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(*b*. Geneva, Switzerland, 18 July 1768; *d*. Paris, France, 13 August 1822)

mathematics.

Biographical data on Argand are limited. It is known that he was the son of Jacques Argand and Eves Canac; that he was baptized on 22 July (a date given by some for his birth); that he had a son who lived in Paris and a daughter, Jeanne-Francoise-Dorothée-Marie-Élizabeth, who married Félix Bousquet and lived in Stuttgart.

Argand, a Parisian bookkeeper, apparently never belonged to any group of mathematical amateurs or dilettantes. His training and background are so little known that he has often been confused with a man to whom he probably was not even related, Aimé Argand, a physicist and chemist who invented the Argand lamp.

It is remarkable that Argand's single original contribution to mathematics, the invention and elaboration of a geometric representation of complex numbers and the operations upon them, was so timed and of such importance as to assure him of a place in the history of mathematics even among those who credit C.F. Gauss with what others call the Argand diagram.

Other circumstances make Argand's story unusual. His system was actually anticipated by Caspar Wessel, a Norwegian, in 1797, but Wessel's work was without significant influence because it remained essentially unknown until 1897. Argand's own work might have suffered the same fate, for it was privately printed in 1806 in a small edition that did not even have the author's name on the title page. He received proper credit for it through a peculiar chain of events and the honesty and generosity of J.F. Français, a professor at the École Impériale d'Artillerie et du Génie, who published a similar discussion in 1813.

Argand had shown his work to A. M. Legendre before its publication, and Legendre mentioned it in a letter to Français's brother. Français saw the letter among his dead brother's papers, and was so intrigued by the ideas in it that he developed them further and published them in J. D. Gergonne's journal *Annales de mathématiques*. At the end of his article Français mentioned the source of his inspiration and expressed the hope that the unknown "first author of these ideas" would make himself known and publish the work he had done on this project.

Argand responded to this invitation by submitting an article that was published in the same volume of the *Annales*. In it he recapitulated his original work (with a change in notation) and gave some additional applications. A key to his ideas may be presented by a description and analysis of Figures 1 and 2. Figure 1 accompanies his initial discussion of a geometric representation of . His motivation for this can be traced back to John Wallis' *Treatise of Algebra* (1685). In it Wallis suggested that since is the mean proportional between +1 and -1, its geometric representation could be a line constructed as the mean proportional between two oppositely directed unit segments.

Argand began his book, *Essai sur une manieère de repréenter les quantités imaginaires daps les constructions géomériques*, with a brief discussion of models for generating negative numbers by repeated subtraction; one used weights removed from a pan of a beam balance, the other subtracted francs from a sum of money. From these examples he concluded that distance may be considered apart from direction, and that whether a negative quantity is considered real or "imaginary" depends upon the kind of quantity measured. This initial use of the word "imaginary" for a negative number is related to the mathematical-philosophical debates of the time as to whether negative numbers were numbers, or even existed. In general, Argand used "imaginary" for multiples of , a practice introduced by Descartes and common today. He also used the term "absolute" for distance considered apart from direction.

Argand then suggested that "setting aside the ratio of absolute magnitude we consider the different possible relations of direction" and discussed the proportions +1; +1:: -1; -1 and +1; -1:: -1: +1. He noted that in them the means have the same or opposite signs, depending upon whether the signs of the extremes are alike or opposite. This led him to consider 1:x::x:-1. In this proportion he said that x cannot be made equal to any quantity, positive or negative; but as an analogy with his original models he suggested that quantities which were imaginary when applied to "certain magnitudes" became real when the idea of direction was added to the idea of absolute number. Thus, Figure 1, if *KA* taken as positive unity with its direction from *K* to *A* is written to distinguish it from the segment *KA*, which is an absolute distance, then negative unity will be . The classical construction for the geometric mean would determine and on the unit circle with center at *K*. Argand did not mention the geometric construction, but merely stated that the condition of the proportion will be met by perpendiculars and , which represent and , respectively. Analogously, Argand inserted and as the mean proportionals between and by bisecting angle *AKE*.

Argand's opening paragraphs included the first use of the word "absolute" in the sense of the absolute value of a positive, negative, or complex number; of the bar over a pair of letters to indicate what is today called a vector; and of the idea that , Later in the *Essai* Argand used the term "modulus" (*module*) for the absolute value or the length of a vector representing a complex number. In this Argand anticipated A.L. Cauchy, who is commonly given credit for originating the term.

Argand's notation in his original essay is of particular interest because it anticipated the more abstract and modern ideas, later expounded by W.R. Hamilton, of complex numbers as arbitrarily constructed new entities defined as ordered pairs of real numbers. This modern aspect of Argand's original work has not been generally recognized. One reason for this no doubt, is that in later letters and journal articles he returned to the more standard notation. In his book, however, Argand suggested omitting , deeming it no more a factor of a than is +1 in +a. He wrote ~a and ≁a for a and respectively. He then observed that both $(\sim a)^2$ and$(\nsim a)^2$were negative. This led him to the rule that if in a series of factors every curved line has a value of 1 and every straight line a value of 2-thus ~ = 1, 1- = 2, ≁ = 3, + = 4—then the sign of the product of any series of factors can be determined by taking the residue modulo, 4, of the sum of the values of the symbols associated with the factors. Here he recognized the periodicity of the powers of the imaginary unit.

Argand generalized the insertion of geometric means between two given vectors to the insertion of any number of means, *n*, between the vectors and by dividing the angle between them by *n*. He noted that one could also find the means between and by beginning with the angles *AKB*+360°, and *AKB*+720°. This is a special case of de Moivre's theorem, as is more clearly and completely shown in Argand's explanation of Figure 2. In it *AB, BC,... EN* are n equal arcs. From the diagram Argand reasoned that , , and ; hence , which leads to cos na + .

This result was well known before Argand, as were the uses he made of it to derive infinite series for trigonometric and logarithmic functions. As noted earlier, we know nothing of Argand's education or contacts with other mathematicians prior to 1813. It seems highly probable, however, that he had direct or indirect contact with some of the results of Wallis, de Moivre, and Leonhard Euler. Nevertheless, the purely geometric-intuitive interpretation and reasoning leading to these results seem to have been original with Argand. This geometric viewpoint has continued to be fruitful up to the present day. Argand recognized the nonrigorous nature of his reasoning, but he defined his goals as clarifying thinking about imaginaries by setting up a new view of them and providing a new tool for research in geometry. He used complex numbers to derive several trigonometric identities, to prove Ptolemy's theorem., and to give a proof of the fundamental theorem of algebra.

Argand's work contrasts with Wessel's in that the latter's approach was more modern in its explicit use of definitions in setting up a correspondence between and vectors referred to a rectangular coordinate system (which neither Wessel nor Argand ever explicitly mentioned or drew). Wessel stressed the consistency of his assumptions and derived results without regard for their intuitive validity. he did not present as many mathematical consequences as Argand did.

Just as it seems clear that Argand's work was entirely independent of Wessel's, so it also seems clear that it was independent of the algebraic approach published by Suremain de Missery in 1801. Argand refuted the suggestion that he knew of Buée's work published in the *Philosophical Transactions of the Royal Society* in 1806 by noting that since academic journals appear after the dates which they bear, and that his book was printed in the same year the journal was dated, he could not have known of Buée's work at the time he wrote the book. Buée's ideas were not as clear, extensive, or well developed as Argand's.

There are obvious connections between Argans's geometric ideas and the later work of Moebius, Bellavitis, Hermann Grassmann, and others, but in most cases it is as difficult to establish direct outgrowths of his work as it is to establish that he consciously drew on Wallis, de Moivre, or Euler.

Two of the most important mathematicians of the early nineteenth century, Cauchy and Hamilton, took care to note the relationship of Argand's work to some of their own major contributions, but claimed to have learned of his work only after doing their own. Gauss probably could have made a similar statement, but he never did. Cauchy mentioned Argand twice in his "Mémoire sur les quantités géométriques," which appeared in *Exercices d'analyse et de physique matheématique* (1847). He cited Argand as the originator of the geometric interpretation of imaginary quantities, which he suggested would give clarity, a new precision, and a greater generality to algebra than earlier theories of imaginary quantities had. he also cited Argand and A.M. Legendre as authors of proofs of what Gauss termed the "fundamental theorem of algebra," Argand's proof involved considering the modulus of

when $x = a + bi$. He noted that if $|P(x)| = 0$ the theorem was true, and argued geometrically that if $|P(x)| > 0$ one could find $x' = a' + b'i$ such that $|P(x')| < |P(x)|$. Servois objected that this only showed that $P(x)$ was asymptotic to 0 for some sequence of x's. Argand replied that such behavior was associated with hyperbolas having zeros at infinity, not with polynomials. Cauchy asserted that a proof proposed by Legendre reduced to Argand's but left much to be desired, while his own method for approximating roots of $P(x) = 0$ could be used to demonstrate their existence, Gauss had published a proof of this existence in his thesis (1799). Although the geometric representation of complex numbers was implicit in this thesis, Gauss did not actually publish a discussion of it until 1832 in his famous paper "Theoria residuorum biquadraticorum." Argand, however, was the first mathematician to assert that the fundamental theorem also held if the coefficients of $P(x)$ were complex.

Hamilton used lengthy footnotes in the first edition of his *Lectures on Quaternions* (1853) to assert the priority and quality of Argand's work, especially with respect to the "multiplication of lines." He traced the roots of his own development of the algebra of couples and of quaternions, however, to John Warren's *A Treatise on the Geometrical Representation of the Square Roots of Negative Quantities (1828). This, like C.V. Mourrey's Lavraie théorie des quantité negatives et des quantités prétendues imaginaires* (1828), seems to have been free of any dependence on Argand's work.

Argand's later publications, all of which appeared in Gergonne's *Annales*, are elaborations of his book or comments on articles published by others. His first article determined equations for a curve that had previously been described in the *Annales* (**3** ,243). Argand went on to suggest an application of the curve to the construction of a thermometer shaped like a watch. His analysis of probable errors in such a mechanism showed familiarity with the mechanics of Laplace, as presented in *Exposition du systéme du monde*.

His fifth article in the *Annales*, defending his proof of the fundamental theorem of algebra, showed his familiarity with the works of Lagrange, Euler, and d'Alembert, especially their debates on whether all rational functions of ($a + bi$) could be reduced to the form $A + Bi$ where a, b, A, and B are real. Argand, oddly enough, did not accept this theorem. He apparently was not familiar with Euler's earlier reduction of , for he cited this as an example of an expression that could not be reduced to the form $A + Bi$.

His last article appeared in the volume of *Annales* dated 1815–1816 and dealt with a problem in combinations. In it Argand devised the notation (m, n) for the combinations of m things taken n at a time and the notation $Z(m, n)$ for the number of such combinations.

Argand was a man with an unknown background, a nonmathematical occupation, and an uncertain contact with the literature of his time who intuitively developed a critical idea for which the time was right. He exploited it himself. The quality and significance of his work were recognized by some of the geniuses of his time, but breakdowns in communication and the approximate simultaneity of similar developments by other workers force a historian to deny him full credit for the fruits of the concept on which he labored.

BIBLIOGRAPHY

I.Original Works. There have been three editions of Argand's book (his first publication), *Essai sur une maniére de représenter les quantités imaginaires dans les constructions géométriques*. The first edition (Paris, 1806) did not bear the name of the author; the second edition, subtitled *Précédé d'une préface par M.J. Hoüel et suivie d'une appendice contenant des extraits des Annales de Gergonne, relatifs à la question des imaginaires* (Paris, 1874), cites the author as "R. Argand" on the title page but identifies him as Jean-Robert Argand on page xv. The Essai was translated by Professor A.S. Hardy as *Imaginary Quantities*: *Their Geometrical Interpretation* (New York, 1881). Argand's eight later publications all appeared in Vols. **4,5** , and **6** (1813–1816) of J.D. Gergonne's journal *Annales de mathématiques pures et appliquées*. Hoüel lists them at the end of his preface to the second edition of the *Essai*.

II.Secondary Literature. Data on Argand's life were included by Hoüel with the second edition of the *Essai*. Verification of the dates of his birth and death is given by H. Fehr in *Intermédiare des mathématiciens*, **9** (1902), 74, Niels Nielsen, *Géométres français sous la Révolution* (Copenhagen, 1929), pp.6–9, discusses Argand with reference to Wessel, Français, and others. William Rowan Hamilton gives a comparative analysis of contemporary work with complex numbers while praising Argand in *Lectures on Quaternions* (Dublin, 1853), pp.31–34, 56, 57. Augustin Louis Cauchy's appraisal is found in "Mémoire sur les quantités géométriques," in *Exercices d'analyse et de physique mathématiques*, IV (Paris, 1847), and in *Oeuvres*, 2nd series, XIV (Paris, 1938), 175–202, J.F. Français's development of Argand's ideas contained in a letter to his brother, "Nouveaux principes de géométrie de position, et interpretation géométrique des symboles imaginaires," is *Annales de mathématiques*, **4** (1813–1814), 61–71.

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