

Al-Khwārizmī | Encyclopedia.com

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(*b.* before 800; *d.* after 847)

mathematics, astronomy, geography.

Only a few details of al-Khwārizmī's life can be gleaned from the brief notices in Islamic bibliographical works and occasional remarks by Islamic historians and geographers. The epithet "al-Khwārizmī" would normally indicate that he came from Khwārizm (Khorezm, corresponding to the modern Khiva and the district surrounding it, south of the [Aral Sea](#) in central Asia). But the historian al-Tabarī gives him the additional epithet "al-Qutrubbullī," indicating that he came from Qutrubbullī, a district between the Tigris and Euphrates not far from Baghdad,¹ so perhaps his ancestors, rather than he himself, came from Khwārizm; this interpretation is confirmed by some sources which state that his "stock" (*asl*) was from Khwārizm.² Another epithet given to him by al-Tabarī, "al-Majūsī," would seem to indicate that he was an adherent of the old Zoroastrian religion. This would still have been possible at that time for a man of Iranian origin, but the pious preface to al-Khwārizmī's *Algebra* shows that he was an orthodox Muslim, so al-Tabarī's epithet could mean no more than that his forebears, and perhaps he in his youth, had been Zoroastrians.

Under the Caliph al-Ma'mūn (reigned 813-833) al-Khwārizmī became a member of the "House of Wisdom" (Dār al-Hikma), a kind of academy of scientists set up at Baghdad, probably by Caliph Harūn al-Rashīd, but owing its preeminence to the interest of al-Ma'mūn, a great patron of learning and scientific investigation. It was for al-Ma'mūn that al-Khwārizmī composed his astronomical treatise, and his *Algebra* also is dedicated to that ruler. We are told that in the first year of his reign (842) Caliph al-Wāthiq sent al-Khwārizmī on a mission to the chief of the Khazars, who lived in the northern Caucasus.³ But there may be some confusion in the source here with another "Muhammad ibn Mūsā the astronomer," namely, one of the three Banū Mūsā ibn Shākir. It is almost certain that it was the latter, and not al-Khwārizmī, who was sent, also by al-Wāthiq, to the Byzantine empire to investigate the tomb of the Seven Sleepers at Ephesus.⁴ But al-Khwārizmī survived al-Wāthiq (*d.* 847), if we can believe the story of al-Tabarī that he was one of a group of astronomers, summoned to al-Wāthiq's sickbed, who predicted on the basis of the caliph's horoscope that he would live another fifty years and were confounded by his dying in ten days.

All that can be said concerning the date and order of composition of al-Khwārizmī's works is the following. The *Algebra* and the astronomical work, as we have seen, were composed under al-Ma'mūn, in the earlier part of al-Khwārizmī's career. The treatise on Hindu numerals was composed after the *Algebra*, to which it refers. The treatise on the Jewish calendar is dated by an internal calculation to 823-824. The *Geography* has been tentatively dated by Nallino ("al-Khwārizmī," p. 487) to soon after 816-817, since one of the localities it mentions is Qiman, an Egyptian village of no importance whatever except that a battle was fought there in that year; but the inference is far from secure. The *Chronicle* was composed after 826, since al-Tabarī quotes it as an authority for an event in that year.⁵

The *Algebra* is a work of elementary practical mathematics, whose purpose is explained by the author (Rosen trans., p. 3) as providing "what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, lawsuits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computations, and other, objects of various sorts and kinds are concerned." Indeed, only the first part of the work treats of algebra in the modern sense. The second part deals with practical mensuration, and the third and longest with problems arising out of legacies. The first part (the algebra proper) discusses only equations of the first and second degrees. According to al-Khwārizmī, all problems of the type he proposes can be reduced to one of six standard forms. These are (here, as throughout, we use modern notation, although al-Khwārizmī's exposition is always rhetorical) the following:

(1) $ax^2 = bx$

(2) $ax^2 = b$

(3) $ax = b$

(4) $ax^2 + bx = c$

(5) $ax^2 + c = bx$

(6) $ax^2 = bx + c$,

where a , b , and c are positive integers. Such an elaboration of cases is necessary because he does not recognize the existence of negative numbers or zero as a coefficient. He gives rules for the solution of each of the six forms—for instance, form (6) is solved by

$$x^2 = (b/a)x + c/a,$$

He also explains how to reduce any given problem to one of these standard forms. This is done by means of the two operations *al-jabr* and *al-muqābala*. *Aljabr*, which we may translate as “restoration” or “completion,” refers to the process of eliminating negative quantities. For instance, in the problem illustrating standard form (1) (Rosen trans., p. 36), we have

$$x^2 = 40x - 4x^2.$$

By “completion” this is transformed to

$$5x^2 = 40x.$$

Al-muqābala, which we may translate as “balancing,” refers to the process of reducing positive quantities of the same power on both sides of the equation. Thus, in the problem illustrating standard form (5) (Rosen trans., p. 40), we have

$$50 + x^2 = 29 + 10x.$$

By *al-muqābala* this is reduced to

$$21 + x^2 = 10x.$$

These two operations, combined with the arithmetical operations of addition, subtraction, multiplication, and division (which al-Khwārizmī also explains in their application to the various powers), are sufficient to solve all types of problems propounded in the *Algebra*. Hence they are used to characterize the work, whose full title is *al-Kitāb al-mukhtasar fī hisāb al-jabr wa'l-muqābala* (“The Compendious Book on Calculation by Completion and Balancing”). The appellation *al-jabr wa'l-muqābala*, or *al-jabr* alone, was commonly applied to later works in Arabic on the same topic; and thence (via medieval Latin translations from the Arabic) is derived the English “algebra”.

In his *Algebra* al-Khwārizmī employs no symbols (even for numerals) but expresses everything in words. For the unknown quantity he employs the word *shay'* (“thing” or “something”). For the second power of a quantity he employs *māl* (“wealth,” “property”), which is also used to mean only “quantity.” For the first power, when contrasted with the second power, he uses *jidhr* (“root”). For the unit he uses *dirham* (a unit of coinage). Thus the problem

$$(x/3 + 1)(x/4 + 1) = 20$$

and the first stage in its resolution.

$$x^2 / 12 + x/3 + x/4 + 1 = 20,$$

appear, in literal translation, as follows:

A quantity: I multiplied a third of it and a *dirham* by a fourth of it and a *dirham*: it becomes twenty. Its computation is that you multiply a third of something by a fourth of something: it comes to a half of a sixth of a square (*māl*). And you multiply a *dirham* by a third of something: it comes to a third of something: and [you multiply] a *dirham*. Thus its total, [namely] a half of a sixth of a square and a third of something and a quarter of something and a *dirham*, is equal to twenty *dirhams*.⁶

After illustrating the rules he has expounded for solving problems by a number of worked examples, al-Khwārizmī, in a short section headed “On Business Transactions,” expounds the “rule of three,” or how to determine the fourth member in a proportion sum where two quantities and one price, or two price and one quantity, are given. The next part concerns practical mensuration. He gives rules for finding the area of various plane figures, including the circle, and for finding the volume of a number of solids, including cone, pyramid, and truncated pyramid. The third part, on legacies, consists entirely of solved problems. These involve only arithmetic or simple linear equations but require considerable knowledge of the complicated [Islamic law](#) of inheritance.

We are told that al-Khwārizmī’s work on algebra was the first written in Arabic.⁷ In modern times considerable dispute has arisen over the question of whether the author derived his knowledge of algebraic techniques from Greek or Hindu sources. Both Greek and Hindu algebra had advanced well beyond the elementary stage of al-Khwārizmī’s work, and none of the known works in either culture shows much resemblance in presentation to al-Khwārizmī’s. But, in favor of the “Hindu hypothesis,” we may note first that in his astronomical work al-Khwārizmī was far more heavily indebted to a Hindu work than to Greek sources; second, that his exposition is completely rhetorical, like Sanskrit algebraic works and unlike the one

surviving Greek algebraic treatise, that of Diophantus, which has already developed quite far toward a symbolic representation; third that the “rule of three” is commonly enunciated in Hindu works but not explicitly in any ancient Greek work; and fourth that in the part on mensuration two of the methods he gives for finding the circumference of the circle from its diameter are specifically Hindu.⁸

On the other hand, in his introductory section al-Khwārizmī uses geometrical figures to explain equations, which surely argues for a familiarity with Book II of Euclid’s *Elements*. We must recognize that he was a competent enough mathematician to select and adapt material from quite disparate sources in order to achieve his purpose of producing a popular handbook. The question of his sources is further complicated by the existence of a Hebrew treatise, the *Mishnat ha-Middot*, which is closely related in content and arrangement to the part of al-Khwārizmī’s work dealing with mensuration. If we adopt the conclusion of Gandz, the last editor of the *Mishnat ha-Middot*, that it was composed about A.D. 150,⁹ then al-Khwārizmī must be the borrower, either through an intermediary work or even directly—his treatise on the Jewish calendar (see below) shows that he must have been in contact with learned Jews. But the Hebrew treatise may be a later adaptation of al-Khwārizmī’s work. Gad Sarfatti (*Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages*, Jerusalem, 1968, 58-60) argues on linguistic grounds that the *Mishnat ha-Middat* belongs to an earlier Islamic period.

Al-Khwārizmī wrote on the use of the Hindu numerals, which has not survived in Arabic but has reached us in the form of a Latin translation (probably much altered from the original). The Arabic title is uncertain; it may have been something like *Kitāb hisāb al-‘adad al-hindi* (“Treatise on Calculation With the Hindu Numerals”),¹⁰ or possibly *Kitāb al-jam‘wa’ l-tafriq bi hisāb al-hind* (“Book of Addition and Subtraction by the Method of Calculation of the Hindus”).¹¹ The treatise, as we have it, expounds the use of the Hindu (or, as they are misnamed, “Arabic”) numerals 1 to 9 and 0 and the place-value system, then explains various applications. Besides the four basic operations of addition, subtraction, multiplication, and division, it deals with both common and sexagesimal fractions and the extraction of the square root (the latter is missing in the unique manuscript but is treated in other medieval works derived from it). In other words, it is an elementary arithmetical treatise using the Hindu numerals. Documentary evidence (eighth-century Arabic papyri from Egypt) shows that the Arabs were already using an alphabetic numeral system similar to the Greek (in which 1, 2, 3, . . . 9, 10, 20, 30, . . . 90, 100, 200, . . . 900 are each represented by a different letter). The sexagesimal modified placevalue system used in Greek astronomy must also have been familiar, at least to learned men, from the works such as Ptolemy’s *Almagest* which were available in Arabic before 800. But it is likely enough that the decimal place-value system was a fairly recent arrival from India and that al-Khwārizmī’s work was the first to expound it systematically. Thus, although elementary, it was of seminal importance.

The little of al-Khwā’ astronomical work was *Zij al-sindhind*.¹² This was appropriate, since it is based ultimately on a Sanskrit astronomical work brought to the court of Caliph al-Mansūr at Baghdad soon after 770¹³ by a member of an Indian political mission. That work was related to, although not identical with, the *Brāhmasphutasiddhānta* of Brahmagupta. It was translated into Arabic under al-Mansūr (probably by al-fazāri), and the translation was given the name *Zij alsindhind*. *Zij* means “set of astronomical tables”; and *sindhind* is a corruption of the Sanskrit *siddhānta*, which presumably was part of the title Hindu source work. This translation formed the basis of astronomical works (also called *Zij al-sindhind* by al-Fazāri and Ya‘qūb ibn Tāriq in the late eighth century. Yet these astronomers also used other sources for their work, notably the *Zij al-shāh*, a translation of a Pahlavi work composed for the Sassanid ruler Khosrau I (Anūshirwān) about 550, which was also based on Hindu sources.

Al-Khwārizmī’s work is another “revision” of the *Zij al-sindhind*. Its chief importance today is that it is the first Arabic astronomical work to survive in anything like entirety. We are told that there were two editions of it; but we know nothing of the differences between them, for it is available only in a Latin translation made by [Adelard of Bath](#) in the early twelfth century. This translation was made not from the original but from a revision executed by the Spanish Islamic astronomer al-Majrīṭī (d.1007-1008) and perhaps further revised by al-Majrīṭī’s pupil Ibn al-Saffār (d. 1035).¹⁴ We can, however, get some notions of the original form of the work from extracts and commentaries made by earlier writers.¹⁵ Thus from the tenth-century commentary or Ibn al-muthannā we learn that al-Khwārizmī constructed his tables of sines to base 150 (a common Hindu parameter), whereas in the extant tables base 60 (more usual in Islamic sine tables) is employed. From the same source we learn that the epoch of the original tables was era Yazdegerd (16 June 632) and not the era Hijra (14 July 622) of al-Majrīṭī’s revision.¹⁶

The work as we have it consists of instructions for computation and use of the tables, followed by a set of tables whose form closely resembles that made standard by Ptolemy. The sun, the moon, and each of the five planets known in antiquity have a table of mean motion (s) and a table of equations. In addition there are tables for computing eclipses, solar declination and right ascension, and various trigonometrical tables. It is certain that Ptolemy’s tables, in their revision by Theon of Alexandria, were already known to some Islamic astronomers; and it is highly likely that they influenced, directly or through intermediaries, the form in which al-Khwārizmī’s tables were cast.

But most of the basic parameters in al-Khwārizmī’s tables are derived from Hindu astronomy. For all seven bodies the mean motions, the mean positions at epoch, and the positions of the apogee and the node all agree well with what can be derived from the *Brāhmasphutasiddhānta*. The maximum equations are taken from the *Zij al-shāh*. Furthermore, the method of computing the true longitude of a planet by “halving the equation” prescribed in the instructions is purely Hindu and quite alien to Ptolemaic astronomy.¹⁷ This is only the most notable of several Hindu procedures found in the instructions. The only tables (among those that can plausibly be assigned to the original *Zij*) whose content seems to derive from Ptolemy are the tables of solar declination, of planetary stations, of right ascension, and of equation of time. Nowhere in the work is there any

trace of original observation or of more than trivial computations by the author. This appears strange when we learn that in the original introduction (the present one must be much altered) al-Khwārizmī discussed observations made at Baghdad under al-Mamūn to determine the obliquity of the ecliptic.¹⁸ The value found, 23° 33', was fairly accurate. Yet in the tables al-Khwārizmī adopts the much worse value of 23° 51' from Theon. Even more inexplicable is why, if he had the Ptolemaic tables available, he preferred to adopt the less accurate parameters and obscure methods of Hindu astronomy.

The *Geography*, *Kitāb sūrat al-ard* ("Book of the Form of the Earth"), consists almost entirely of lists of longitudes and latitudes of cities and localities. In each section the places are arranged according to the "seven climata" (in many ancient Greek geographical works the known world was divided latitudinally into seven strips known as "climata", each clima being supposed to enjoy the same length of daylight on its longest day), and within each clima the arrangement is by increasing longitude. Longitudes are counted from an extreme west meridian, the "shore of the western ocean." The first section lists cities; the second, mountains (giving the coordinates of their extreme points and their orientation); the third, seas (giving the coordinates of salient points on their coastlines and a rough description of their outlines); the fourth, islands (giving the coordinates of their centers, and their length and breadth); the fifth, the central points of various geographical regions; and the sixth, rivers (giving their salient points and the towns on them).

It is clear that there is some relationship between this work and Ptolemy's *Geography*, which is a description of a world map and a list of the coordinates of the principal places on it, arranged by regions. Many of the places listed in Ptolemy's work also occur in al-Khwārizmī's, with coordinates that are nearly the same or systematically different. Yet it is very far from being a mere translation or adaptation of Ptolemy's treatise. The arrangement is radically different, and the outline of the map which emerges from it diverges greatly from Ptolemy's in several regions. Nallino is surely right in his conjecture that it was derived by reading off the coordinates of a map or set of maps based on Ptolemy's but was carefully revised in many respects. Nallino's principal argument is that al-Khwārizmī describes the colors of the mountains in a way which could not possibly represent their physical appearance but might well represent their depiction on a map. To this we may add that in those areas where al-Khwārizmī agrees, in general, with Ptolemy, the coordinates of the two frequently differ by 10, 15, 20, or more minutes, up to one degree of arc; such discrepancies cannot be explained by scribal errors but are plausible by supposing a map as intermediary. The few maps accompanying the sole manuscript of al-Khwārizmī's *Geography* are crude things; but we know that al-M'mūn had had constructed a world map, on which many savants worked. According to al-Mas'ūdi, the source of this information, al-Ma'ū's map was superior to Ptolemy's.¹⁹ Nallino makes the plausible suggestion that al-Khwārizmī's *Geography* is based on al-Ma'mūn's world map (on which al-Khwārizmī himself had probably worked), which in turn was based on Ptolemy's *Geography*, which had been considerably revised.

The map which emerges from al-Khwārizmī's text is in several respects more accurate than Ptolemy's, particularly in the areas ruled by Islam. Its most notable improvement is to shorten the grossly exaggerated length of the Mediterranean imagined by Ptolemy. It also corrects some of the distortions applied by Ptolemy to Africa and the Far East (no doubt reflecting the knowledge of these areas brought back by Arab merchants). But for Europe it could do little more than reproduce Ptolemy; and it introduces errors of its own, notably the notion that the Atlantic is an inland sea enclosed by a western continent joined to Europe in the north.

The only other surviving work of al-Khwārizmī is a short treatise on the Jewish calendar, *Istikhrāj al-yahūd* ("Extraction of the Jewish Era"). His interest in the subject is natural in a practicing astronomer. The treatise describes the Jewish calendar, the 19-year intercalation cycle, and the rules for determining on what day of the week the first day of the month Tishrī shall fall; calculates the interval between the Jewish era (creation of Adam) and the Seleucid era; and gives rules for determining the mean longitude of the sun and moon using the Jewish calendar. Although a slight work, it is accurate, well informed, and of importance as evidence for the antiquity of the present Jewish calendar.

Al-Khwārizmī wrote two works on the astrolabe, *Kitāb 'amal al-asturlāb* ("Book on the Construction of the Astrolabe") and *Kitāb al-'amal bl' asturlāb* ("Books on the Operation of the Astrolabe"). Probably from the latter is drawn the extract found in a Berlin manuscript of a work of the ninth-century astronomer al-Farghānī. This extract deals with the solution of various astronomical problems by means of the astrolabe—for instance, determination of the sun's altitude, of the ascendant, and of one's terrestrial latitude. There is nothing surprising in the content, and it is probable that al-Khwārizmī derived it all from earlier works on the subject. The astrolabe was a Greek invention, and we know that there were once ancient Greek treatises on it. Astrolabe treatises predating al-Khwārizmī survive in Syriac (by Severus Sebokht, seventh century) and in Arabic (now only in Latin translation, by Māshā'illāh, late eighth century).

The *Kitāb al-tārikh* ("Chronicle") of al-Khwārizmī does not survive, but several historians quote it as an authority for events in the Islamic period. It is possible that in it al-Khwārizmī (like his contemporary Abū Ma'shar) exhibited an interest in interpreting history as fulfilling the principles of astrology.²⁰ In that case it may be the ultimate source of the report of Hamza al-Isfahānī about how al-Khwārizmī cast the horoscope of the Prophet and showed at what hour Muhammad must have been born by astrological deduction from the events of his life.²¹ Of a book entitled *Kitāb al-rukhāma* ("On the Sundial") we know only the title, but the subject is consonant with his other interests.

Al-Khwārizmī's scientific achievements were at best mediocre, but they were uncommonly influential. He lived at a time and in a place highly favorable to the success of his works: encouraged by the patronage of the caliphs, Islamic civilization was beginning to assimilate Greek and Hindu science. The great achievements of Islamic science lay in the future, but these early works which transmitted the new knowledge ensured their author's lasting fame. Between the ninth and twelfth centuries

algebra was developed to a far more sophisticated level in Islamic lands, aided by the spread of knowledge of Diophantus' work. But even such advanced algebraists as al-Karājī (d. 1029) and 'Umar al-Khayyāmī (d. 1123-1124) still used the rhetorical exposition popularized by al-Khwārizmī.

Al-Khwārizmī's *Algebra* continued to be used as a textbook and praised highly (see, for instance, the quotations in Hajī Khalfā, V, 67-69). The algebraic part proper was twice translated into Latin in the twelfth century (by Robert of Chester and by [Gerard of Cremona](#)) and was the chief influence on medieval, European algebra, determining its rhetorical form and some of its vocabulary (the medieval *cossa* is a literal translation of Arabic *shay'*, and *census of māl*). The treatise on Hindu numerals, although undoubtedly important in introducing those useful symbols into more general use in Islamic lands, achieved its greatest success only when introduced to the West through Latin translation in the early twelfth century (occasional examples of the numerals appeared in the West more than a century earlier, but only as isolated curiosities). The work quickly spawned a number of adaptations and offshoots, such as the *Liber alghoarismi* of John of Seville (ca. 1135), the *Algorismus* of John of Seville (ca. 1250), and the *Liber ysagogarum Alchorizmi* (twelfth century). In fact, al-Khwārizmī's name became so closely associated with the "new arithmetic" using the Hindu numerals that the Latin form of his name, *algorismus*, was given to any treatise on that topic. Hence, by a devious path, is derived the Middle English "augrim" and the modern "algorism" (corrupted by false etymology to "algorithm").

The other works did not achieve success of such magnitude; but the Zīj continued to be used, studied, and commented on long after it deserved to be superseded. About 900 al-Battānī published his great astronomical work, based on the *Almagest* and tables of Ptolemy and on his own observations. This is greatly superior to al-Khwārizmī's astronomical work in nearly every respect, yet neither al-Battānī's opus nor the other results of the prodigious astronomical activity in Islamic lands during the ninth and tenth centuries drove al-Khwārizmī's Zīj from the classroom. In fact it was the first such work to reach the West, in Latin translation by [Adelard of Bath](#) in the early twelfth century. Knowledge of this translation was probably confined to England (all surviving manuscripts appear to be English), but many of al-Khwārizmī's tables reached a wide audience in the West via another work, the *Toledan Tables*, a miscellaneous assembly of astronomical tables from the works of al-Khwārizmī, al-Battānī, and al-Zarqāl which was translated into Latin, probably by [Gerard of Cremona](#), in the late twelfth century and which, for all its deficiencies, enjoyed immense popularity throughout Europe for at least 100 years.

The *Geography* too was much used and imitated in Islamic lands, even after the appearance of good Arabic translations of Ptolemy's *Geography* in the later ninth century caused something of a reaction in favor of that work. For reasons rather obscure the medieval translators of Arabic scientific works into Latin appear to have avoided purely geographical treatises, so al-Khwārizmī's *Geography* was unknown in Europe until the late nineteenth century. But some of the data in it reached medieval Europe via the lists of longitudes and latitudes of principal cities, which were commonly incorporated into ancient and medieval astronomical tables.²²

NOTES

1. Al-Tabarī, de Goeje ed., III, 2, 1364.

2. E.g., *Fihrist*, Flügel ed., I, 274; followed by Ibn al-Qiftī, Lippert, ed., p. 286.

3. Al-Muqaddasi, de Goeje ed., p. 362.

4. The story is found in several sources, all of which call the envoy "Muhammad ibn Mūsā the astronomer." Only one, al-Mas'ūdī, *Kitāb al-tanbīth*, de Goeje ed., p. 134, adds "ibn Shākir." For the full story see Nallino, "Al-Khwārizmī, pp. 465-466.

5. Al-Tabarī, de Goeje ed., III, 2, 1085.

6. Rosen, text, p. 28, somewhat emended. The translation is mine.

7. E.g., Hajī Khalfā, Flügel ed., V, 67, no. 10012.

8. These are $c = \sqrt{10} d^2$ and $c = 62832 d/20000$. See Rosen's note on pp. 198-199 of his ed. The second value, which is very accurate, is attested for the later *Paulīśasiddhānsiddhānta* and also, significantly, for Ya'qūb ibn Tāriq, al-Khwārizmī's immediate predecessor, by al-Bīrūnī, *India*, Sachau trans., I, 168-169. Paulīśa presumably derived it from the *āryabhatīya* (see the *āryabhatīya*, Clark ed., p. 28).

9. See Gandz's ed. of the *Mishnat ha-Middot*, pp. 6-12.

10. Some such title seems to be implied by Ibn al-Qiftī, Lippert ed., pp. 266-267.

11. As conjectured by Ruska, "Zur ältesten arabischen Algebra," pp. 18-19.

12. *Fihrist*, Flügel ed., I, 274.
13. See, e.g., al-Būni, *India*, Sachau trans., II, 15.
14. For the latter revision see Ibn Ezra, *Libro de los fundamentos*, p. 109.
15. For a list of these see Pingree and Kennedy, commentary on al-Hāshimī's *Book of the Reasons Behind Astronomical Tables.*, sec. II; sec also in biblio. (below).
16. For the value 150 see, e.g., Goldstein, *Ibn al-Muthanná*, p. 178. For the epoch, *ibid.*, p. 18.
17. On "halving the equation" see Neugebauer, *al-Khwārizmī*, pp. 23-29.
18. Ibn Yūnus, quoted by Nallino, "Al-Khwārizmī," p. 469.
19. Al-Mas'ūdi, *Kitāb al-tanbih*, de Goeje ed., p. 33.
20. On Abū Ma'shar see especially Pingree, *The Thousands of Abū Ma'shar*.
21. Hamza, *Ta'rikh*, Beirut ed., p. 126. However, Hamza quotes this not directly from the *Chronicle* (which he use elsewhere, *ibid.*, p. 144) but from Shān's book of Abū Ma'shar' table talk, so the ultimate source might be a conversation between al-Khwārizmī and Abū Ma'shar.
22. The list in the *Toledan Tables*, which is certainly in part related to al-Khwārizmī's *Geography*, is printed with commentary in Toomer, "Toledan Tables," pp. 134-139.

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