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(*b*. Krakow, Poland, 30 March 1892; *d*, Lvov, Ukrainian S.S.R., 31 August 1945)

Mathematics.

Banach's father, a railway official, and mother turned their son over to a laundress, who became his foster mother and gave him her surname. From the age of fifteen he supported himself by giving private lessons. After graduating from secondary school in Krakow in 1910, Banach studied at the Institute of Technology in Lvov but did not graduate. He returned to Krakow in 1914, and from 1916, when he met H. Steinhaus, he devoted himself to mathematics. His knowledge of the field was already fairly extensive, although it probably was not very systematic. Banach's first paper, on the convergence of Fourier series, was written with Steinhaus in 1917 and was published two years later. Also in 1919 he was appointed lecturer in mathematics at the Institute of Technology in Lvov, where, in addition, he lectured on mechanics. In the same year he received his doctorate with an unusual exemption from complete university education. Banach's thesis, "Sur les operations dans les ensembles abstraits et leur application aux equations integrales," appeared in Fundamenta mathematicae in 1922. The publication of this thesis is sometimes said to have marked the birth of functional analysis.

"In 1922 Banach became a Dozent on the basis of a paper on measure theory (published in 1923). Soon afterward he was made associate professor, and in 1927 he became full professor at the University of Lvov. In 1924 he was elected corresponding member of the Polish Academy of Sciences and Arts. Banach's research activity was intense, and he had a number of students who later became outstanding mathematicians: S. Mazur, W. Orlicz, J. Schauder, and S. Ulam, among others. Banach and Steinhaus founded the journal Studia mathematica, but often Banach had little time left for scientific work because the writing of both college texts (of which the book on mechanics is of special importance) and secondary-school texts took most of his time and effort."

From 1939 to 1941 Banach was dean of the faculty at Lvov and was *elected a member of the Ukrainian Academy of Sciences. In the summer of 1941, Lvov was occupied by the German army, and for three years Banach was compelled to feed lice in a German institute that dealt with infectious diseases. After the liberation of Lvov in the autumn of 1944, he resumed his work at the university. His health was shattered, however, and he died less than a year later.

Banach's scientific work comprises about fifty papers and the monograph Theorie des operations lineaires (1932). Although he laid the foundations of contemporary functional analysis, most of his papers are closely connected with the field but are not precisely in it.

Banach made a significant contribution to the theory of orthogonal series, and his theorem on locally meager sets is of lasting importance in general topology. In the descriptive theory of sets and mappings, he extended to mappings some theorems previously known only for numerical functions. A number of results, many of which can now be found in textbooks, concern derivation and absolute continuity, as well as related properties. Banach made a substantial contribution to the theory of measure and integration, results that stimulated a great number of papers and, apparently, the discovery of the Radon-Nikodym theorem. The questions of the existence of measures investigated by Banach have proved to be closely connected with the axiomatic theory of sets.

Despite the great importance of these results and the unusual lucidity and force of mathematical thinking manifested in them, functional analysis is Banach's most important contribution. His work started, of course, from what was achieved during the decades following Vito Volterra's papers of the 1890's on integral equations. Before Banach there were either rather specific individual results that only much later were obtained as applications of general theorems, or relatively vague general concepts. Papers on the so-called general analysis (mainly by E. H. Moore) formed a significant trend, but these were none too comprehensible and, for that period at least, much too general. Ivar Fredholm's and David Hilbert's papers on integral equations marked the most substantial progress. The concepts and theorems they had discovered later became an integral part of functional analysis, but most of them concern only a single linear space (later called Hilbert space).

Later, more or less simultaneoa,ly with Banach, several mathematicians-O. Hahn. L. Freshet, E. Helly, and Norbert Wiener, among others-attained many of the concepts and theorems forming the basis of Banach's theory. None of them, however, succeeded in creating as comprehensive and integrated a system of concepts and theorems and their applications as that of Banach, his co-workers, and his students.

Form 1922 on, Banach introduced through his papers the concept of formed linear spaces and inmcestigated them (and metric linear speaces). Particularly regarding the assumption of completeness (complete normed linear spaces are now generally known as Banach spaces). The concept was introduced at almost the same time by Norbert Wiener (who about a quarter of a century later founded cybernetics), but he did not develop the theory, perhaps because he did not see its possible application. Banach proved three fundamental theorems of normed linear spaces: the theorem on the extension of containuous linear functional, now called the Hahn-Bananch theorem (they proved it independently. and Hahn actually did so first); the theorem on bounded families of mappings, now called the Banach-steinhaus theorem; and the theorem on continuos linear mappings of Banach spaces. He also introduced and examined the concept of weak convergence and weak closure, and gave a series of applications of the general theorems on formed linear spaces.

Further development of functional analysis proved that metric linear spaces are not sufficient for the needs of analysis and that it is essential to use more general, and also richer and more special, objects and structures. Nevertheless, the theory of Banach spaces has—often in combination with other methods—numerous applications in analysis, The theory of these spaces is both an indispensable tool and the basis of contemporary theory of more general linear spaces; it also provided the stimulus and the starting point for other branches of functional analysis.

The fact that functional analysis originated as late as Banach and his school, although favorable conditions seemingly existed at the beginning of the century, is due largely to the way mathematics had developed until then. In fact, sufficiently detailed knowledge about the different concrete instances of linear spaces was not achieved until the 1920's. Also, by that time the applications of some methods of the theory of sets, such as transfinite construction, were clarified, and some theorems on general topology (e.g., Baire's theorem on complete metric spaces and some propositions from the descriptive theory of sets) became widely known and applied.

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11.Secondary Literature. A short biography, a practically complete list of scientific papers, and an analysis of Banach's work are in *Colloquium mathaematicum,***1** , no 2 (1948), 65-102. Also see H, Steinhaus, "Stefan Banach," in *Studia Mathematica special series,***1** (1963), 7-15; and s. Ulam, "Stefan Banach 1892-1945," in *Bulletin of the America Mathematical society*, 52 (1946,) 600-603,

Miroslav KatÉtov