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(*b*. Cremona, Italy, 16 November 1835; *d*. Rome, Italy, 18 February 1900)

mathematics.

Beltrami was born into an artistic family: his grandfather. Giovanni, was an engraver of precious stones, especially cameos.; his father, Eugenio, Painted miniatures. Young Eugenio studied mathematics from 1853 to 1856 at the University of Pavia, where Francesco Brioschi was his teacher. Financial difficulties forced Beltrami to become secretary to a railroad engineer. first in Verona and then in Milan. In Milan he continued his mathematical studies and in 1862 published his first mathematical papers, which deal with the differential geometry of curves.

After the establishment of the Kingdom of Italy in 1861. Beltrami was offered the chair of complementary algebra and analytic geometry at Bologna, which he held from 1862 to 1864; from 1864 to 1866 he held the chair of geodesy in Pisa, where Enrico Betti was his friend and colleogue. From 1866 to 1873 he was back in Bologna, where he occupied the chair of rational mechanics. After Rome had become the capital of Italy in 1870, Beltrami became professor of rational mechanics at the new University of Rome, but served there only from 1873 to 1876, after which he held the chair of mathematical physics at Pavia, where he also taught higher mechanics. In 1891 he returned to Rome, where he taught until his death. He became the president of the Accademia dei Lincei in 1898 and, the following year, a senator of the kingdom. A lover of music, Beltrami was interested in the relationship between mathematics was interested in the relationship between mathematics and music.

Beltrami’s works can be divided into two main groups: those before *ca*. 1872, which deal with differential geometry of curves and surfaces and were influenced by Gauss, Lamé, and Riemann, and the later ones, which are concerned with topics in applied mathematics that range from elasticity to electromagnetics. His most lasting work belongs to this first period, and the paper “Saggio di interpretazione della geometria non-euclidea” (1868) stands out, In a paper of 1865 Beltrami had shown that on surfaces of constant curvature, and only on them. the line element $ds^2 = Edu^2 + 2Fdudv + Gdv^2$ can be written in such a form that the geodesics, and only these. are represented by linear expressions in u and v. For positive curvature R^{-2} this form is

$$ds^2 = R^2[(v^2 + a^2)du^2\ 2uvdudv + (u^2 + a^2)dv^2] \times (u^2 + v^2 + a^2)^{-2}.$$

The geodesics in this case behave, locally speaking, like the great circles on a sphere. It now occurred to Beltrami that, by changing R to iR and a to ia the line element thus obtained.

$$ds^2 = R^2[(a^2 - v^2 + 2uvdudv + (u^2 + a^2)dv^2] \times (a^2 - u^2 - v^2)^{-2}.$$

which defines surfaces of constant curvature $-R^{-2}$. offers a new type of geometry for its geodesics inside the region $u^2 + v^2 < a^2$. This geometry is exactly that of the so-called non-Euclidean geometry of Lobgachevski, if goedesics on such a surface are identified with the “straight lines” of non-Euclidean geometry.

This geometry. developed between 1826 and 1832. was known to Beltrami through some of Gauss’s letters and some translations of the work of Lobachevski. Few mathematicians, however, had paid attention to it. Beltramml now offered a representation of this geometry in terms of the acceptable Euclidean geometry: “We have tried to find a real foundation [*substrato*] to this doctrine, instead of having to admit for it the necessity of a new order of entities and concepts.” He showed that all the concepts and formulas of Lobachevski’s geometry are realized for geodesics on surfaces of constant negative curvature and, in particular, that there are rotation surfaces of this kind. The simplest of this kind of “pseudospherical” surface (Beltrami’s term) is the surface of rotation of the tractrix about its asymptote, now usually called the pseudosphere, which Beltrami analyzed more closely in a paper of 1872.

Thus Beltrami showed how possible contradictions in non-Euclidean geometry would reveal themselves in the Euclidean geometry of surfaces; and this removed for most, or probably all, mathematicians the feeling that non-Euclidean geometry might be wrong. Beltrami, by “mapping” one geometry upon another, made non-Euclidean geometry “respectable.” His method was soon followed by others, including Felix Klein, a development that opened entirely new fields of mathematical thinking.

Beltrami pointed out that his representation of non-Euclidean geometry was valid for two dimensions only. In his “Saggio” he was hesitant to claim the possibility of a similar treatment of non-Euclidean geometry of space. After he had studied

Riemann's *Über die Hypothesen welche der Geometrie zu Grunde liegen*, just published by Dedekind, he had no scruples about extending his representation of non-Euclidean geometry to manifolds of $n > 2$ dimensions in "Teoria fondamentale degli spazi di curvatura costante."

In a contribution to the history of non-Euclidean geometry, Beltrami rescued from oblivion the Jesuit mathematician and logician Giovanni Saccheri (1667–1733), au'hor of *Euclides ab omni naevo vindicatus*, which foreshadowed non-Euclidean geometry but did not achieve it.

In his "Ricerche di analisi applicata alla geometria,"Beltrami, following an idea of Lamé's, showed the power of using so-called differential parameters in surface theory. This can be considered the beginning of the use of invariant methods in differential geometry.

Much of Beltrami's work in applied mathematics shows his fundamental geometrical approach, even in his analytical investigations. This trait characterizes the extensive "Richerche sulle cinematica dei fluidi" (1871–1874) and his papers on elasticity. In these he recognized how Lamé's fundamental formulas depend on the Euclidean character of space, and he sketched a non-Euclidean approach (1880-1882). He studied potential theory, particularly that of ellipsoids and cylindrical discs; wave theory in connection with Huygens' principle; and further problems in thermodynamics, optics, and conduction of heat that led to linear partial differential equations. Some papers deal with Maxwell's theory and its mechanistic interpretation, suggesting a start from d' Alembert's principle rather than from that of Hamilton(1889).

BIBLIOGRAPHY

1. Original Works. Beltrami's works are collected in *Opere matematiche*, 4 vols. (Milan, 1902–1920). Important individual works are "Saggio di interpretazione della geometria non-euclidea,"*in Giornale di matematiche*, **6** (1868), 284–312, and *Opere*, I, 374–405, also translated into French in *Annales scientifiques de I'École Normale Supérieure*, **6** (1869), 251–288; "Richerche di analisi applicata alla geometria," *in Giornale di matematiche*, **2** (1864)and **3** (1865), also in *Opere*, I, 107–206; a paper on surfaces of constant curvature, in *Opere*, I, 262–280; "Teoria fondamentale degli spazi di curvatura costante,"*in Annali di matematica, ser*. 2 (1868–1869), 232–255, and *Opere*, I 406–429; "Richerche sulle cinematica dei fluidi,"in *Opere*, II, 202-379; a paper on the pseudosphere, in *Opere*, II, 394–409; a non-Euclidean approach to space, in *Opere*, III, 383–407; "Sulla teoria della scala diatonica,"in *Opere*, III, 408–412; an article on Saccheri, in *Rendiconti della Reale Accademia dei Lincei, ser*. 4, 5 (1889), 441–448, and *Opere*, IV, 348-355; and papers on Maxwell's theory, in *Opere*, IV, 356–361.

II. Secondary Literature. There is a biographical sketch by L Cremona in *Opere*, I. ix-xxii. See also L. Bianchi, "Eugenio Beltrami," in *Enciclopedia italiana*, VI (1930), 581; G.H. Bryan, "Eugenio Beltrami," in *Proceedings of the London Mathematical Society*, **32** (1900), 436–439: and G. Loria, "Eugenio Beltrami e le sue opere matematiche," in *Bibliotheca mathematica*, ser. 3, 2 (1901), 392–440. On Beltrami's contribution to non-Euclidean geometry, consult among others, R. Bonola, *Non-Euclidean Geometry* (Chicago, 1912; New York, 1955), pp. 130–139, 234-236.

D. J. Struik