Al-T?usi, Mu?ammad Ibn Mu?ammad Ibn Al-?asan Usually Known as Nas?ir Al-Din | Encyclopedia.com

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(*b*. Tüs, Persia, 18 February 1201; *d*. Kadhimain, near Baghdad, 26 June 1274)

astronomy, mathematics, mineralogy, logic, philosophy, ethics, theology.

Life . Nasīr al-Dīn, known to his compatriots as Muhaqqiq-i Ṭūsī, Khwāja-yi Ṭūsī, or Khwāja Nasīr, is one of the best-known and most influential figures in Islamic intellectual history. He studied the religious sciences and elements of the "intellectual sciences" with his father, a jurisprudent of the Twelve Imām school of Shīism at Tūs. He also very likely studied logic, natural philosophy, and metaphysics with his maternal uncle in the same city. During this period he also received instruction in algebra and geometry. Afterward he set out for Nīshāpūr, then still a major center of learning, to complete his formal advanced education; and it was in this city that he gained a reputation as an outstanding scholar. His most famous teachers were Farīd al-Dīn al-Dāmād, who through four intermediaries was linked to Ibn Sīnā and his school and with whom Ṭūsī studied philosophy: Qutb al-Dīn al-Masrī, who was himself the best known student of Fakhr al-Dīn alRāzī (1148-1209), with whom al-Ṭūsī studied medicine, concentrating mostly on the text of Ibn Sīnā's *Canon:* and Kamāl al-Dīn ibn Yūnus (1156-1242), with whom he studied mostly mathematics.

This period was one of the most tumultuous in Islamic history: Mongols were advancing toward Khurasan from Central Asia. Therefore, although already a famous scholar, al-Ṭūsī could not find a suitable position and the tranquillity necessary for a scholarly life. The only islands of peace at this time in Khurasan were the Ismā'īlī forts and mountain strongholds, and he was invited to avail himself of their security by the Ismā'īli ruler, Nasīr al-Dīn Muhtashim. Al-Ṭūsī accepted the invitation and went to Quhistan, where he was received with great honor and was held in high esteem at the Ismā-īlī court, although most likely he was not free to leave had he wanted to. The date of his entrance into the sevice of the Ismā-īlī rulers is not known exactly but was certainly sometime before 1232, for it was during that year that he wrote his famous *Akhlāq-i nāsirī* for the Ismā-īlī ruler. During his stay at the various Ismā-īlī strongholds. including Alamut, al-Ṭūsī wrote a number of his important ethical, logical, philosophical, and mathematical works, including *Asās al-iqtibās* (on logic) and *Risāla-yi mu'iniyya* (on astronomy). His fame as a scholar reached as far as China.

Hūlāgū ended the rule of the Ismā-īlīs in northern Persia in 1256. His interest in astrology, and therefore his respect for astronomers, combined with al-Ṭūsī fame in this field, made Hūlāgū especially respectful toward him after he had captured Alamut and "freed" al-Ṭūsī from the fort. Hence forth al-Ṭūsī remained in the service of Hūlāgū as his scientific adviser and was given charge of religious endowments (*awqāf*) and religious affairs. He accompanied Hūlāgū on the expedition that led to the conquest of Baghdad in 1258 and later visited the Shī'ite centers of Iraq. such as Hilla.

Having gained the full confidence of Hūlāgū, and benefiting from his interest in astrology, al-Ṭūsī was able to gain his approval to construct a major observatory at Marāgha. Construction began in 1259, and the Īlkhānī astronomical tables were completed in 1272 under Abāqā, after the death of Hūlāgū. In 1274, while at Baghdad, al-Ṭūsī fell ill and died a month later. He was buried near the mausoieum of the seventh Shī'ite, imām, mūsā al-Kāzim, a few miles from Baghdad.

Works. Nearly 150 treatises and letters by Nasīr al-Dīn al-Ṭūsī are known, of which twenty-five are in Persian and the rest in Arabic. There is even a treatise in geomancy that al-Ṭūsī wrote in Arabic, Persian, and Turkish, demonstrating his mastery of all three languages. It is said that he also knew Greek. His writings concern nearly every branch of the Islamic sciences, from asatronomy to philosophy and from the occult sciences to theology. Of the two, Ibn Sīnā was the better physician and al-Ṭūsī the greater mathematican and more competent writer in Persian. But otherwise their breadth of knowledge and influence can be compared very favorably. Moreover, the writings of al-Ṭūsī are distinguished by the fact that so many became authoritative works in the Islamic world.

Al-Ṭūsī composed five works in logic, of which *Asās al-iqtibās* ("Foundations of Inference"), written in Persian, is the most important. In fact, it is one of the most extensive of its kind ever written. surpassed only by the section on logic of Ibn Sīnā's al-Shifā' In mathematics al-Ṭūsī composed a series of recensions (*taḥrīr*) upon the works of Autolycus, Aristarchus, Euclid, Apollonius, Archimedes, Hypsicles. Theodosius, Menelaus, and Ptolemy. The texts studied by students of mathematics between Euclid's *Elements* and Ptolemy's *Almagest* were known as the "intermediate works" (mutawassitāt): and the collection of al-Ṭūsī's works concerning this "intermediate" body of texts became standard in the teaching of mathematics,

along with his recensions of Euclid and Ptolemy. He also wrote many original treatises on arithmetic, geometry, and trigonometry, of which the most important are *jawāmi' al-hisāb bi'l-takht wa'l turāb* ("The Comprehensive Work on Computation with Board and Dust"), *al-Risāla al-shāfiya* ("The Satisfying Treatise"), and *Kashf al-qinā' fi asrār shakl al-qitā'*. known as the *Book of the Principle of Transversal*. which was translated into Latin and influenced Regiomontanus. The best-known of al-Ṭūsī' numerous astronomical works is *Zīj-i īlkhānī* ("The Īlkhānī Tables"), written in Persian and later translated into Arabic and also partially into Latin, by John Greaves, as *Astronomia quaedam ex traditions Shah Cholgii Persae una cum hypothesibus planetarum* (London 1650). Other major astronomical works are *Tadhkirah* ("Treasury of Astronomy") and his treatises on particular astronomical subjects, such as that on the astrolabe. He also translated the *Suwar al-Kawākib* ("Figures of the Fixed Stars") of 'Abd al-Rahmān al-Sū from Arabic into Persian. In the other sciences al-Ṭūsī produced many works. of which *Tanksūkh-nāma* ("The Book if precious Materials") is particularly noteworthy. He also wrote on astrology.

In philosophy, ethics, and thelogy al-Ṭūsī composed a commentary on al-Ishārāt wa'l-tanbīhāt ("The Book of Directives and Remarks") of Ibn Sīnā; the *Akhlāq-i nāsirī (Nasiren Ethics),* the best-known ethical work in the Persian language. and the *Tajrīd* ("Catharsis"), the main source book of Shi'ite theology, upon which over 400 commentaries and glosses have been composed. Al-Ṭūsī wrote outstanding expositions of Ismā'īlī doctrine, chief among them the Tasawwurāt ("Notions") and composed mystical treatises, such as Awsāf al-ashrāf ("Qualifications of the Noble").

Al-Ṭūsī also composed lucid and delicate poetry, molt in Persian.

Scientific Achievements . In logic al-Ṭūsī followed the teachings of Ibn Sīnā but took a new step in studying the relation between logic and mathematics. He also elucidated the conditional conjunctive (*iqtirānī*) also syllogism better than his predecessor. He converted logical terms into mathematical sign and clarified the mathematical signs employed by Abu'l-Barakāt in his *Kitāb al-mu'tabar* ("The Esteemed Book") Al-Ṭūsī distinguished between the meaning of "substance" in the philosophical sense and its use as a scientific term, and clarified the relation of the categories with respect to metaphysics and logic.

In mathematics al-Ṭūsī contributions were mainly in arithmetic, geometry, and trigonometry. He continued the work of al-Khayyāmī extending the meaning of number to include irrational. In his *Shakl al-gitā'* he showed the commutative property of multiplication between pairs of ratios (which are real numbers) and stated that every ration is a number. Jairāmi' al-hisāb, which marks an important stage in the development of the Indian numerals, contains a reference to Pascal's triangle and the earliest extant method of extracting fourth and higher roots of numbers. In collaboration with his colleagues at Marāgha, al-Ṭūsī also began to develop computational mathematics, which was pursued later by al-Kāshī and other mathematicians of the Tīmūrid period.

In geometry al-Ṭūsī also followed the work of al-Khayyāmī and in his al-Risāla al-shāfiya he examined Euclid's fifth postulate. His attempt to prove it through Euclidean geometry was unsuccessful. He demonstrated that in the quadrilateral *ABCD*, in which *AB* and *DC* are equal and both perpendicular to *BC*, and the angles *A* and *D* are equal, if angles *A* and *D* are acute, the sum of the angles of a triangle will be less than 180° [1] This is characteristic of the geometry of Lobachevski and shows that al-Ṭūsī like al-Khayyāmī, had demonstrated some of the properties of the then unknown non-Euclidean geometry. The quadrilateral associated with Saccheri was employed centuries before him by Thābit ibn Qurra,al-Ṭūsī, and al-Khayyāmī.

Probably al-Ṭūsī most outstanding contribution to mathematics was in trigonometry. In Shakl algitā', which follows the earlier work of Abu'l-Wafā', Mansūr ibn 'Irāq, and al-Bīrūnī,al-Ṭūsī for the first time, as far as modern research has been able to how, developed trigonometry without using Menelaus' theorem or astronomy. This work is really the first in history on trigonometry as an independent branch of pure mathematics and the first in which all six cases for a right-angled spherical triangle are set forty. If c = the hypotenuse of a spherical trangle. then:

$\cos c = \cos a \cos b \cot A = \tan b \cot c$

$\cos c = \cot A \cot B \sin b = \sin c \sin B$

$\cos A = \cos a \sin B \sin b = \tan a \cot A.$

He also presents the theorem of sines:

It is described clearly for the first time in this book, a landmark in the history of mathematics.

Al-Ṭūsī is best-known as an astronomer. With Hūlāgū's support he gained the necessary financial assistance and supervised the construction of the first observatory in the modern sense. Its financial support, based upon endowment funds; its lifespan, which exceeded that of its funds; its use as a center of instruction in science and philosophy; and the collaboration of many scientists in its activities mark this observatory as a major scientific institution in the history of sciences. The observatory was staffed by Qutb al-Dīn al-Shīrāzī, Muhyi'l-Dīn al-Maghribī, Fakhr al-Dīn al-Marāghī, Mu'ayyad al-Dīn al-'Urdi . 'Ali ibn 'Umar al-Qazwīnī, Najm al-Dīn Dabīrān al-Kātibī al- Qazwīnī, Athīr al-Dīn al-Abharī, al-Ṭūsī's sons Asīl al-Dīn and Sadr al-Dīn, the Chinese scholar Fao Mun-ji, and the librarian Kamāl al-Din al-Aykī It had excellent instruments made by Mu'ayyad

al-Din al-'Urdi in 1261 – 1262, including a gaint mural quadrant, an armillary sphere with five rings and an alidade, a solstitial armill, an azimuth ring with two quadrants, and a parallactic ruler. It was also equipped with a fine library with books on all the sciences. Twelve years of observation and calculation led to the completion of the *Zīj-i īlkhānī*in 1271, to which Muhyī'1-Dīn al-Maghribī later wrote a supplement. The work of the observatory was not confined to astronomy, however; it played a major role in the revival of all the sciences and philosophy.

Al-Ṭūsī's contributions to astronomy, besides the *Zīj* and the recession of the *Almagest* consists of a criticism of Ptolemaic astronomy in his *Tadhkira*, which is perhaps the most thorough exposition of the shortcomings of Ptolemaic astronomy in medieval times, and the proposal of a new theory of planetary motion. The only new mathematical model to appear in medieval astronomy, this theory influenced not only Qutb al-Dīn al-Shīrāzī and Ibn al-Shātir but also most likely Copernicus, who followed closely the planetary models of Nasīr al-Dīn's students. In chapter 13 of the second treatise of the *Tadhkira*, al-Ṭūsī proves that "if one circle rolls inside the periphery of a stationary circle, the radius of the first being half the second, then any point on the first describes a straight line, a diameter of the second,"[2] E. S. Kennedy, who first discovered this late medieval planetary theory issuing from Marāgha, interprets it as "a linkage of two equal length vectors, the second rotating with constant velocity twice that of the first and in a direction opposite the first,"[3] He has called this the "Ṭūsī-couple" and has demonstrated (see Figures 1 and 2) its application by al-Ṭūsī, Qutb al-Dīn Ibn al-shātir to planetary motion and its comparsion with the Ptolemaic model.[4]

This innovation, which originated with al-Ṭūsī, is without doubt the most important departure from Ptolemaic astronomy before modern times. Except for the heliocentric thesis, the "novelty" of Copernicus' astronomy is already found in the works of al-Ṭūsī and his followers, which probably reached Copernicus through Byzantine intermediaries.

The most important mineralogical work by al-Ṭūsī is *Tanksūkh-nāma*, written in Persian and based on many of the earlier Muslim sources, such as the works of Jābir ibn Hayyān, al-Kindī, Muhammad ibn Zakariyyā', al-Rāzī, 'Utārid ibn Muhammad, and especially al-Bīrūnī, whose *Kitāb al-jamāhir fi ma'rifat al-jawāhir* ("The book of Multitudes Concerning the Knowledge of Precious Stones") is the main source of al-Ṭūsī's work. In fact the *Tanksūkh-nāma,* which derives its name from the Turco-Mongolian word meaning "Something precious," probably is second in importance in the annals of Muslim mineralogy only to al-Bīrūnī's masterpiece.

Al-Ṭūsī' work comprises four chapters. In the first he discusses the nature of compounds; the four elements, their mixture, and the coming into being of a "fifth quality" called temperament *(mizāj)*, which can accept the forms of different species; and the role of vapors and the rays of the sun in their formation, in all of this following closely the theories of Ibn Sīnā's *De mineralibus*. An interesting section is devoted to colors, which al-Tūsī believes result from the mixture of white and blank. In jewels, colors are due to the mixture of earthy and watery elements contained in the substance of the jewel.

The second chapter is devoted exclusively to jewels, their qualities, and their properties. Special attention is paid to rubies, the medical and occult properties of which are discussed extensively. In the third chapter al-Ṭūsī turns to metals and gives an alchemical theory of metallic formation, calling sulfur the father and mercury the mother of metals. He also enumerates the seven traditional metals, including *khārsīnī*. Like so many Muslim philosopher-scientists, al-Ṭūsī accepts the cosmological and mineralogical theories of alchemy concerning the formation of metals without belonging to the alchemical tradition or even discussing the transmutation of base metal into gold. A section on perfumes ends the book, which is one of the major sources of Muslim mineralogy and is valuable as a source of Persian scientific vocabulary in this field.

Of all the major fields of science, al-Ṭūsī was least interested in medicine, which he neverthless studied, generally following the teachings of Ibn Sīnā. He also composed a few works on medicine including *Qawānīn al-tibb* ("Principles of Medicine") and a commentary on Ibn Sīnā's *Canon*, and exchanged letters with various medical authorities on such subjects as breathing and temperament. He expressed certain differences of opinion with Ibn Sīnā concerning the temperament of each organ of the body but otherwise followed his teachings. Al-Tāsī's view of medicine was mainly philosophical; and perhaps his greatest contribution was in psychosomatic medicine, which he discusses, among other places, in his ethical writings, especially *Akhlāq-i nāsirī* (*Nasirean Ethics*).

Al-Ṭūsī was one of the foremost philosophers of Islam, reviving the Peripatetic (*Mashshā'ī*) teachings of Ibn Sīnā after they had been eclipsed for

nearly two centuries by *Kalām*. He wrote a masterful commentary on the *Ishārāt wa'l-tanbīhāt* of Ibn Sīnā, which Fakhr al-Dīn al-Rāzī had attacked severely during the previous century. In this work, which is unusual among Muslim philosophical works for its almost mathematical precision, al-Ṭūsī succeeded in rekindling the light of philosophy in Islam. But while claiming in this work to be a mere follower of Ibn Sīnā, in several places questions of God's knowledge of particulars, the nature of space, and the createdness of the physical world clearly shows his debt to Shihāb al-Dīn al-Suhra-wardī and some of the Muslim theologians. Al-Ṭūsī in fact marks the first stage in the gradual synthesis of the Peripatetic and Illuminationist (*ishrāqī*) schools, a tendency that became clearer in the writings of his foremost student, Qutb al-Dīn al-Shīrāzī. He also wrote many philosophical treatises in Persian, so that his prose in this field must be considered, along with the writings of Nāsir-i Khusraw, Suhrawardī, and Afdal al-Dīn al-Kāshānī, as the most important in the Persian language.

In ethics al-Ṭūsī composed two major works, both in Persian: the *Akhlāq-i muhtashimī* ("The Muhtashimī Ethics") and the much better-known *Nasirean Ethics*, his most famous opus. Based upon the *Tahdhīb al-akhlāq* ("The Refinement of

Character") of Muskūya (Miskawayh), the *Nasirean Ethics* expounds a philosophical system combining Islamic teachings with the ethical theories of the Aristotelian and, to a certain extent, the Platonic traditions. The work also contains an elaborate discussion of psychology and psychic healing. For centuries it has been the most popular ethical work among the Muslims of India and Persia.

In Twelve Imām Shi'ism, al-Ṭūsī is considered as much a theologian as a scientist and philosopher because of his *Tajrīd*, which is still central to Shi'ite theological education. A work of great intellectual rigor, the *Tajrīd* represents the first systematic treatment of Shi1ite *Kalām* and is therefore the foundation of systematic theology for the Twelve Imām Shi'ites. In the history of Islam, which is known for its multitalented figures of genius, it is not possible to find another person who was at once an outstanding astronomer and mathematician and the most authoritative theologian of a major branch of Islam.

Influence . Al-Ṭūsī's influence, especially in eastern Islam, was immense. Probably, if we take all fields into account, he was more responsible for the revival of the Islamic sciences than any other individual. His bringing together so many competent scholars and scientists at Marāgha resulted not only in the revival of mathematics and astronomy but also in the renewal of Islamic philosophy and even theology. Al-Ṭūsī's works were for centuries authoritative in many fields of Islamic learning; and his students, such as Qutb al-Dīn and 'Allāma Hillī, became outstanding scholars and scientists. His astronomical activities influenced the observatories at Samarkand and Istanbul and in the West to a much greater extent than was thought to be the case until recently; and his mathematical studies affected all later Islamic mathematics. In fact, the work of al-Ṭūsī and his collaborators at Marāgha moved eastward to influence Chinese science, which, as a result of the Mongol invasion, had a much closer relationship with Islam. The school of al-Ṭūsī also influenced later Indian science as cultivated under the Moguls and even as late as the eighteenth century, as can be seen in the observatory constructed by Jai Singh II, which indirectly reflects the observatory of Marāgha.

In the West al-Ṭūsī is known almost entirely as an astronomer and mathematician whose significance, at least in these fields, is becoming increasingly evident. In the Muslim East he has always been considered as a foremost example of the "wise man" (*ḥakīm*), one who, while possessing an acute analytical mind, which he devoted to mathematical, astronomical, and logical studies, extended the horizon of his thought to embrace philosophy and theology and even journeyed beyond the limited horizon of all mental activity to seek ultimate knowledge in the ecstasy provided by gnosis (*'irfān*) and Sufism.

NOTES

1. E. S. Kennedy, "The Exact Sciences in Iran Under the Selijuqs and Mongols," 664

2. E. S. Keenedy, "Late Medieval Planetary Theory," 369.

3.*Ibid*.

4.*Ibid*., 369, 367.

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