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(*b*. Czestochowa, Poland, 5 May 1895; *d*. Palo Alto, California, 6 June 1977)

mathematics.

Stefan Bergman was the son of the Jewish merehant Bronislaw Bergman and his wife, Tekla. He graduated from the local gymnasium in 1913 and studied in the schools of engineering in Breslau and Vienna, receiving a degree as *Diplomingenieur* in 1920.

In 1921 Bergman entered the Institute for Applied Mathematics at the University of Berlin. Richard von Mises, the founder and director of the institute, was a leading theoretician in fluid dynamics and probability, and influenced Bergman during his whole career. Bergman worked on various problems of potential theory as applied to electrical engineering, elasticity, and fluid flow. To obtain a large number of harmonic functions in space, he applied and generalized the Whittaker method to create such functions by means of integrals over analytic functions. Using algebraic-logarithmic analytic functions as generators in the integral he created harmonic functions that are multivalued in space and have closed branch lines. This led him further to a general theory of integral operators that map arbitrary analytic functions into solutions of various partial differential equations. He devoted many years of work to this topic, producing a monograph in 1969.

The decisive influence on Bergman's scientific development came from Erhard Schmidt, who, with David Hilbert, had developed an elegant and seminal approach to the theory of integral equations with symmetric kernel. The eigenfunctions $ø_v(x)$ of such equations over an interval (a, b) form an orthonormal system, that is, one has

Bergman generalized this concept in a very original manner. Let D be a domain in the complex z-plane ($z = x + iy$) and consider analytic functions $f(z)$ in D such that

Consider a system $\{ø_v (z)\}$ of the type such that

and such that every f(z) can be written as

where the series converges uniformly in each closed subdomain of *D*. Such a system is called complete and orthonormal.

To a given domain D there exists an infinity of possible systems of this kind, but Bergman made the surprising discovery that the combination

Converges uniformly in each closed subdomain of D and is independent of the particular system used in its construction. For each $f(z)$ we have the identity

Therefore, Bergman called K(z, ζ) the reproducing kernel of the domain; it is now called the Bergman kernel. It has a simple covariance behavior under conformal mapping and is a very useful tool in the theory of analytic functions. Bergman's thesis in 1922 summarized his researches and led to his doctor's degree.

He soon realized that his method worked equally well when applied to analytic functions $f(z_1, z_2, \ldots, z_n)$ of n-complex variables. In the early 1920's the theory of such functions was in its initial stages, and Bergman was forced to do much pioneering work in this field. He may be considered as one of the founders of this theory, which is today an important field of research. The kernel function plays a very useful role in the theory of "Pseudo-conformal" mapping that carries a domain D in the space of $z_1 \ldots z_n$ into a domain Δ in the space $w_1 \ldots w_n$ by the relation $w_i = f_i (z_1 \ldots z_n)$, i = l, …, n. Again the kernel function has the same covariance behavior, and one can construct invariants from it and introduce a metric in D that is invariant under pseudo-conformal mapping. It is a special case of an important class, called "Kähler metrics," which was much later defined to deal with Riemannian manifolds.

Several important concepts in the theory of analytic functions of n-complex variables are due to Bergman. He discovered, for example, that for a large class of domains, an analytic function in it is completely determined by its values on a relatively small part of its boundary. He called it the "distinguished boundary" of *D*, and it now goes by the name "Bergman-Shilov boundary." Bergman used this concept to give a generalization of the Cauchy integral formula and to represent the value of a function at an interior point of *D* in terms of its values on the distinguished boundary.

In 1930 Bergman became a *Privatdozent* at the University of Berlin with a habitation thesis on the behavior of the kernel function on the boundary of its domain. In 1933 the Nazi seizure of power forced him out of his position and out of Germany. From 1934 to 1937 Bergman taught in Russia (Tomsk, 1934 -1936; Tbilisi, 1936–1937). From 1937 to 1939 he worked at the Institut Henri Poincaré in Paris. where he wrote a two-volume monograph on the kernel function and its applications in complex analysis.

Just before the outbreak of World War II, Bergman moved to the United States. He taught at MIT, Yeshiva College, and Brown University. In 1945 he joined his old teacher and friend von Mises at Harvard Graduate School of Engineering. He worked on various problems of fluid dynamics, using his methods on orthonormal developments, on integral operators, and on functions of several complex variables.

He settled into a more leisurely life. In 1950 he married Adele Adlersberg. He found time to summarize his results on the kernel function in a monograph. He started a collaboration with M. Schiffer, who has shown the close connection of the kernel function of a plane domain with its harmonic Green's function. They extended the kernel function concept to the case of elliptic partial differential equations. One orthonormalizes solutions of such equations in an appropriate metric, forms the kernel function in an analogous way, and constructs from it the fundamental solutions of the equation for the given domain. This was summarized in *Kernel Functions and Elliptic Differential Equations in Mathematical Physics* (1953). In 1952 Bergman accepted a position as professor at Stanford University, where he taught and did active research until his death.

BIBLIOGRAPHY

I. Original Works. Bergman's writings are given in *Poggendorff*. Important hooks are *Sur les fonctions orthagonales de plusieurs variables complexes aver les applications à la théorie des fonctions analytiques* (Paris, 1947); *Sur la fonction-noyau d'un domaine et ses applications dans la théorie des transformations pseudo-conformes* (Pans, 1948); *Kernel Functions and Elliptic Differential Equations in Mathematical Physics* (New York, 1953), with M. Schiffer: *Integral Operators in the Theory of Linear Partial Differential Equations*, 2nd rev. ed. (New York, 1969); and *The Kernel Function and Conformal Mapping*, 2nd rev. ed. (Providence, R.I., 1970).

II. Secondary Literature. Obituaries are in *Applicable Analysis*, **8** , no. 3 (1979), 195–199 (by Menahem Schiffer and Hans Samelson); and *Annales polonici mathematics*, **39** (1981), 5–9 (by M. M. Schiffer).

M. M. Schiffer