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(*fl*. Tarentum [now Taranto], Italy, *ca*. 375 b.c.)

philosophy, mathematics, physics.

After the Pythagoreans had been driven out of most of the cities of southern Italy by the Syracusan tyrant Dionysius the Elder at the beginning of the fourth century B.C., Tarentum remained their only important political center. Here Archytas played a leading role in the attempt to unite the Greek city-states against the non-Greek tribes and powers. After the death of Dionysius the Elder, he concluded, through the agency of Plato, an alliance with his son and successor, Dionysius the Younger.

Archytas made very important contributions to the theory of numbers, geometry, and the theory of music. Although extant ancient tradition credits him mainly with individual discoveries, it is clear that all of them were connected and that Archytas was deeply concerned with the foundations of the sciences and with their interconnection. Thus he affirmed that the art of calculation (*λογιστικ´η*) is the most fundamental science and makes its results even clearer than those of geometry. He also discussed mathematics as the foundation of astronomy.

A central point in Archytas' manifold endeavors was the theory of means (*μεσότητες*) and proportions. He distinguished three basic means: the arithmetic mean of the form $a - b = b - c$ or $a + c = 2b$; the geometric mean of the form $a: b = b: c$ or $ac = b^2$; and the harmonic mean of the form $(a-b):(b-c) = a: c$. Archytas and later mathematicians subsequently added seven other means.

A proposition and proof that are important both for Archytas' theory of means and for his theory of music have been preserved in Latin translation in Boethius' *De musica*. The proposition states that there is no geometric mean between two numbers that are in "superparticular" (*´επιμόριος*) ratio, i.e., in the ratio $(n + 1): n$. The proof given by Boethius is essentially identical with that given for the same proposition by Euclid in his *Sectio canonis* (Prop. 3). It presupposes several propositions of Euclid that appear in *Elements* VII as well as VIII, Prop. 8. Through a careful analysis of Books VII and VIII and their relation to the above proof, A. B. L. Van der Waerden has succeeded in making it appear very likely that many of the theorems in Euclid's *Elements* VII and their proofs existed before Archytas, but that a considerable part of the propositions and proofs of VIII were added by Archytas and his collaborators.

Archytas' most famous mathematical achievement was the solution of the "Delian" problem of the duplication of the cube. A generation before Archytas, Hippocrates of Chios had demonstrted that the problem can be reduced to the insertion of two mean proportionals between the side of the cube and its double length: If a is the side of the cube and $a: x = x: y = y: 2a$, then x is the side of the doubled cube. The problem of the geometrical construction of this line segment was solved by Archytas through a most ingenious three-dimensional construction. In the figure below, everything, according to the custom of the ancients, is projected into a plane.

Let *AD* and *C* be the two line segments between which the two mean proportionals are to be constructed. Let a circle *ABDF* be drawn with *AD* as diameter and *AB* = *C* as a chord. Then let the extension of *AB* cut the line tangent to the circle at *D* at point *P*. Let *BEF* be drawn parallel to *PDO*. Next, imagine a semicylinder above the semicircle *ABD* (i.e., with *ABD* as its base) and above *AD* another semicircle located perpendicularly in the rectangle of the semicylinder (i.e., with *AD* as diameter in a plane perpendicular to plane *ABDF*). Let this semicircle be rotated around point *A* in the direction of *B*. Being rotated in this way, it will cut the surface of the semicylinder and will describe a curve.

If *ΔAPD* is turned around the axis *AD* in the direction opposite to that of the semicircle, its side *AP* will describe the surface of a circular cone and in doing so will cut the aforementioned curve on the surface of the semicylinder at a point. At the same time, point *B* will describe a semicircle in the surface of cone . Then, at the moment in which the aforementioned curves cut one another, let the position of the moving semicircle be determined by points *AKD'* and that of the moving triangle by points *ALD*, and let the point of intersection be called *K*. The semicircle described by *B* will then be *BMF* (namely, *B* at the moment of intersection of the curves being in *M*). Then drop a perpendicular from *K* to the plane of semicircle *ABDF*. It will fall on the circumference of the circle, since the cylinder is a right cylinder. Let it meet the circumference in I, and let the line drawn from *I* to *A* cut *BF* at *H*, and let line *AL* meet semicircle *BMF* in *M* (see above) Also let the connecting lines *KD' MI*, and *MH* be drawn.

Since each of the two semicircles *AKD'* and *BMF* is then perpendicular to the underlying plane (*ABDF*), their common intersection *MH* is also perpendicular to the plane of that circle, i.e., *MH* is perpendicular to *BF*, *HF*, and likewise that determined by *AH*, *HI*, is equal to the square on *HM*, Hence Δ*AMI* is similar to *HMI* and *AHM* and ∠*AMI* is a right angle. But

$\angle D'KA$ is also a right angle. Hence $KD' \parallel MI$.. Therefore $D'A:AK = KA:AI = AI = AI: AM$ because of the similarity of the triangles. The four line segments $D'A$, AK, AI, and AM are therefore in continuous proportion. Thus, between the two given line segments AD and C (i.e., AB) two mean proportionals, AK and AI, have been found.

Tannery suspected and Van der Waerden, through an ingenious interpretation of *Epinomis* 990E, tried to show how the theory of means also served Archytas as a basis for his theory of music. Starting from the octave 1:2 or 6:12, one obtains the arithmetic mean 9 and the harmonic mean 8. The ratio 6:9= 8:12 is 2:3 or, in musical terms, the fifth. The ratio 6:8 = 9:12 is 3:4 or, in musical terms, the fourth. Forming in like manner the arithmetic and harmonic means of the fifth, one obtains the ratios 4:5 and 5:6 or, in musical terms, the intervals of the major third and the minor third. Using the same procedure with the fourth, one obtains 6:7, or the diminished minor third, and 7:8, or an augmented whole tone. On these intervals Archytas built his three musical scales: the enharmonic, the chromatic, and the diatonic. The theory is also related to the theorem (mentioned above) that there is no geometric mean between two numbers in superparticular ratio. Since all the basic musical intervals are in that ratio, they can be subdivided by means of the arithmetic and the harmonic means but not by the geometric mean.

Archytas also elaborated a physical theory of sound which he expounded in the longest extant fragment of his works. He starts with the observation that arithmetic, geometry, astronomy, and the theory of music are all related, but then proceeds to draw conclusions from empirical observations that are not subjected to mathematical analysis within the fragment. The fundamental observation is that faster motion appears to produce higher sounds. Thus, the bull-roarers used at certain religious festivals produce a higher sound when swung around swiftly than when swung more slowly. There are other easy experiments that confirm this observation. For instance, when the holes of a flute that are nearest the mouth of the flutist are opened, a higher sound is produced than when the farther holes are opened. Archytas reasoned that the air pressure in the first case ought to be higher, and therefore the air motion should be faster than in the second case. This is true of the frequencies of the air impulses produced, but Archytas appears to have concluded that the higher sounds reach the ear of the listener more quickly than the lower ones. Thus, he can hardly have applied his arithmetical theory of music consistently to his theory of the production of sound, or he would almost certainly have discovered his error.

Archytas is also credited with the invention of a wooden dove that could fly.

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