Side of the Heptagon [in the Book at the End of Which He Mentioned the Heptagon]”). Composed before III 74. German trans. by C. Schoy in *Die trigonometrischen Lehren des persischen Astronomen Abu ’l-Raihān Muh. ibn Ahmad al Bîrûnî, dargestellt nach al-Qânûn al-Mas‘ ûdî* (Hannover, 1927), pp. 85–91.

III 43 (Br. 10, Kr. 9). *Qawl fī Qismat al-khatt alladhī ista‘malahu Arshimīdis fī Kitāb al-Kura wa ’l-ustuwāna* (“On the Division of the Line Used by Archimedes in His Book on the Sphere and Cylinder”). Concerned with prop. 4 of bk. II in Archimedes’ work. French trans. by F. Woepcke, *L’algébre d’Omar Alkhayâmî* (Paris, 1851), pp. 91–93.

III 44 (Br. 46, Kr. 28). *Qawl fī Istikhrāj khatt nisf al-nahār bi-zill wāhid* (“Determination of the Meridian by Means of One Shadow”).

III 46 (Br. 26). *M.f. al-Majarra* (“On the Milky Way”). German trans. by E. Wiedemann, “Über die Lage der Milchstrasse nach ibn al Haiṯam,” in *Sirius*, **39** (1906), 113–115.

III 48 (Br. 24, Kr. 5). *M.f. Adwā’ al-kowākib* (“On the Light of the Stars”). Composed before III 49. Published as no. 1 in *Rasā’il*. Abridged German trans. by E. Wiedemann, “Über das Licht der Sterne nach Ibn Al Haitham,” in *Wochenschrift für Astronomie, Meteorologie und Geographie*,, n.s. **33** (1890), 129–133. English trans. by W. ʿArafat and H. J. J. Winter, “The Light of the Stars—a Short Discourse by Ibn al-Haytham,” in *British Journal for the History of Science*, **5** (1971), 282–288.

III 49 (BR. 37). *M.f. al-Athar alladhī* [*Yurā*] *fī* [*wajh*] *al-qamar* (“On the Marks [Seen] on the [Face of the] Moon”). Composed after III 3, III 6, and III 48, to all of which it refers. German trans. by C. Schoy, as *Abhandlung des Schaichs Ibn ʿAli al-Ḥasan ibn al-Ḥasan ibn al-Haitham: Über die Natur der Spuren* [*Flecken*], *die man auf der Oberfläche des Mondes sieht* (Hannover, 1925).

III 53 (Br. 35). *M.f. al-Tahlīl wa ’l-tarkīb* (“On Analysis id Synthesis”). Brockelmann lists a Cairo MS. Another is Dublin, Chester Beatty 3652, fols. 69v-86r, dated 612 a.h. (1215). Composed before III 54 (to which it is closely related) and after III 2.

III 54 (Br. 11). *M.f. al-Maʿlūumāt* (“On the Known Things [Data]”). Trans. of the enunciations of its propositions by L. A. Sédillot, in “Du *Traité* des connues géométriques de Hassan ben Haithem,” in *Journal asiatique*, **13** (1834). 435–458.

III 55. *Qawl fī Hall Shakk fi ʼl-maqāla ʼl-thāniya ʿashar min Kitāb Uqlīdis* ("Solution of a Difficulty in Book XII of Euclid's Book"). Possibly a part of the work listed below as "Additional," no. 1.

III 56, *M.f. Ḥall shukūk al-maqāla ʼl-ūlā min Kitāb Uqlīdis* ("Solution of the Difficulties in Book I of Euclid's Book"). Possibly part of the work listed below as "Additional Works," no. 1.

III 60 (Br. 32, Kr. 4). *M.* (or *Qawl*) *f. al-Ḍawʾ* ("A Discourse on Light"). Composed after III 3. Printed as no. 2 in *Rasāʾil*. J. Baarmann published an ed. of the Arabic text together with a German trans. as "Abhandlung über das Licht von Ibn al-Haiṯam," in *Zeitschrift der Deutschen morgenländischen Gesellschaft*, **36** (1882), 195–237. (See remarks on this ed. by E. Wiedemann, *ibid*., **38** (1884), 145–148.) A Cairo ed. by A. H. Mursī correcting Baarmann's text appeared in 1938. There is now a critical French trans. by R. Rashed: "Le 'Discours de la lumiére' d'Ibn al-Haytham," in *Revue d'histoire des sciences et de leurs applications*, **21** (1968), 198–224. A recension is in *Tanqīḥ*, II, 401–407. A German trans. of this recension (*taḥrīr*) is E. Wiedemann, "Ueber 'Die Dartegung der Abhandlung Über das Licht' von Ibn al Haiṯam," in *Annalen der Physik und Chemie*, n.s. **20** (1883), 337–345.

III 63 (Br. 19, Kr. 29). *M.f. Ḥall shukūk ḥarakat al-iltifāf* ("Solution of Difficulties Relating to the Movement of Iltifāf"). A reply to an unnamed scholar who raised objections against an earlier treatise by Ibn al-Haytham (III 61: "On the Movement of Iltifāf") which is now lost. In the reply Ibn al-Haytham revealed an intention he had entertained to write a critique of Ptolemy's *Almagest, Planetary Hypotheses* (*Kitāb al-Iqtiṣāṣ*), and *Optics* (MS Pet. Ros. 192, fols. 19v–20r)—almost certainly a reference to III 64.

III 64 (Br. 30). *M.f. al-Shukūk ʿalā Baṭlamyūs* ("Dubitationes in Ptolemaeum"). Composed after III 63; see preceding note. There is a critical ed. by A. I. Sabra and N. Shehaby (Cairo, 1971). English trans. of part of this work by A. I. Sabra, "Ibn al-Haytham's Criticism of Ptolemy's *Optics," in Journal of the History of Philosophy*, **4** (1966), 145–149.

III 65. *M.f. al-Juzʾ alladhī la yatajazzaʾ* ("On Atomic Parts"). A unique copy belonging to a private collection in Aleppo is recorded in P. Sbath, *Al-Fihris* (cited above), 1, 86, no. 724.

III 66 (Br. 40, Kr. 17). *M.f. Khuṭūuṭ al-sāʿāt* ("On the Lines of the Hours [i.e., on sundials]"). "Al-sāʿāt" has sometimes been misread as "*al-shuʿāʿāt*" (rays). The treatise refers to a work by Ibrāhīm ibn Sinān, "On Shadow Instruments." See note for III 11 above.

III 67. *M.f. al-Qarasṭūn* ("On the *Qarasṭūn"*). A unique copy belonging to a private collection in Aleppo is recorded in P. Sbath, *Al-Fihris* (cited above), I, p. 86, no. 726.

III 68 (Br. 12, Kr. 11). *M.f. al-Makān* ("On Place"). Later in composition than III 26. Published as no. 5 in *Rasāʾil*. A short account is given by Wiedemann in "Kleiners Arbeiten...," cited under III 15 above, pp. 1–7.

III 69 (Br. 18). *Qawl fī Istikhrāj aʿmidat al-jibāl* ("Determination of the Altitudes of Mountains,"), A longer title is *Fī Maʿrifat irtifāʿ al-ashkhāṣ al-qāʾima wa-aʿmidat al-jibāl wa irtifāʿ al-ghuyūm* ("Determination of the Height of Erect Objects and of the Altitudes of Mountains and of the Height of Clouds"). A German trans. is H. Suter. "Einige geometrische Aufgahen bei arabisehen Mathematiker." in *Bibliotheca mathematica*, 3rd ser., **8** (1907), 27–30. A short account by Wiedemann is in "Kleinere Arbeiten...," cited under III 15 above, pp. 27–30.

III 71 (Br. 38). *M.f. Aʿmidat al-muthalahāt* ("On the Altitudes of Triangles"). (An alternate title is *Khawāṣṣ al-muthallath min jihat al-ʿamūd* ("Properties of the Triangle in Respect of Its Altitude"). Published as no, 9 in *Rasāʾil*.

III 73, (Br. 13. Kr. 3). *M.f. Shakl Banū Mūsā*. ("On the Proposition of Banū Mūsā [proposed as a lemma for the *Conics* of Apollonius]"). Published as no. 6 in *Rasāʾil*, An account of it is in Wiedemann, "Kleinere Arbeiten...," cited under III 15 above, pp. 14–16.

III 74 (Br. 48, Kr. 30). *M.f. ʿAmal al-musabbaʿ fi ʼl-dāʾira* ("On Inscribing a Heptagon in a Circle"). Composed after III 42, to which it refers. As well as referring to Archimedes it mentions al-Qūhī, whose treatise on the subject has been published and trans. by Y. Dold-Samplonius, "Die Konstruklion des regel-mässigen siebenecks nach Abû Sahl al-Qûhî," in *Janus*. **50** (1963), 227–249.

III 75 (Br. 25, Kr. 1). *M.f. Irtfāʿ al-quṭb ʿalā ghāyat al-taḥqīq* ("Determination of the Height of the Pole With the Greatest Precision"). A German trans. is C. Schoy, "Abhandlung des Ḥasan ben al-Ḥasan ben al-Haiṯam über eine Methode, die Polhöhe mit grösster Genanigkeit zu bestimmen," in *De Zee*, **10** (1920), 586–601.

III 76 (Br. 47, Kr. 31). *M.f. ʿAmal al-binkām* ("On the Construction of the Water Clock").

III 77 (Br. 33b, Kr. 32). *M.f. al-Kura ʼl-muḥriqa* ("On the Burning Sphere"). Written after III 3 and III 66. A recension by Kamāl al-Dīn is in *Tanqīḥ*, II, 285–302. A German trans. of this recension is in E. Wiedemann. "Brechung des Lichtes in

Kugeln naeh Ibn al Haiṯam und Kamâl al Dîn al Fârisî," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **42** (1910), 15–58, esp. 16–35.

III 78 (Br. 15). *M.f. Masʾala ʿadadiyya mujassama* ("On an Arithmetical Problem in Solid Geometry").

III 79 (Br. 5). *Qawl fī Masʾala handasiyya* ("On a Geometrical Problem"). A German trans. is in C. Schoy, "Behandlung einiger geometrischen Fragenpunkte durch muslimische Mathematiker," in *Isis*, **8** (1926), 254–263, esp. 254–259.

III 80 (Br. 20, Kr. 8). *M.f. Sūrat al-kusūf* ("On the Shape of the Eclipse"). Composed after III 3. A recension by Kamāl ai-Dīn is *Tanqīḥ*. II, 381–401. A German trans. of the original text from the India Office MS is in E. Wiedemann, "Über die Camera obscura bei Ibn at Haiṯam," in *Sitzungberichte der Physikalisch-medizinischen Sozietät in Erlangen*. **46** (1914), 155–169.

III 82 (Br. 21, Kr. 13). *M.f. Ḥarakat al-qamar* ("On the Motion of the Moon"). A vindication of Ptolemy's account of the mean motion of the moon in latitude.

III 83 (Br. 4). *M.f. Masāʾil al-talāqī* ("On Problems of Talāqī"). These are problems involving the solution of simultaneous linear equations. There is an account by E. Wiedemann, in "Über eine besondere Art des Gesellschaftsrechnens besondere nach Ibn al Haiṯam," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*. **58–59** (1926–1927), 191–196.

III 92 (Br. 16). *Qawl fī Istikhrāj masʾala ʿadadiyya* ("Solution of an Arithmatical Problem"). An account is given by Wiedemann in "Kleinere Arbeiten...," cited under III 15 above, pp. 11–13.

Additional Works. These are extant works whose titles do not appear in list III.

Add. 1 (Br. 7, Kr. 6). *Kitāb fī Ḥall shukūk Kitāb Uqlīdis fi 'l-Uṣūl wa-sharḥ maʿānīh* ("A Book on the Solution of the Difficulties in Euclid's *Elements* and an Explanation of Its Concepts"). This seems to be a different work from I*a* 1: *Sharḥ Uṣūl Uqlīdis fi 'l-handasa wa 'l-ʿadad wa talkhīṣuhu* ("A Commentary on and Summary of Euclid's *Elements of Geometry and Arithmetic"*).

The absence of this comprehensive work from list III may perhaps be explained by supposing III 39, III 55, and III 56 to be parts of it. It refers to III 2 and also to "our treatise on crescent-shaped figures," which is either III 20 or III 21. An Istanbul MS that is not recorded in Broekelmann or in Krause is üniversite 800, copied before 867 a.h. (1462–1463). The MS has 182 fols. but is not complete.

Add, 2. *Kalām fī tawṭiʾat muqaddamāt li-ʿamal al-quṭūʿ ʿalā saṭḥ mā bi-ṭarīq ṣināʿī* ("A Passage in Which Lemmas Are Laid Down for the Construction of [Conic] Sections by Mechanical Means"). MS Florence, Biblioteca Medicea-Laurenziana, Or. 152, fols. 97v–100r. No author is named, and the "lemmas" follow immediately after a copy of Ibn al-Haytham's III 19 ("On Paraboloidal Burning Mirrors"), which also does not bear the author's name. Since Ibn al-Haytham refers in III 19 to a treatise of his on the mechanical construction of conic sections, it is very likely that the "passage" we have here is a fragment of that treatise which the copyist found joined to III 19.

Add. 3. "Commentary on the *Almagest*," Istanbul MS, Ahmet III 3329, copied in Jumādā II 655 (1257), 123 fols. Probably written after III 36, to which it appears to refer (fol. 90r).

Spurious Works. Ibn al-Haytham is not the author of the *Liber de crepusculis*, the work on dawn and twilight translated by Gerard of Cremona and included in Risner's *Opticae thesaurus* (see A. I. Sabra, "The Authorship of the *Liber de crepusculis*," in *Isis*, **58** [1967], 77–85: above, III 3). An astrological work, *De imaginibus celestibus*, Vatican MS Urb. Lat. 1384, fols. 3v–26r, has also been mistakenly ascribed to him (*ibid*., p. 80, n. 14).

Two more writings are listed in Brockelmann (nos. 49, 50) which may or may not be genuine.

I am grateful to M. Clagett for showing me a microfilm of MS Bruges 512 (III 3) and to M. Schramm for showing me microfilms of the following MSS: Kastamonu 2298 (III 1), Üniversite 800 (Add. 1), and Ahmet III 3329 (Add. 3). For the last three MSS and for other Arabic MSS not hitherto recorded, see the appropriate volume of F. Sezgin, *Geschichte des arabischen Schrifttums* (Leiden, 1967–).

II. Secondary Literature. Sources for she biography of Ibn al-Haytham are Ibn al-Qifṭī, *Taʾrīkh al-ḥukamāʾ*, J. Lippert, ed. (Leipzig, 1903), pp. 155–168 (see corrections of this ed. by H. Suter in *Bibliotheca mathematica*, 3rd ser.,**4** [1903], esp. 295–296); ʿAlī ibn Zayd al-Bayhaqī, *Tatimmat ṣiwān al-ḥikma*, M. Shafīʿ, ed., fasc. I: Arabic text (Lahore, 1935), 77–80 (analysis and partial English trans. of this work by M. Meyerhof in *Osiris*, **8** [1948], 122–216, see esp. 155–156); Ibn Abī Uṣaybiʿa. *Ṭabaqāt al-aṭibbāʾ*, A. Müller ed. (Cairo–Königsberg, 1882–1884), II, 90–98 (German trans. in E. Wiedemann, "Ibn al-Haiṯam, ein arabischer Gelehrter," cited below): Ṣāʿid al-Andalusī, *Ṭabaqāt al-umam*, L. Cheikho, ed. (Beirut, 1912), p. 60 (French trans. by R. Blachère [Paris, 1935] p. 116). The account in Abu'l-Faraj ibn al-ʿIbrī, *Taʾrīkh mukhtaṣar al-duwal*, A. Ṣālḥānī, ed. (Beirut, 1958), pp. 182–183, derives from Ibn al-Qifṭī, In addition to the works by Suter (*Mathematiker*) and

Brockelmann (*Geschichte*) already cited, see M. Steinschneider, "Vite di matematici arabi, tratte da un' opera inedita di Bernardino Baldi, con note di M.S.," in *Bullettino di bibliografia e di storia delle scienze noatematiche et fisiche*, **5** (1872), esp. 461–468, also printed separately (Rome, 1874); M. J. de Goeje, "Notice biographique d'Ibn al-Haitham," in *Archives néerlandaises des sciences exactes et naturelles*, 2nd ser., **6** (1901), 668–670; E. Wiedemann, "Ueber das Leben von Ibn al Haiṯam und al Kindî," in *Jahrbuch für Photographie und Reproduktiontechnik*, **25** (1911), 6–11 (not important).

The literary relationship of Ibn al-Haytham's autobiography to Galen's *De libris propriis* is discussed by F. Rosenthal in "Die arabische Autobiographie," in *Studia arabica I*, Analecta Orientalia, no. 14 (Rome, 1937), 3–40, esp. 7–8. There is a discussion of the autobiography in G. Misch, *Geschichte der Autobiographie*, III, pt. 2 (Frankfurt, 1962), 984–991. Lists I*a* and III of the works of Ibn al-Haytham are translated from Ibn Abī Uṣaybiʿa in F. Woepcke, *L'algèbre d'Omar Alkhayyāmī* (Paris, 1851), pp. 73–76; but see H. Suter's corrections in *Mathematiker*, pp. 92–93. There is a German trans. of Ibn al-Haytham's autobiography and of Lists I–III in E. Wiedemann, "Ibn al Haiṯam, ein arabischer Gelehrter," in *Festschrift* [*fü*] *J. Rosenthal* (Leipzig, 1906), pp. 169–178. M. Schramm discusses the chronological order of some of Ibn al-Haytham's works in *Ibn al-Haythams Weg zur Physik* (Wiesbaden, 1962), pp. 274–285.

The most complete study of Ibn al-Haytham's optical researches is M. Naẓīf, *Al-Ḥasan ibn al-Haytham, buḥūthuhu wa-kushūfuhu al-baṣariyya* ("Ibn al-Haytham, His Optical Researches and Discoveries"), 2 vols. (Cairo, 1942–1943)—reviewed by G. Sarton in *Isis*, **34** (1942–1943), 217–218. Based on the extant MSS of *Kitāb al-Manāẓir* and on Ibn al-Haytham's other optical works, this voluminous study (more than 850 pages) is distinguished by clarity, objectivity, and thoroughness. It is particularly valuable as a study of the mathematical sections of Ibn al-Haytham's works. M. Schramm, *Ibn al-Haythams Weg zur Physik*, is the most substantial single study of Ibn al-Haytham in a European language, and it has the merit of drawing on MS sources not previously available. In analyzing Ibn al-Haytham's attempt to combine Aristotelian natural philosophy with a mathematical and experimental approach, Schramm illuminates other important treatises of Ibn al-Haytham besides the *Optics*.

The question of mathematizing Aristotelian physics is also discussed in S. Pines, "What Was Original in Arabic Science," in A. C. Crombie, ed., *Scientific Change* (London, 1963), pp. 181–205, esp. 200–202. It is taken up afresh by R. Rashed in "Optique géometrique et doctrine optique chez Ibn al Haytham," in *Archive for History of Exact Sciences*, **6** (1970), 271–298. For further discussions of the concept of experiment in Arabic optics generally and in the work of Ibn al-Haytham in particular, see M. Schramm, "Aristotelianism: Basis and Obstacle to Scientific Progress in the Middle Ages," in *History of Science*, **2** (1963), 91–113, esp. 106, 112; and "Steps Towards the Idea of Function: A Comparison Between Eastern and Western Science of the Middle Ages," *ibid*, **4** (1965), 70–103, esp. 81, 98; A. I. Sabra, "The Astronomical Origin of Ibn al-Haytham's Concept of Experiment," in *Actes du XII^e Congrès international d'histoire des sciences*, Paris, 1968, III A (Paris, 1971), 133–136.

General accounts mainly based on the *Optics* are J. B. J. Delambre, "Sur l'*Optique* de Ptolémée comparée à celle qui porte le nom d'Euclide et à celle d'Alhazen et de Vitellion," in *Historie de l'astronomie ancienne*, II (Paris, 1817), 411–432; E. Wiedemann, "Zu Ibn al Haiṯams Optik," in *Archiv für Geschiclue der Naturwissensehaften und der Technik*, **3** (1910–1911), 1–53, an account of Kamāl al-Dīn's revision (*Tanqīḥ*) of Ibn al-Haytham's *Optics*, based on a Leiden MS; includes an abbreviated trans. of *Optics*, bk. I, chs. 1–3, as reported by Kamāl al-Dīn; L. Schnaasse, *Die Optik Alhazens* (Stargard, 1889); V. Ronchi, "Sul contributo di Ibn al-Haitham alle teorie della visione e della luce," in *Actes du VII^e Congrès international d'histoire des sciences* (Jerusalem, 1953), pp. 516–521; and *The Nature of Light*, a trans. of *Storia della luce* (2nd ed., Bologna, 1952) (London, 1970), pp. 40–57; H. J. J. Winter, "The Optical Researches of Ibn al-Haitham," in *Centaurus*, **3** (1953–1954), 190–210, which includes accounts of treatises other than the *Optics*.

Studies of particular aspects of Ibn al-Haytham's optical work are A. Abel, "La sélénographie d'Ibn al Haitham (965–1039) dans ses rapports avec la science grecque," in *Comptes rendus, II^e Congrés national des sciences* (Brussels, 1935), pp. 76–81 (concerned with III 49); J. Lohne, "Zur Geschichte des Brechungsgesetzes," in *Sudhoffs Archiv für Geschichte der Medizin und der Naturwissenschaften*, **47** (1963), 152–172, esp. 153–157; R. Rashed, "Le modéle de la sphére transparente et l'explication de 1'arc-en-ciel: Ibn al-Haytham, al-Fārisī," in *Revue d'histoire des sciences et de leurs applications*, **23** (1970), 109–140; E. Wiedemann, "Ueber den Apparat zur Untersuchung und Brechung des Lichtes von Ibn al Haiṯam," in *Annalen der physik und Chemie*, n.s. **21** (1884), 541–544; "Über die Erfindung der Camera obscura," in *Verhandlung der Deutschen physikalischen Gesellschaft*, **12** (1910), 177–182; and Über die erste Erwähnung der Dunkelkammer durch Ibn al Haiṯam." in *Jahrbuch für Photographie und Reproduktiontechnik*, **24** (1910), 12–13; J. Würschmidt. "Zur Theorie der Camera obscura bei Ibn al Haiṯam," in *Situngsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **46** (1914), 151–154; and "Die Theorie des Regenbogens und das Halo bei Ibn al Haiṯam und bei Dietrich von Freiberg," in *Meteorologische Zeitschrift*, **13** (1914), 484–487. Apart from Vescovini's *Studi* (see below) there is one account of Ibn al-Haytham's psychological ideas as expounded in bk. II of the *Optics:* H. Bauer, *Die Psychologie Alhazens auf Grund von Alhazens Optik dargestellt*, in the series Beiträge zur Geschichte der Philosophie des Mittelalters, 10, no. 5 (Münster in Westfalen, 1911). The physiological aspect of vision is discussed in M. Schramm, "Zur Entwicklung der physiologischen Optik in der arabischen Literatur," in *Sudhoffs Archiv für Geschichte der Medizin und der Naturwissenschaften*, **43** (1959), 289–316, esp. 291–299.

The following are concerned with the transmission of Ibn al-Haytham's optical ideas to the West; they include comparisons with Hebrew and Latin medieval, Renaissance, and seventeenth-century writers: M. Steinschneider, "Aven Natan e le teorie sulla origine della luce lunare e delle stelle, presso gli autori ebrei del medio evo," in *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, **1** (1868), 33–40; E. Narducci, "Nota intorno ad una traduzione italiana fatta nel secolo decimoquarto, del trattato d'*Ottica* d'Alhazen, matematico del secolo undecimo, e ad altri lavori di questo scienziato," *ibid*., **4**

(1871), 1–48; and “Giunte allo scritto intitolato ‘Intorno ad una traduzione italiana, fatta nel secolo decimoquarto, dell’ *Ottica* di Alhazen,’” *ibid*., pp. 137–139; A. I. Sabra, “Explanation of Optical Reflection and Refraction: Ibn al-Haytham, Descartes and Newton,” in *Actes du Xe Congrés international d’histoire des sciences*, Ithaca, 1962 (Paris, 1964), I, 551–554: and *Theories of Light From Descartes to Newton* (London, 1967), pp. 72–78. 93–99 (concerned with the theories of reflection and refraction); G. F. Vescovini, *Studi sulla prospettiva medievale* (Turin, 1965), which reveals the influence of Ibn al-Haytham’s *Optics* on the development of empiricist theories of cognition in the fourteenth century; and “Contributo per la storia della fortuna di Alhazen in Italia: 11 volgarizzamento del MS. Vat. 4595 e il ‘Commentario terzo’ del Ghiberti,” in *Rinascimento*, 2nd ser., **5** (1965), 17–49; D. Lindberg, “Alhazen’s Theory of Vision and Its Reception in the West,” in *Isis*, **58** (1968), 321–341; and “The Cause of Refraction in Medieval Optics,” in *British Journal for the History of Science*, **4** (1968), 23–38. See also G. Sarton, “The Tradition of the *Optics* of Ibn al-Haitham,” in *Isis*. **29** (1938), 403–406.

The following are studies relating to Ibn al-Haytham’s astronomical works, particularly his treatise on *The Configuration of the World* (III 1): M. Steinschneider, “Notice sur un ouvrage astronomique inédit d’Ibn Haitham,” in *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, **14** (1881), 721–736; and “Supplément á la ‘Notice sur un ouvrage inédit d’Ibn Haitham,’” *ibid*., **16** (1883), 505–513-*Extrait du Bullettino*..., containing the “Notice” and the “Supplement” (Rome, 1884), includes many corrections of the earlier publications; E. Wiedemann, “Ibn al Haiṯam und seine Bedeutung für die Geschichte der Astronomie,” in *Deutsche Literaturzeitung*, **44** (1923), 113–118; P. Duhem, *Systéme du monde*, II (Paris, 1914), 119–129; W. Hartner, “The Mercury Horoscope of Marcantonio Michiel of Venice, a Study in the History of Renaissance Astrology and Astronomy,” in A. Beer, ed., *Vistas in Astronomy* (London–New York, 1955), pp. 84–138, esp. 122–127;” S. Pines, “Ibn al-Haytham’s Critique of Ptolemy,” in *Actes du Xe Congrès international d’histoire des sciences*, Ithaca, 1962 (Paris, 1964), I, 547–550 (concerned with Ibn al-Haytham’s criticism of the equant in the *Dubitationes in Ptolemaeum*, III 64); M. Schramm, *Ibn al-Haythams Weg zur Physik*, esp. pp. 63–69. 88–146.

For discussions of “Alhazen’s problem,” see P. Bode, “Die Alhazensche Spiegelaufgabe in ihrer historischen Entwicklung nebst einer analytischen Lösung des verallgemeinerten Problems,” in *Jahresbericht des Physikalischen Vereins zu Frankfurt am Main*, for 1891–1892 (1893), pp. 63–107; M. Baker, “Alhazen’s Problem. Its Bibliography and an Extension of the Problem,” in *American Journal of Mathematics*, **4** (1881), 327–331; M. Nazīf, *Al-Ḥasan ibn al-Haytham*..., pp. 487–589; J. A. Lohne, “Alhazens Spiegelproblem,” in *Nordisk matematisk tidskrift*, **18** (1970), 5–35 (with bibliography).

A general survey of Ibn al-Haytham’s work in various fields is M. Schramm, “Ibn al-Haythams Stellung in der Geschichte der Wissenschaften,” in *Fikrun wa fann*, no. 6 (1965), 2–22. See also M. Nazīf and P. Ghalioungui, “Ibn at Haitham, an 11th-Century Physicist,” in *Actes du Xe Congrés international d’histoire des sciences*, Ithaca, 1962 (Paris, 1964), I, 569–571. No. 2 of the Publications of the Egyptian Society for the History of Science (Cairo, 1958) includes articles in Arabic by M. Nazīf, M. Madwar, M. ʿAbd al-Rāziq, M. Ghālī, and M. Hijāb on various aspects of Ibn al-Haytham’s thought. Some of these articles are reprints of previous publications. For a detailed table of contents see *Isis*, **51** (1960), 416.

Many of the European translations of Ibn al-Haytham’s works, cited in the first part of the bibliography, include historical and critical notes.

A. I. Sabra