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(*b*. London, England, 17 October 1888; *d* Zurich, Switzerland, 18 September 1977)

mathematical logic, set theory.

Bernays came from a distinguished German-Jewish family of scholars and businessmen. His great-grandfather, Isaac ben Jacob Bernays, chief rabbi of Hamburg, was known for both strict Orthodox views and modern educational ideas. His grandfather, Louis Bernays, a merchant, traveled widely before helping to found the Jewish community in Zurich, while his great-uncle, Jacob Bernays, was a *Privatdozent* at the University of Bonn. In 1887 his father, Julius Bernays, a businessman, married Sara Brecher, who had likewise descended from Isaac Bernays. Their first child, Paul, born in London as a Swiss citizen and a *Bürger* of Zurich, was followed by a brother and three sisters.

After living in Paris for a time, the family settled in Berlin, where Paul Bernays had what he later described as a happy childhood. From 1895 to 1907 he attended the Köllnisches Gymnasium in Berlin. At an early age his talent as a pianist attracted attention, and he began to try his hand at composing. While at school, he added to his musical interests a growing attraction to ancient languages and mathematics. At eighteen, after hesitating between a career in music and one in mathematics, he opted for engineering (which his parents regarded as an eminently practical way to use his mathematical talent) and spent the summer semester of 1907 studying that subject at the Technische Hochschule in Charlottenburg. This experience made it clear to him that his future lay in pure mathematics rather than in its applications, Consequenily, in the winter he transferred to the University of Berlin. For the next two years he studied there the three subjects of central interest to him: mathematics, mainly under Issai Schur, Edmund Landau (including lectures on set theory), Leo Frobenius, and Friedrich H. Schottky; philosophy under Alois Riehl. Carl Stumpf, and Ernst Cassirer: and physics under Max K. E. L. Planck. Then, from 1910 to 1912, he attended lectures at Göttingen, chiefly by David Hilbert, Landau, Hermann Weyl, and Felix Klein in mathematics; by Woldemar Voigt and Max Born in physics: and by Leonard Nelson in philosophy. Göttingen would always remain his spiritual home.

In 1912 he received his doctorate ai Göttingen with a thesis, supervised by Landau, on the analytic number theory of binary quadratic forms. After his *Habilitationsschrift* on modular elliptic functions was accepted at the University of Zurich the following year, he served as assistant to Ernst F. F. Zermelo (who was then professor there) and as a *Privatdozent* until the spring of 1919. When Zermelo left the University of Zurich in 1916 (both for reasons of health and because of disagreements with the university). Bernays took over his courses. Although Bernays lectured primarily on topics in analysis, during his last year in Zurich (probably influenced by Hilhert), he gave courses on the foundations of geometry and on set theorv. While at Zurich, he became both a friend and a colleague of Pólya, had conversations with Einstein, and was received socially at the home of Hermann Weyl.

When Hilbert came to Zurich to lecture on “Axiomatisches Denken” in the fall of 1917, he invited Bernays to come to Göttingen as his assistant and to help him to resume investigations of the foundations of arithmetic. During 1918 Bernays quickly wrote for Göttingen a second *Habilitationsschrift*, establishing in it the completeness of propositional logic. But he continued to leach ai Zurich during both the summer semester and the winter semester of 1918. When he moved to Göttingen in 1919. Bernays received the *venia legendi*, which permitted him to lecture. He served as a *Privatdozzent* until he was made untenured extraordinary professor in March 1922. As at Zurich, he lectured chiefly on topics in analysis, but, beginning in 1922, he also gave courses on the foundations of geometry and (jointly with Hilhert) on the foundations of arithmetic. During the winter semester of the year 1929–1930 he first lectured on mathematical logic, giving at the end a version of his axioimatization for set theory. Believing that he learned better orally than through reading, he attended lectures by Emmy Noether. van der Waerden, and Herglotz. During the academic holidays he traveled regularly to Berlin to visit his family. His lather died in 1916.

On 28 April 1933 the dean at Göttingen ordered Bernays, as a “non-Aryan,” to stop teaching pending a final decision on his official status by the minister of education. In August he was relieved of his position as an assistant at the Mathematical Institute, and a month later his right to teach was officially withdrawn by the minister. For six months during this period Hilbert employed Bernays privately as his assistant. Finally Bernays and his family, having remained Swiss citizens, returned to Zurich.

From time to time, beginning with the summer semester of 1934, Bernays held a temporary teaching position at the Eidgenössische Technische Hochschule (E.T.H.) in Zurich. Meanwhile, he visited the Institute for Advanced Study at Princeton during the academic year 1935–1936 (as he was to do again during the academic year 1959–1960). In October 1939 the E.T.H. granted him the *venia legendi* for four years, renewing it in 1943. At last, in October 1945, he received a half-time

appointment as extraordinary professor at the E.T.H., a position that he continued to hold until 1959, when he became professor emeritus. During the spring semester of 1956, and again during 1961 and 1965, he was visiting professor at the University of Pennsylvania and gave lectures at several American universities. Although some mathematicians thought that the E.T.H. had not granted Bernays a position commensurate with his abilities, he himself always remained grateful to the E.T.H. for its support under difficult circumstances.

Bernays, who remained mathematically active until the end of his life, held a variety of positions: corresponding member in the Academy of Sciences of Brussels and in that of Norway, president of the International Academy of the Philosophy of Science, and honorary chairman of the German Society for Mathematical Logic and Foundational Research in the Exact Sciences. In addition he served on the editorial board of *Dialectica* and was a coeditor for both the *Journal of Symbolic Logic* and *Archiv für mathematische Logik und Grundlagenforschung*. In 1976 he received an honorary doctorate from the University of Munich for his work in proof theory and set theory. After a brief illness he died of a heart condition at the age of eighty-eight.

Bernays' earliest publications, beginning in 1910, were devoted to philosophy. They showed the direct influence of his teacher Leonard Nelson, head of the neo-Friesian school, which had revived the philosophy of Jacob Fries and extended it to ethics. Except for his doctoral thesis and his Zurich *Habilitationsschrift*, Bernays published no mathematical articles until 1918. His attempt during that period to extend the special theory of relativity was preempted when Einstein introduced general relativity.

Then, answering Hilbert's call to Göttingen to collaborate on foundational (especially proof-the-oretic) questions, Bernays began the work in mathematical logic that molded his career. The first product was his 1918 *Habilitationsschrift*, "Beiträge zur axiomatischen Behandlung des Logik-Kalküls," which was devoted to the metamathematics of the propositional calculus (the heir of George Boole's logic) and was a contribution to Hilbert's program. In contrast to most earlier logicians, Bernays had a firm grasp of the difference between syntax and semantics, and he distinguished carefully between provable formulas and valid formulas. By establishing the completeness theorem for propositional logic, he showed these two notions to be equivalent in that context. Further, he gave a partial solution to Hilbert's decision problem (*Entscheidungsproblem*) by stating a decision procedure for validity in this part of logic. He also demonstrated that one of the axioms for propositional logic in *Principia Mathematics* was redundant while the four remaining axioms were independent. In these independence proofs, many-valued logic was utilized for the first time. Although in 1926 his *Habilitationsschrift* of 1918 was published in part. Post had independently published the same completeness result in 1921— with the effect that, outside the Hilbert school. Post was usually given credit for it.

The most enduring achievement of the collaboration between Hilbert and Bernays was their *Grundlagen der Mathematik*, published in two volumes in 1934 and 1939, which for decades remained the standard work on proof theory. It appears that, while the overall approach was due to Hilbert, the specific contents and the actual writing came from Bernays. There Bernays developed the ε-calculus and the ε-theorems for eliminating quantifiers so as to give a decision procedure for various theories. Moreover, he provided the first detailed proof of Gödel's second incompleteness theorem and supplied the first correct proof for Herbrands's theorem (Herbrand's proof was faulty). Finally, he demonstrated a proof-theoretic version of Gödel's completeness theorem for first-order logic.

In 1937, when Bernays published the first of his seven-part article on his axiomatization of set theory, he pointed out that his aim was to modify von Neumann's axiom system (based on function and argument rather than on set and membership) so as to make it resemble more closely Zermelo's original system, and thereby to use some of the set-theoretic concepts of Schröder and Whitehead-Russell, while expressing the theory in first-order logic. Bernays chose his groups of axioms in order to render them analogous, whenever possible, to Hilbert's (1899) groups of axioms for geometry. As Hilbert had done, Bernays explored (in 1941 and 1942) the consequences of various axioms within a group, and showed which of his axioms were needed to develop number theory on the one hand and analysis (up to Lebesgue measure) on the other. While doing so, he formulated the principle of dependent choices (the form of the axiom of choice sufficient for analysis), later independently rediscovered by Tarski. In the last two sections (published in 1948 and 1954) he investigated the independence of his axioms, mainly by using number-theoretic models in the spirit of Ackermann. Bernays' axiom system, slightly modified by Gödel, is now generally known as Bernays-Gödel set theory.

"In philosophy," Bernays wrote in his autobiography about his return to Zurich in 1934, "I came into closer contact with Ferdinand Gonseth.… Because of my interior dialogues on the philosophy of Kant, Fries, and Nelson, I had come very close to Gonseth's views, and so I joined his school of philosophy." Bernays was sympathetic, in particular, to Gonseth's "open philosophy," with its emphasis on dialogue between opposing view-points, and encouraged tolerance between diverse foundational positions (such as Platonism and intuitionism). In 1946 Bernays, Gonseth, and Karl Popper founded the Internationale Gesellschaft zur Pflege der Logik und Philosophie der Wissenschaft, which started the journal *Dialectica* the following year. Bernays' philosophical writings, so refreshingly undogmatic, are models of clarity.

Bernays directed a number of doctoral theses, At Göttingen he was involved with Haskell Curry's and Gerhard Gentzen's, and even supervised Saunders Mac Lane's (1934), though Weyl conducted Mac Lane's oral examination since the Nazis had already dismissed Bernays. At the E.T.H. in Zurich he directed theses by Martin Altwegg (1948), Hugh Ribeiro (1949), J. Richard Büchi (1950), Walter Strickler (1955), Erwin Engeler (1958), and Hersz Wermus (1961), as well as serving as *Korreferent* for six other dissertations.

Bernays' precise contributions are often difficult to ascertain because of his preference for collaboration (especially with Hilbert) and his modesty. Engeler, his former student, described him as a "great scholar and kind man." Yet Hilbert's 1922 letter recommending Bernays for an extraordinary professorship at Göttingen remains the most fitting tribute to his life's work:

Bernays' publications extend over the most diverse fields of mathematics… [and] are all marked by thoroughness and reliability…. He is distinguished by a deep-seated love for science as well as a trustworthy character and nobility of thought, and is highly valued by everyone. In all matters concerning foundational questions in mathematics, he is the most knowledgeable expert and, especially for me, the most valuable and productive colleague.

BIBLIOGRAPHY

I. Original Works. A bibliography of Bernay's publications up to 1976 can he found in Gert H. Müller. ed., *Sets and Classes: On the Work by Paul Bernays* (New York, 1976) and is supplemented in Müllers 1981 article mentioned below. The same book, which contains a photo of Bernays, reprints his seven-part article giving his axiom system for set theory and includes an English translation of his 1961 article on strong axioms of infinity. Fourteen of his philosophical essays (eight of them on logic or foundational questions) are reprinted in his *Abhandlunngen zur Philosophie der Mathematik* (Darmstadt, 1976). He also edited the later editions of David Hilberi's *Grundlangen der Geometrie* (Stuttgart, 1977) and was an editor of Leonard Nelson's *Gesammelte Schriften*.

The *Wissenschaftshistorische Sammlungen* at the Eidgenössische Technische Hochschule in Zurich has a rich collection of material on Bernays, including many of his unpublished lectures and a voluminous correspondence, as well as his lecture notes for courses (generally in Gabelsberger shorthand). Some of Bernays' letters and manuscripts are located at the *Niedersächsische Staatsund Universitätsbibliotek* in Göttingen.

II. Secondary Literature. Bernays wrote a brief autobiography for the Müller volume as well as the *Lebenslauf* at the end of his 1912 dissertation. Obituaries are by Ernst Specker and Erwin Engeler, in *New Zürcher Zeitung* (26 September 1977); and Gert H. Müller, in the *Mathematical Intelligencer*, **1** (1978) 27–28. More extensive discussions of his life and work can he found in Erwin Engeler, "Zum logischen Werk von Paul Bernays," in *Dialectia*, **32** (1978), 191–200; Abraham Fraenkel, "Paul Bernays and die Begründung der Mengenlehre," in *Dialectica*. **12** (1958), 274–279; Henri Lauener, "Wissenschaftstheorie in der Schweiz," and "Paul Bernays (1888–1977)," in *Zeitschriff fût Allgemeine Wissenschaftstheorie*, **2** , no. 2 (1971), 294–299, and 9, no. 1 (1978), 13–20 Gert H. Müller, "Framingham Mathematics," in *Epistemologia*, **4** (1981), 253–285; Andrés R. Raggio, "Die Rolle der Analogie in Bernays' Philosophie der Mathematik," in *Dialectica*, **32** (1978), 201–207; Ernst Specker, "Paul Bernays," in Maurice Boffa *et al*. eds. *Logic Colloquium' 78* (1979), 381–389; Gaisi Takeuti. "Work of Paul Bernays and Kurt Gödel," in L. Jonathan Cohen *et al*., eds., *Logic, Methodology and Philosophy of Science*, VI (1982), 77–85.

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