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(*fl*. Baghdad, ca. 830)

mathematics, astronomy.

Al-Jawharī was one of the astronomers in the service of the ʻAbbāsid Caliph al-Ma'm–n (813-833). He participated in the astronomical observations which took place in Baghdad in 829-830 and in those which took place in Damascus in 832-833. Ibn al-Qiftī (d. 1248) describes him as an expert in the art of *tasyīr* (αφεσℓδ, "probagation"), the complex astrological theory concerned with determining the length of life of individuals (Ptolemy, Tetrabiblos III, 10), and adds that he was in charge of (qayyimāa) the construction of astronomical instruments. According to **I** bn al-Nadī (*fl*.987), he worked mostly (*al-ghālib ʻalayh*) in geometry.

Ibn al-Nadīm lists two works by al-Jawharī: *Kitāb Tafsīr Kitāb Uqlīdis* ("A Commentary on Euclid's Elements") and Kitāb al-Ashkā allatī zādahā fi rsquo;-maqāla '-ūlā min Uqlīdis ("Propositions Added to Book I of Euclid' Elements"). To this list Ibn al-Qiftī adds Kitāb al-Zīj ("A Book of Astronomical Tables"), which, he says, was well known among astronomers, being based on the observations made in Baghdad. None of these works has survived.

Nasīr al-Din al-T.ūsī (d. 1274), in his work devoted to Euclid's theory of parallels, al-Risāla '-shāfiya *'an al-shakk fī 'l-khutūt al-mutawāziya, ascribes to al-jawharī an "Emendation of the Elements" (Islāh li-Kitab al-Usūl)*, which may be identical with the "Commentary" *(Tafsīr)* mentioned by Ibn al-Nadīm and Ibn al-Qiftī. According to al-T.ūsī, this work included additions by al-Jawharī to the premises and the theorems of the *Elements*, the added theorems totaling"nearly fifty propositions."From among these al-Tūsī quotes six propositions constituting al-Jawharī's attempt to prove Euclid's parallels postulate.

Al-Jawharī's is the earliest extant proof of the Euclidean postulate written in Arabic. As a premise (which his book included among the common notions) al-Jawharī lays down a rather curious version of the so-called Eudoxus-Archimedes axiom: If from the longer of two unequal lines a half is cut off, and from the [remaining] half another half is cut off, and from many times; and if to the shorter line an equal line is added, and to the sum a line equal to it is added, and so on many times; there will remain of the halves of the longer line a line shorter than the multiples *(adʻāf)* of the shorter line. The axiom, which in different forms became a common feature of many Arabic proofs of the postulate, had already been applied in the same context in a demonstration attributed by Simplicius to an associate *(sāhib)* of his named Aghānīs or Aghānyns (Agapius[?]). This demonstration was known to mathematicians in Islam through the Arabic translation of a commentary by Simplicius on the premises of Euclid's *Elements*.The exact date of this translation is unknown, but it was available to al-Nayrīzī(*fl*. 895) and could have been made early in the ninth century.

The six propositions making up al-Jawharī's proof are the following:

(1)If a straight line falling on two straight lines makes the alternate angles equal to one another, then the two lines are parallel to one another; and if parallel to one another, then the distance from every point on one to the corresponding *(nazīra)* point on the other is always the same, that is, the distance from the first point in the first line to the first point in the second line is the same as that from the second point in the first line to the second point in the second line, and so on.

(2) If each of two sides of any triangle is bisected and a line is drawn joining the dividing points, then the remaining side will be twice the joining line.

(3) For every angle it is possible to draw any number of bases (sing.*qā'ida*).

(4) If a line divides an angle into two parts *(biqismayn)* and a base to this angle is drawn at random, thereby generating a triangle, and from each of the remainders of the sides containing the angle a line is cut off equal to either side of the generated triangle, and a line is drawn joining the dividing points, then this line will cut off from the line dividing the given angle a line equal to that which is drawn from the [vertex of the] angle to the base of the generated triangle.

(5) If any angle is divided by a line into two parts and a point is marked on that line at random, then a line may be drawn from that point on both sides. [of the dividing line] so as to form a base to that given angle.

(6) If from one line and on one side of it two lines are drawn at angles together less than two right angles, the two lines meet on that side.

Proposition (6) is, of course, Euclid's parallels postulate. Proposition (5) is, essentially, an attempt to prove a statement originally proposed by Simplicius, as we learn from a thirteenth-century document, a letter from 'Alam al-Dīn Qaysar to Nasīr al-Dīn al-Tūsī, which is included in manuscripts of the latter's al-*Risāla '-shāfiya*. The attempted proof, which makes use of the Eudoxus-Archimedes axiom, rests on proposition (4) and ultimately depends on (1) and (2). Proposition (3), used in the deduction of (4), also formed part of Simplicius' attempted demonstration. The first part of proposition (1) is the same as Euclid I, 27, and does not depend on the parallels postulate. To prove the second part al-Jawharī takes *HO = TO* on the two parallel lines by the transversal *HT* (figure 1). The alternate angles *AHT*,

HTD being equal, it follows that the corresponding angles and sides in the triangles *OHT, HTQ* are equal. He then takes *HL=TS* and similarly proves the congruence of the triangles *OST, QLH*, and hence the equality of the corresponding sides *OS, QL*. As the lines *OS, QL* join the extremities of the equal segments *Ol, SQ*, they may be said to join "corresponding points"of the latter, parallel lines, and they have been shown to be equal.

As al-Tūsī remarked, the proof fails to establish the intended general case: it does not establish the equality of lines joining "corresponding points" on the same side of the transversal or at unequal distances on either side of the transversal, nor does it show the equality of either *OS* or *LQ* to the transversal *HT* itself, even if one takes *HL*=TQ= *OH*= *ST*=. It is this failure which is overlooked in proving proposition (2), which, in turn, forms the basis of (4).

It seems clear that al-Jawharī took his starting point from Simplicius, although he himself appears to have been responsible for propositions (1) and (2). His attempt should therefore be grouped with those Arabic proofs clustered round Simplicius' propositions. Another proof belonging to this group was one proposed in the thirteenth century by Muhyi 'l-Dīn al-Maghribī, and still another is the anonymous treatise on parallel lines in Istanbul MS Carullah 1502, fols. 26v-27r, dated A.H. 894 (A.D. 1488-1489).

Also extant by al-Jawharī are some "additions" (ziyādāt) to book V of the *Elements*. Istanbul *MS* Feyzullah 1359, fols. 239v-240v, dated A. H. 868 (A. D. 1464-1465), contains only a fragment consisting of three propositions taken either from a longer work on that part of Euclid's book or, probably, from al-Jawharīs comprehensive commentary on or emendation of the Elements. The first of these propositions "proves" Euclid' definition of proportionals (book V, def.5), the second is the counterpart of the first, and the third is the same as Euclid's definition of "to have a greater ratio" (book V, def.7) Further, al-Tūsī quotes from the "Emendation" one proposition which al-Jawharī added after Euclid I, 13: If three straight lines are drawn from any point in different directions, the three angles thus contained by the three lines are together equal to four right angles.

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