Al-Khujandī, Ab | Encyclopedia.com

8-11 minutes

(*d*. 1000)

mathematics, astronomy.

Little is known of al-Khujandī's life. NāŞir al-Dīn al-Ţūsī states that he had the title of khan, which would lead one to believe that he was one of the khans of Khujanda on the Syr Darya, or jaxartes, in Transozania. For a time he lived under the patronage of the Buwayhid ruler Fakhr al-Dawla (976-997). He died in 1000.

Hājjī Khalī, Suter, and Brockelmann ascribe the following scientific works to al-Khujandi: *Risāla fi'l mayl wa'arḍ balad* ("On the Obliquity of the Ecliptic and the Latitude of the lands"), a text on geometry, and *Fi amal al-āla al-amma or al-ā al-shāmila* ("The Comprehensive Instrument").

According to NāŞir al-Dīn al-Ţūsī, al-Khujandī discovered *qānũn al-haiya*, the sine theorem relative to spherical triangles; it displaced the so-called theorem of menelaus. Abu'l-Wafā and Abũ NaŞr ibn Ali ibn Irāq (tenth century) also claimed to have discovered the sine theorem.

Al-Ţūsī, in his *Shakl al-Qaţţā*, gives al-Khujandīs solution related to the sine theorem.

Given the spherical triangle ABC whose sides AC and AB are completed into quadrants. RA, RD, RE, and RB are joined and form radii of the sphere.

RA ⊥plane of the circle *DE*

At the time

RA ⊥radii *RE* and *RD*

Erect the perpendicular *CF* on the plane of the circle *DE*. The perpendiculars *FN* and *CS* are erected on the plane *ABE*. *CFNS* is a rectangle, and *DE FN*.

The perpendicular *CT* is erected on the plane of the circle *AR* and is parallel to *RS*,

angle R = angle T = 90°

CF ⊥ RH

angle CFR = 90°

Therefore

CFRT forms a rectangle

In geometry al-Kjujandī proved (imperfectly) that the sum of two cubic numbers cannot be a cubic number.

Under the patronage of Fakhr al-Dawla, al-Khujandi constructed, on a bill called *jabal Tabũk*, in the vicinity of Rayy, an instrument called *al-suds al-Fakhrī* ("sixth of a circle") for the measurement of the obliquity of the ecliptic. The instrument can be described as follows.

Two walls, parallel to the meridian and 40 *zira* in height, are constructed. Near the southern wall, there is an arched ceiling with an aperture about three inches in diameter.

The floor directly underneath this aperture is excavated to a depth of forty *zira*. A wooden arc of 60°, forty *zira* in diameter and covered with sheets of copper, is placed between the two walls. Each degree of the arc is divided into sixty minutes and each minute into ten parts.

Since the sun's rays projected through the aperture form a cone, an instrument is needed to find the center of its base. This instrument, a circle with two diameters intersecting at right angles, coincides with base of the cone. It is moved as the cone moves until its center is at the meridian. The are between the plumbline and the altitude of the sun is equal to the cosine of the altitude of the sun.

Al-Khujandī says that this instrument is his own invention and adds, "We have attained to the degrees, minutes, and seconds with this instrument." According to Al-Bīrūnī, on this instrument each degree was subdivided into 360 equal parts and each ten-second portion was distinguished on the scale. It should be noted that before al-Khujandī the instruments did not indicate the seconds.

Before al-Khujandī a domed building was used to make solar measurements. According to Al-Bīrūnī, Abũ Sahl al-Kũhī (tenth century), at the Sharaf al-Dawla observatory (built in 988) constructed a domed building with an aperture on the top. This structure was a section of a sphere with a radius of 12.5 meters. Solar rays entered through the aperture and traced the daily trajectory of the sun.

After al-Khujandī, an instrument like *suds al-Fakhri* was constructed at the Maraāgha observatory (built in 1261). The huge meridian arc of the Samarkand observatory (built in 1420) apparently was similar to al-Khujandī's *suds al-Fakhrāi*.

The astronomers of Islam tried to increase the precision of their instruments and to make it possible to read the smaller fractions of a degree. For this purpose they increased the size of the instruments. Al-Khujandi and Ulugh Beg represent the extreme examples of this tendency. Increased size, however, causes slight displacement. Al-BiũrŪnī says that the apearture of *sude al-Fakhrī* sank by about one span because of the weight of the instrument. Experience with several large instruments proved disappointing and may have led to some doubts about the advisability of continuing to build them.

For the observations of the planets al-Khujandī constructed an armillary sphere and other instruments. He also built a universal instrument called *al-āla al-shāmila* (comprehensive instrument), which was used instead of the astrolabe or the quadrant. It could, however, be used for only one latitude. Al-Badi al-Asturlābi al-Baghaddi al-Isfahani (first half of the twelfth century) constructed an astrolabe used for all latitudes.

Al-Khujandī observed the sun and the planets and determined the obliquity of the ecliptic and the latitude of Rayy. He says that these observations were made in the presence of a group of distinguished astronomers and that they gave their written testimony concerning the observations. Using these observations, he compiled his *Zīj al-Fakhrī*. There is in the Library of the Iranian Parliament (Teheran MS 181) an incomplete copy of a *zīj* written in Persian about two centuries after the death of al-Khujandī, which may have been on his observations.

Al-Khujandī observed the meridian altitude of the sun on two consecutive days, 16 and 17 June 994, and found it to be 77°57'40". According to this result the entrance of the sun into the summer solstice must have taken place at midnight.

He then observed the sun on 14 December 994 and found the meridian altitude to be 30°53'35". On the following two days the weather was cloudy, and on the third day he found the meridian altitude of the sun to be 30°53'32". The entrance of the sun into the winter solstice must have taken place between these two observations. But the second observation is 3 "less than the first. Al-Khujandī calculated from them that its least meridian altitude must have been 30°53'2.30" (the least altitude of the sun).

Half of the difference between the greatest and the least altitudes of the sun is equal to the obliquity of the ecliptic:

1/2(77°57'40" - 30°53'2") = 23°32'19".

Al-Khujandī says that the Indians found the greatest obliquity of the ecliptic, 24°; Ptolemy, 23°51'; and he himself, 23°32'19". These divergent values cannot be due to defective instruments. Actually the obliquity of the ecliptic is not constant; it is a decreasing quantity.

Al-Khujandī calculated the latitude of Rayy by 76 adding the obliquity of the ecliptic (23°32'18.45") to the least altitude of the sun (30°53'2.30") and subtracting the result from 90°(90°—54°25'21.15" = 35°34'38.45").

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Sevim Tekeli