

Aristarchus Of Samos | Encyclopedia.com

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(ca. 310–230 b.c.)

mathematics, astronomy.

Aristarchus is celebrated as being the first man to have propounded a heliocentric theory, eighteen centuries before Copernicus. He was born on the island of Samos, close by Miletus, cradle of Ionian science and philosophy. Little is known of Aristarchus' subsequent habitation. He was a pupil of Strato of Lampsacos, third head of the Lyceum founded by Aristotle. It is more likely that he studied under Strato at Alexandria than at Athens after the latter's assumption of the headship of the Lyceum in 287 b.c. Aristarchus' approximate dates are determined by Ptolemy's record (*Syntaxis* 3.2) of his observation of the summer solstice in 280 b.c. and by Archimedes' account of his heliocentric theory in a treatise, *The Sand-Reckoner*, which Archimedes composed before 216 b.c. The sole surviving work of Aristarchus is the treatise *On the Sizes and Distances of the Sun and Moon*.

To his contemporaries Aristarchus was known as "the mathematician"; the epithet may merely have served to distinguish him from other men of the same name, although *On Sizes and Distances* is indeed the work of a highly competent mathematician. The Roman architect Vitruvius lists him with six other men of rare endowment who were expert in all branches of mathematics and who could apply their talents to practical purposes. Vitruvius also credits him with inventing the *skaphe*, a widely used sundial consisting of a hemispherical bowl with a needle erected vertically in the middle to cast shadows. Speculations as to why a reputable mathematician like Aristarchus should interest himself in the true physical orientation of the [solar system](#) thus appear to be idle. Some have pointed to the possible influence of Strato, who was known as "the physical philosopher." There is no evidence however, to indicate that Aristarchus got his physical theories from Strato. A more likely assumption is that *On Sizes and Distances* gave him an appreciation of the relative sizes of the sun and earth and led him to propound a heliocentric system.

The beginnings of heliocentrism are traced to the early Pythagoreans, a religiophilosophical school that flourished in southern Italy in the fifth century b.c. Ancient tradition ascribed to Pythagoras (ca. 520 b.c.) the identification of the Morning Star and the Evening Star as the same body. Philolaus (ca. 440 b.c.) gave the earth, moon, sun, and planets an orbital motion about a central fire, which he called "the hearth of the universe." According to another tradition, it was Hicetas, a contemporary of Philolaus, who first gave a circular orbit to the earth. Hicetas was also credited with maintaining the earth's axial rotation and a stationary heavens. More reliable ancient authorities, however, associate the hypothesis of the earth's diurnal rotation with [Heraclides of Pontus](#), a pupil of Plato, who is also explicitly credited with maintaining (ca. 340 b.c.) an epicyclic orbit of Venus—and presumably that of Mercury also—about the sun. Some Greek astronomer may have taken the next logical step toward developing a complete heliocentric hypothesis by proposing the theory advanced in modern times by [Tycho Brahe](#), which placed the five visible planets in motion about the sun, and the sun, in turn, in motion about the earth. Several scholars have argued that such a step was indeed taken, the most notable being the Italian astronomer Schiaparelli, who ascribed the Tychonic system to Heraclides; but evidence of its existence in antiquity is lacking.

Ancient authorities are unanimous in attributing the heliocentric theory to Aristarchus. Archimedes, who lived shortly afterward, says that he published his views in a book or treatise in which the premises that he developed led to the conclusion that the universe is many times greater than the current conception of it. Archimedes, near the opening of *The Sand-Reckoner*, gives a summary statement of Aristarchus' argument:

His hypotheses are that the fixed stars and the sun are stationary, that the earth is borne in a circular orbit about the sun, which lies in the middle of its orbit, and that the sphere of the fixed stars, having the same center as the sun, is so great in extent that the circle on which he supposes the earth to be borne has such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface.

Plutarch (ca. a.d. 100) gives a similar brief account of Aristarchus' hypothesis, stating specifically that the earth revolves along the ecliptic and that it is at the same time rotating on its axis.

After reporting Aristarchus' views, Archimedes criticizes him for setting up a mathematically impossible proportion, pointing out that the center of the sphere has no magnitude and therefore cannot bear any ratio to the surface of the sphere. Archimedes intrudes the observation that the "universe," as it is commonly conceived of by astronomers, is a sphere whose radius extends from the center of the sun to the center of the earth. Accordingly, as a mathematician he imputes to the mathematician Aristarchus a proportion that he feels is implicit in his statement, namely, that the ratio that the earth bears to the universe, as it

is commonly conceived, is equal to the ratio that the sphere in which the earth revolves, in Aristarchus' scheme, bears to the sphere of the fixed stars.

Modern scholars have generally supposed that Aristarchus did not intend to have his proportion interpreted as a mathematical statement, that instead he was using an expression conventional with Greek mathematical cosmographers—"having the relation of a point"—merely to indicate the minuteness of the earth's orbit and the vastness of the heavens. Sir Thomas Heath points to similar expressions in the works of Euclid, Geminus, Ptolemy, and Cleomedes, and in the second assumption of Aristarchus' extant treatise *On Sizes and Distances* (see below). Heath feels that Archimedes' interpretation was arbitrary and sophistical and that Aristarchus introduced the statement to account for the inability to observe stellar parallax from an orbiting earth. Neugebauer defends the proportion that Archimedes ascribes to Aristarchus,

$$r: R_e = R_e: R_f,$$

as mathematically sound and providing finite dimensions for the sphere of the fixed stars: the earth's radius (r) is so small in comparison with the sun's distance (R_e) that no daily parallax of the sun is discernible for determining R_e ; according to Aristarchus' hypothesis, the earth moves in an orbit whose radius is R_e and no annual parallax of the fixed stars is discernible.

Why did the Greeks, after evolving a heliocentric hypothesis in gradual steps over a period of two centuries, allow it to fall into neglect almost immediately? Only one man, Seleucus of Seleucia (*ca.* 150 b.c.), is known to have embraced Aristarchus' views. The common attitude of deploring the "abandonment" of the heliocentric theory as a "retrogressive step" appears to be unwarranted when it is realized that the theory, however bold and ingenious it is to be regarded, never attracted much attention in antiquity. Aristarchus' system was the culmination of speculations about the physical nature of the universe that began with the Ionian philosophers of the sixth century, and it belongs to an age that was passing away. The main course of development of Greek astronomy was mathematical, not physical, and the great achievements were still to come—the exacting demonstrations and calculations of [Apollonius of Perga](#), Hipparchus, and Ptolemy. These were based upon a geocentric orientation.

To a mathematician the orientation is of no consequence; in fact it is more convenient to construct a system of epicycles and eccentrics to account for planetary motions from a geocentric orientation. A heliocentric hypothesis neatly explained some basic phenomena, such as the stations and retrogradations of superior planets; but a circular orbit for the earth, about a sun in the exact center, failed to account for precise anomalies, such as the inequality of the seasons. In explanation of this inequality, Hipparchus determined the eccentricity of the earth's position as 1/24 of the radius of the sun's circle and he fixed the line of apsides in the direction of longitude $65^\circ 30'$. Ptolemy adopted Hipparchus' solar data without change, unaware that the sun's orbit describes a revolving eccentric, the shift being $32'$ in a century. The Arab astronomer al-Battānī (a.d. 858–929) discovered this shift. Epicyclic constructions had two advantages over eccentric constructions: they were applicable to inferior as well as superior planets and they palpably demonstrated planetary stations and retrograde motions. By the time of Apollonius it was understood that an equivalent eccentric system could be constructed for every epicyclic system. Henceforth, combinations of epicycles and eccentrics were introduced, all from a geocentric orientation. Aristarchus, too, had used a geocentric orientation in calculating the sizes and distances of the sun and moon.

It is not hard to account for the lack of interest in the heliocentric theory. The *Zeitgeist* of the new Hellenistic age was set and characterized by the abstruse erudition of the learned scholars and the precise researches of the astronomers, mathematicians, and anatomists working at the library and museum of Alexandria. Accurate instruments in use at Alexandria were giving astronomers a better appreciation of the vast distance of the sun. Putting the earth in orbit about the sun would lead to the expectation that some variation in the position of the fixed stars would be discernible at opposite seasons. Absence of displacement would presuppose a universe of vast proportions. The more precise the observations, the less inclined were the astronomers at Alexandria to accept an orbital motion of the earth. It is the opinion of Heath that Hipparchus (*ca.* 190–120 b.c.) usually regarded as the greatest of Greek astronomers, in adopting the geocentric orientation "sealed the fate of the heliocentric hypothesis for so many centuries."

The intellectual world at large was also disinclined to accept Aristarchus' orientation. Aristotle's doctrine of "natural places," which assigned to earth a position at the bottom or center among the elements comprising the universe, and his plausible "proofs" of a geocentric orientation, carried great weight in later antiquity, even with the mathematician Ptolemy. Religious minds were reluctant to relinquish the central position of man's abode. According to Plutarch, Cleanthes, the second head of the Stoic school (263–232 b.c.), thought that Aristarchus ought to be indicted on a charge of impiety for putting the earth in motion. Astrology, a respectable science in the eyes of many leading intellectuals, was enjoying an extraordinary vogue after its recent introduction. Its doctrines and findings were also based upon a geocentric orientation.

It is interesting to note in passing that Copernicus' disappointment at being anticipated by Aristarchus has recently come to light. Copernicus deliberately suppressed a statement acknowledging his awareness of Aristarchus' theory; the statement, deleted from the autograph copy of the *De revolutionibus*, appears in a footnote in the Thorn edition (1873) of that work. Elsewhere Copernicus tells of his search for classical precedents for his novel ideas about the heavens and of his finding in Plutarch the views of Philolaus, Heraclides, and Ecphantus; but he omits mention of the clear statement about Aristarchus' theory that appears a few pages earlier. Lastly, Copernicus' almost certain acquaintance with Archimedes' *The Sand-Reckoner*, the work containing our best account of Aristarchus' theory, has recently been pointed out.

His accomplishments as an astronomer have tended to detract attention from Aristarchus' attainments as a mathematician. Flourishing a generation after Euclid and a generation before Archimedes, Aristarchus was capable of the same sort of rigorous and logical geometrical demonstrations that distinguished the work of those famous mathematicians. *On Sizes and Distances* marks the first attempt to determine astronomical distances and dimensions by mathematical deductions based upon a set of assumptions. His last assumption assigns a grossly excessive estimate to the apparent angular diameter of the moon (2°). We are told by Archimedes in *The Sand-Reckoner* that Aristarchus discovered the sun's apparent angular diameter to be $1/720$ part of the zodiac circle ($1/2^\circ$), a close and respectable estimate. Aristarchus uses a geocentric orientation in *On Sizes and Distances* and concludes that the sun's volume is over 300 times greater than the earth's volume. For these reasons it is generally assumed that the treatise was an early work, antedating his heliocentric hypothesis.

Aristarchus argues that at the precise moment of the moon's quadrature, when it is half-illuminated, angle *SME* is a right angle; angle *SEM* can be measured by observation; therefore it is possible to deduce angle *MSE* and to determine the ratio of the distance of the moon to the distance of the sun (Figure 1). Two obvious difficulties are involved in his procedures: the determination with any exactitude (1) of the time of the moon's dichotomy and (2) of the measurement of angle *SEM*. A slight inaccuracy in either case would lead to a grossly inaccurate result. Aristarchus assumes angle *SEM* to be 87° , when in actuality it is more than $89^\circ 50'$, and he derives a distance for the sun of 18 to 20 times greater than the moon's distance (actually nearly 400 times greater). His mathematical procedures are sound, but his observational data are so crude as to make it apparent that Aristarchus was interested here in mathematical demonstrations and not in physical realities.

Aristarchus' treatise begins with six assumptions:

- (1) That the moon receives its light from the sun.
- (2) That the earth has the relation of a point and center to the sphere of the moon.
- (3) That when the moon appears to us to be exactly

at the half the [great circle](#) dividing the light and dark portions of the moon is in line with the observer's eye.

- (4) That when the moon appears to us to be at the half its distance from the sun is less than a quadrant by $1/30$ part of a quadrant (87°).
- (5) That the breadth of the earth's shadow (during eclipses) is that of two moons.
- (6) That the moon subtends $1/15$ part of a sign of the zodiac (2°).

He then states that he is in a position to prove three propositions:

- (1) The distance of the sun from the earth is more than eighteen times but less than twenty times the moon's distance (from the earth); this is based on the assumption about the halved moon.
- (2) The diameter of the sun has the same ratio to the diameter of the moon (i.e., assuming that the sun and moon have the same apparent angular diameter).
- (3) The diameter of the sun has to the diameter of the earth a ratio greater than 19:3, but less than 43:6; this deduction follows from the ratio between the distances thus discovered, from the assumption about the shadow, and from the assumption that the moon subtends $1/15$ part of a sign of the zodiac.

Then follow eighteen propositions containing the demonstrations. Heath has edited and translated the complete Greek text, together with Pappus' comments on the treatise, in his *Aristarchus of Samos* (pp.352–414), and presents a summary account of the treatise in *A History of Greek Mathematics* (Vol. II).

Anticipating trigonometric methods that were to come. Aristarchus was the first to develop geometric procedures for approximating the sines of small angles. He deals with angles expressed as fractions of right angles and ratios of the sides of triangles, determining limits between which actual values lie. In Proposition 7, demonstrating that the distance of the sun is more than eighteen times but less than twenty times the distance of the moon, which would be expressed trigonometrically $1/18 > \sin 3^\circ > 1/20$, he uses in his proof certain inequalities that he assumes to be known and accepted. These may be expressed trigonometrically. If α and β are acute angles and $\alpha > \beta$, then

$$\tan\alpha/\tan\beta > \alpha/\beta > \sin\alpha/\sin\beta.$$

If Aristarchus had had a correct measurement of the angle *SEM*— $89\ 5/6^\circ$ instead of 87° —his result would have been nearly correct. A century later Hipparchus was able to obtain a very close approximation of the moon's distance, expressed in terms

of earth radii, by measuring the earth's shadow during lunar eclipses; but an appreciation of the vast distance of the sun had to wait upon the development of modern precision instruments.

Other dimensions deduced by Aristarchus in his treatise, all of them grossly underestimated because of his poor observational data, are:

(Prop. 10) The sun has to the moon a ratio greater than 5,832:1 but less than 8,000:1.

(Prop. 11) The diameter of the moon is less than $\frac{2}{45}$ but greater than $\frac{1}{30}$ of the distance of the centre of the moon from the observer.

(Prop. 16) The sun has to the earth a ratio greater than 6,859:27 but less than 79,507:216.

(Prop. 17) The diameter of the earth is to the diameter of the moon in a ratio greater than 108:43 but less than 60:19.

(Prop. 18) The earth is to the moon in a ratio greater than 1,259,712:79,507 but less than 216,000:6,859.

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