Aleksandrov (or Alexandroff), Pavel Sergeevich | Encyclopedia.com

---

(*b*. Bogorodsk [formerly Noginsk], Russia, 7 May 1896; *d*. Moscow, U.S.S.R., 16 November 1982)

mathematics.

Pavel Sergeevieh Aleksandrov was the youngest of six children (four sons, two daughters) of Sergei Aleksandrovich Aleksandrov, a rural government doctor, and of Tsezariia Akimovna Aleksandrova (née Zdanovskaiah, whose main concern was the education of her children. Both parents instilled in him an intense interest in science and music. His mother taught him German, in which he was as proficient as in his native Russian, and French. Aleksandrov's early education was in the public schools of Smolensk, where his family moved in 1897 when his father became senior doctor in the Smolensk state hospital. The development of Aleksandrov's mathematical abilities and inierest in the fundamental problems of mathematics were encouraged by his grammar school mathematics teacher, Aleksandr Romanovich Eiges. (In 1921 Aleksandrov was married for a brief time to Ekaterina Romanovna Eiges, sister of his teacher.)

Aleksandrov matriculated in the mathematics department of Moscow University in September 1913, intending to become a teacher. In the tall of 1914 he attended a lecture given by the brilliant young mathematician Nikolai Nikolaevich Luzin and became his first student. In 1915 Aleksandrov obtained his first mathematical result on the structure of Borel sets. When it was initially explained to him. Luzin doubted that the method Aleksandrov employed would work and suggested that another approach be taken. Aleksandrov persisted, however. Theresult he obtained may be stated as follows: every nondenumerable Borel set contains a perfect subset.

Enjoying the success of this first project, Aleksandrov energetically embarked on his second project: the continuum hypothesis. It is now known that Cantor's famous hypothesis cannot be proved or disproved within the framework of the theory of sets, so that Aleksandrov's efforts to obtain a definitive result were doomed to failure. His lack of complete success led him to conclude that his mathematical career was ended, and he left the university to go to Novgorod-Severskii, where he worked as a producer in the local theater, and then to Chernigov, where he helped to establish the Chernigov Soviet Dramatic Theater in the spring of 1919.

Life during the years 1918 to 1920 was filled with turmoil following the October Revolution of 1917. Aleksandrov was arrested and jailed for a brief time in 1919 by a group opposing the new Soviet government, but he was released when the Soviet army reoccupied Chernigov. In addition to his work in the theater, he embarked upon a series of public lectures on literature and mathematics. In December 1919, following a six-week illness, he decided to go to Moscow University and resume his study of mathematics.

After returning to Moscow in September 1920, Aleksandrov prepared for his master's examinations by studying with Pavel Samuilovich Uryson. Their association blossomed into a deep friendship. During the summer of 1922 the two P.S.'s (as they were referred to by their fellow students) and several friends rented a dacha on the banks of the Klyaz'ma. The two young mathematicians embarked on the study of topology, a recently formed field of mathematics. Their work of that summer and fall was guided by a mere handful of articles, among them the pioneering work of Maurice Frédict (1906) and Felix Hausdorffs monumental *Grunmdzüge der Mengenlehre* (1914). Their primary concern was to obtain necessary and sufficient conditions for a topological space to be metrizuble. The outcome of their research was the lengthy and authoritative paper "Mémoire sur les espaces topologiques compacts," which, because of various problems, was not published until 1929.

Their search for a metrization theorem was successful, but their formulation made its application difficult. Thesearch for a workable result continued until one of Aleksandrov's students, Yuri M. Smirnov, as well as J. Nagata and R. H. Bing. independently achieved a workable formulation (1951–1952). Using modern terminology, the condition that Aleksandrov and Uryson derived may be stated as follows: A topological space is metrizable if and only if it is paracompact and has a countable refining system of open coverings.

Encouraged by their work, the two young men visited GÖttingen. the intellectual hotbed of German mathematics, (To finance their journey, they gave a series of lectures in and around Moscow on the theory of relativity.) During the summer of 1923. the two presented their results, which were enthusiastically received by such mathematicians as Emmy Noether, Richard Courant, and David Hilbert. This summer not only marked the first time since the revolution that Soviet mathematicians had traveled outside their country, but it also set the stage for mathematical exchanges between Moscow and Göttingen. In fact, Aleksandrov returned to Göttingen every summer until 1932, when such exchanges became impossible because of the restrictiveness of regulations imposed by the German government.

Aleksandrov and Uryson returned to Göttingen in the summer of 1924; they also visited with Felix Hausdorff in Bonn and with L. E. J. Brouwer in Holland, After their time with Brouwer, the two young Soviets went to Paris and then to the Atlantic coast of France for a period of work and relaxation. In Batz, France, their tour ended tragically on 17 August 1924 when Uryson drowned.

The death of his friend seemed to intensity Aleksandrov's interest in topology and in the seminar which the two had begun organizing in the spring of 1924. One of the first students in this seminar was the first of Aleksandrov's students to make substantial contributions to mathematics in general and to pology in particular, Andrei Nikolaevich Tikhonov developed the concept of the product of an infinite number of topological spaces (at least for infinitely many copies of the closed unit interval [0, 1]), He developed the concept to solve a problem posed by Aleksandrov: Is every normal space embeddable as a subspace of a compact Hausdorff space?

After the death of Uryson, Aleksandrov returned to Moscow and made plans to spend the academic year 1925–1926 in Holland with Brouwer. One of the reasons was that during their visit with Brouwer in 1924, Aleksandrov and Uryson had been persuaded by him to have their topological work published in *Verhandelingen der Koninklijke akademie van wetenschappen*. Due to a series of delays, this monumental work was not published until 1929. The original version was in French; since then it has appeared in Russian three times (1950, 1951, 1971), each time with footnotes by Aleksandrov updating contributions by mathematicians answering questions posed by the work.

Aleksandrov formed lifelong friendship during his summers in Göttingen, the most important of which, with Heinz Hopf, began in 1926. Then friendship grew during the academic year 1927–1928, which they spent at Princeton University. When they returned to conduct a topological seminar at Göttingen during the summer of 1928, they were asked by Richard Courant to write a topology book as part of his Yellow Collection for Springer. This request resulted in a seven-year collaboration that culminated in 1935 with the publication of *Topologie*, a landmark textbook on topology. Two additional volumes had originally been planned, but the war prevented completion of the project.

Aleksandrov loved to swim and to take long walks with his students while discussing mathematical problems. His athletic inclinations were restricted by his eyesight, which had been poor from his youth. (He was totally blind during the last three years of his life.)

In 1935 Aleksandrov and his close friend Andrei Nikolaevich Kolmogorov acquired a century-old dacha in the village of Komarovka, outside Moscow. They shared the house and its surrounding garden until death separated them. It not only became a convenient and popular place for Aleksandrov and his students to gather but also sheltered many renowned mathematicians who came to meet and work with Aleksandrov or Kolmogorov.

Aleksandrov's achievements were not limited to pure mathematics. From 1958 to 1962 he was vice president of the International Congress of Mathematicians. He held the chair of higher geometry and topology at Moscow State University, was head of the mathematics section of the university, and served as head of the general topology section of the Steklov Institute of Mathematics of the Soviet Academy of Sciences. For thirty-three years Aleksandrov was president of the Moscow Mathematical Society; he was elected honorary president in 1964. In addition to serving as editor of several mathematical journals, he was editor in chief of *Uspekhi matematicheskikh nauk (Russian Mathematical Surveys)*. In 1929 he was elected a corresponding member of the Soviet Academy of Sciences, and a full member in 1953. He was a member of the Göttingen Academy of Sciences, the Austrian Academy of Sciences, the Leopoldina Academy in Halle, the Polish Academy of Sciences, the Academy of Sciences of the German Democratic Republic, the National Academy of Sciences (United States), and the American Philosophical Society, and an honorary member of the London Mathematical Society. He was awarded honorary doctorates by the Dutch Mathematical Society and Humboldt University in Berlin.

In his last years Aleksandrov supervised the editing of a three-volume collection of what he considered his most important works; *Teoriia funktsii deistvitel'nogo peremennogo i teoriia topologicheskikh prostransty* ("The Theory of Functions of Real Variables and Theory of Topological Spaces"; 1978), *Teoriia razmernosti i smezhnye voprosy: Stat'i obshchego kharaktera* ("Dimension Theory and Related Questions: Articles of a General Nature": 1978), and *Obshehaia teoriia gomologii* ("The General Theory of Homology": 1979).

In addition Aleksandrov wrote his autobiography, portions of which appeared in two parts under the title "Stranitsii avtobiografii" ("Pages from an Autobiography") in *Russian Mathematical Surveys* along with papers presented at an international conference on topology in Moscow (June 1979) of which he was the prime organizer.

Aleksandrov's mathematical results were substantial and diverse, Through his joint work with Uryson he is credited with the definition of compact spaces and locally compact spaces (originally described as bicompact spaces). In 1925 he first formulated the modern definition of the concept of topological space. The concept of compact space undoubtedly led to the definition of a locally finite covering of a space that he used to prove that every open cover of a separable metric space has a locally finite open cover, or, in modern terminology, that every separable metric space is paracompact. This idea appeared later in the result of A. H. Stone that every metric space is paracompact, a fact used by Smirnov Nagata, and Bing in their metrization theorem.

In the period 1925 to 1929 Aleksandrov is credited with laying the foundations of the homology theory of general topological spaces. This branch of topology is a blend of topology and algebra, his study of which was inspired by Emmy Noether during his Göttingen summers and during a visit she made to Brouwer (while Aleksandrov was working with him) in the winter of 1925 to 1926. It was during this visit that Aleksandrov became interested in the concept of a Betti group, which he was to use in his work. (He coined the term "kernel of a homomorphism," which appeared in print for the first time in an algebraic supplement to *Topoloigie*.)

Aleksandrov's arguments used the concept of the nerve of a cover that he had introduced in 1925. Thenerve of a cover ω of a topological space X is a simplicial complex N_ω whose vertexes are in a one-to-one correspondence with the elements of ω. and any vertexes $e_1 \dots e_K$ of N_ω form a simplex in N_ω if and only if the elements of ω corresponding to these vertexes have a nonempty intersection. Based upon this, one may define a simplicial transform (ω' being a cover contained in or succeeding ω), which is called the "projection" of the nerve into N_ω

For a compact space X, the collection of all such projections formed by letting ω range over the directed family of all the finite open covers of X is the projective spectrum S of X. This projective spectrum is the directed family of complexes N_ω which are linked by the projections . The limit space of the projective spectrum is homeomorphic to X, which implies that the topological properties of the space X may be reduced to properties of the complexes and their simplicial mappings. Among other results, this work led to Aleksandrov's theorem that any compact set of a given dimension lying in a Hilbert space can, for any $\in > 0$, be transformed into a polyhedron of equal dimension by means of an$\in$ -deformation, that is, a continuous deformation in which each point is displaced by at most $\in$.

These concepts led to the creation of the homo logical theory of dimension in 1928 to 1930. Aleksandrov's works frequently seemed to be springboards for other mathematicians, including many of his own students: A. N. Tikhonov, L. S. Pontriagin. Y. M. Smirnov, K. A. Sitnikov, A. V. Arkhangel'skii, V. I. Ponomarev, V. I. Zaitsev, and E. V. Shehepin, to name only a few.

In his autobiography Aleksandrov broke down his mathematical life and the associated papers into six periods:

1. The summer of 1915—the structure of Borel sets and the A-operation

2. May 1922—August 1924—basic papers on general topology

3. August 1925—spring 1928—the definition of the nerve of a family of sets and the establishment of the means of the foundations of homology theory of general topological spaces by a method that permitted him to apply the methods of combinatorial topology to point-set topology

4. The first half of 1930—the development of homological dimension theory, which built upon his spectral theory

5. January-May 1942—Because of World War II, Aleksandrov, Kolmogorov, and other scientists were sent to Kazan in July 1941. Although Aleksandrov returned to Moscow for the start of the fall session at the university, he was told to go back to Kazan. There, during the winter of 1941, he wrote a work devoted to the study of the form and disposition of a closed set (or complex) in an enveloping closed set (or complex) by homological means. One important by-product of this paper was the concept of an exact sequence, an important algebraic tool used in many branches of mathematics

6. The winter of 1946—1947—duality theorems for nonelosed sets; he regarded the paper containing these results as his last important work.

In the late 1940's and early 1950's. Aleksandrov and his pupils built upon this last work with the construction of homology theory for nonelosed sets in euclidean spaces. At all times his works seemed motivated and guided by geometric ideas undoubtedly stemming from his youthful fascination with the subject.

One of the pervasive elements in Aleksandrov's works is the theory of continuous mappings of topological spaces, beginning with his theory of the continuous decompositions of compacta, which led to the theory of perfect mappings of arbitrary, completely regular spaces. Included in this development is Aleksandrov's theorem on the representation of each compactum as a continuous image of a perfect Cantor set. This result gave rise to the theorem that every compactum is a continuous image of a zerodimensional compactum of the same weight, and is part of the foundation of the theory of dyadic compacta.

Most of Aleksandrov's life was spent in university teaching and research, and in many ways was structured around education and his students. It included visits to Kamarovka, musical evenings at the university, public talks, and private concerts. The last twenty-five years of his life seem to have been devoted to his students and education in general, as indicated by the survey articles he wrote during this period. He seemed to radiate the same kind of magnetism and contagious fervor for mathematics and life that first drew Aleksandrov, Uryson, and other young students to cluster around Luzin in their student years.

BIBLIOGRAPHY

I. Original Works, “Some Results in the Theory of Topological Spaces, Obtained Within the Last Twenty-five Years.” in *Russian Mathematical Surveys*, **15** , no. 2 (1960), 23–84; *Teoriia funkfsii deistivitel’nogo peremennogo i teoriia topologicheskikh prostransty* (“The Theory of Functions of Real Variables and Theory of Topological Spaces”; Moscow, 1978): *Teoriia razmernosti i smezhnye voprosy: Stat’ i obshchego kharaktera* (“Dimension Theory and Related Questions: Articles of a General Nature” Moscow, 1978); “The Main Aspects in the Development of Set-The oretical Topology,” in *Russian Mathematical Surveys*, **33** , no. 3 (1938), 1–53, with V. V. Fedorchuk; *Obshchaia teoriia gomologii* (“The General Theory of Homology”; Moscow, 1979); “Pages from an Autobiography,” in *Russian Mathematical Surveys*. **34** , no 6 (1979), 267–302, and **35** , no. 3 (1980), 315–358.

II. Secondary Literature. A. V. Arkhangelskii *et al*., “Pavel Sergeevich Aleksandrov (On His 80th BirthdayL” in *Russian Mathematical Surveys*, **31** no. 5 (1976), 1–13; and *Russian Mathematical Surveys*, **21** , no. 4 (1966), an issue dedicated to Aleksandrov, with a biographical introduction by A. N. Kolmogorov *et al*.

Douglas Ewan Cameron