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(*b*. Basel, Switzerland, 27 December 1654; *d*. Basel, 16 August 1705)

mathematics, mechanics, astronomy.

Bernoulli came from a line of merchants. His grandfather, Jakob Bernoulli, was a druggist from Amsterdam who became a citizen of Basel in 1622 through marriage. His father, Nikolaus Bernoulli, took over the thriving drug business and became a member of the town council and a magistrate; his mother, Margaretha Schönauer, was the daughter of a banker and town councillor. Jakob was married in 1684 to Judith Stupanus, the daughter of a wealthy pharmacist; their son Nikolaus became a town councillor and master of the artists' guild.

Bernoulli received his master of arts in philosophy in 1671, and a licentiate in theology in 1676; mean-while, he studied mathematics and astronomy against the will of his father. In 1676 he went as a tutor to Geneva, where in 1677 he began his informative scientific diary, *Meditationes*; he then spent two years in France, familiarizing himself with the methodological and scientific opinions of Descartes and his followers, among whom Was Nicolas Malebranche. Bernoulli's second educational journey, in 1681–1682, took him to the Netherlands, where he met mathematicians and scientists, especially Jan Hudde, and to England, where he met Robert Boyle and Robert Hooke. The scientific result of these journeys was his inadequate theory of comets (1682) and a theory of gravity that was highly regarded by his contemporaries (1683).

After returning to Basel, Bernoulli conducted experimental lectures, concerning the mechanics of solid and liquid bodies, from 1683 on. He sent reports on scientific problems of the day to the *Journal des sçavans* and the *Acta eruditorum*, and worked his way through the principal mathematical work of those days, *Geometria*, the Latin edition of Descartes's *Géométrie*, which had been edited and provided with notes and supplements by Frans van Schooten (2nd ed., Amesterdam, 1659–1661). As a result of this work, Bernoulli contributed articles on algebraic subjects to the *Acta eruditorum*. His outstanding achievement was the division of a triangle into four equal parts by means of two straight lines perpendicular to each other (1687). After these contributions had been extended and supplemented, they were published as an appendix to the *Geometria* (4th ed., 1695).

In four disputations published from 1684to 1686, Bernoulli presented formal logical studies that tended toward the sophistical. His first publication on probability theory dates from 1685. By working with the pertinent writings of John Wallis (those of 1656, 1659, and 1670–1671) and Isaac Barrow (1669–1670), concerning mathematical, optical, and mechanical subjects, Bernoulli was led to problems in infinitesimal geometry.

In the meantime his younger brother Johann began attending the University of Basel after an unsuccessful apprenticeship as a salesman. As respondent to one of Jakob's scholarly logic debates, Johann earned his master of arts degree in 1685 and by order of his father, studied medicine. Simultaneously, however, he secretly studied mathematics under his brother, becoming well versed in the fundamentals of the field. In 1687 Jakob became professor of mathematics at Basel, and with his brother he studied the publications of Leibniz and of Ehrenfried Walther von Tschirnhaus in *Acta eruditorum* (1682–1686), which had in essence been limited to examples and intimations of infinitesimal mathematics and its application to mechanics and dynamics. After much effort, Bernoulli was able to make himself master of those new methods, which he erroneously believed to be merely a computational formalism for Barrow's geometrical treatment of infinitesimals, His mathematical studies reached a first peak about 1689 with the beginnings of a theory of series, the law of large numbers in probability theory, and the specials stress on complete induction.

Bernoulli showed his mastery of the Leibnizian calculus with his analysis (in May 1690) of the solutions given by Huygens in 1681 and by Leibniz in 1689 to the problem of the curve of constant descent in a gravitational field. (It was in that analysis that the term "integral" was first used in its present mathematical sense.) The determination of the curve of constant descent had been posed as a problem by Leibniz in 1687. As a counter problem Bernoulli raised the determination of the shape of the catenary, to which he had, perhaps, been directed by Albert Girard's notes to the *Oeuvres* of Simon Stevin (1634); Girard claimed that the catenary is a parabola. Leibniz promptly referred to the significance of this counterproblem, which he had spontaneously solved (1690) and which was later treated by Johann Bernoulli, Huygeus and himself in the *Acta eruditorem* (1691). Jakob, who found himself at that time in difficulties at the university because of his open criticism of university affairs and saw himself being overshadowed by his brother, did not take part directly, but proposed generalizations of the problem, allowing the links of the chain to be elastic or of unequal weight. He also announced a treatise on the *elastica*, the form of a bent elastic beam, which, under certain conditions, satisfies the differential equation $dy/dx =$. Later he investigated this thoroughly, supposing arbitrary functions of elasticity (1694). In two notable contributions to differential calculus (1691), he examined the parabolic spiral (in polar coordinates: , the elliptical integral for the curve length with its charateristic feature of symmetry) and the logarithmic spiral.

In Johann Bernoulli's study concerning the focal line of incident parallel rays of light on a semicircular mirror (1692), there is reference to Jakob's general procedure for determination of evolutes. This procedure is based on the generation of an algebraic curve as the envelope of its circles of curvature, and this procedure is worked out fully in the case of the parabola. Here Bernoulli corrected a mistake made by Leibniz (1686)—the statement that the circle of curvature meets the curve at four coinciding points—but he himself made a mistake in his assertion that the radius of curvature becomes infinite at every point of inflection. This error, corrected in 1693 by G. F. A. de L'Hospital, was the occasion for Bernoulli's removing of the singularity $a^2x^3 = y^5$ origin (1697). Almost simultaneously, and independently of each other, the brothers recognized that the form of a sail inflated by the wind is described by $(dx/ds)^3 = ad^2y/dx^2$. Jakob made a preliminary report in 1692 and a thorough one in 1695.

Further investigations concerned evolutes and caustics, first of the logarithmic spiral (*spira mirabilis*) and the parabola (1692), and later of epicycloids (1692) and diacaustic surfaces (1693), this in connection with Johann's similar studies. These last were included in his private instruction to L'Hospital (1691–1692). Here, for the first time, public reference was made to the *theorema aureum*, which had been developed in the spring of 1692. The theorem, which gives the radius of curvature as $(ds/dx)^3$: (d^2y/dx^2), was published in 1694. Bernoulli's solution of the differential equation proposed by Johann Bernoulli (1693), $xdy - ydx/yds = a/b$, was completed by Huygens (1693). In treating the paracentric isochrone, a problem proposed by Leibniz (1689) that leads to the differential equation

Bernoulli separated the variables by substituting

$x^2 + y^2 = r^2$, $ay = rt$

and was able to relate the solution to the rectification of the *elastica;* later he found the reduction to the rectification of the lemniscate,

These and other studies—among which the kinetic geometrical chord construction for the solution (1696) of $dy/dx = t(x)/a$ and the solution (1696) of the so called Bernoullian differential equation $y' = p(x)y + q(x)y^n$ (1696) merit special attention—are proof of Bernoulli's careful and critical work on older as well as on contemporary contributions to infinitesimal mathematics and of his perseverance and analytical ability in dealing with special pertinent problems, even those of a mechanical-dynamic nature.

Sensitivity, irritability, a mutual passion for criticism, and an exaggerated need for recognition alienated the brothers, of whom Jakob had the slower but deeper intellect. Johann was more gifted in working with mathematical formulations and was blessed above all with a greater intuitive power and descriptive ability. Johann was appointed a professor at the University of Groningen in 1695, and in 1696 he proposed the problem to determine the curve of quickest descent between two given points, the brachistochrone. In connection with this he replied to the previous gibes of his brother with derisive insinuations. Jakob gave a solution (1697) that was closely related to that given by Leibniz (1697). It is based on the sufficient but not necessary condition that the extreme-value property of the curve in question (a common cycloid) is valid not only for the entire curve but also for all its parts. As a counter problem Jakob set forth the so-called isoperimetric problem, the determination of that curve of given length between the points $A(-c; 0)$, $B(+c; 0)$ for which takes a maximum value. Johann, in the *Histoire desouvrages de savants* (1697), through a misunderstanding of the difficulty of the problem and of its nature (calculus of variations), gave a solution based on a differential equation of the second degree. A differential equation of the third degree is necessary however, After showing that a third-degree equation is required (1701), Jakob was able also to furnish the proof which Johann and Leibniz had been seeking in vain, that the inexpansible and homogeneous catenary is the curve of deepest center of gravity between the points of suspension.

Johann Bernoulli may have comprehended the justification for his brother's argument soon after publication of the dissertation of 1701 (*Analysis magni problematis isoperimetrici*), but he remained silent. Only after Brook Taylor had adopted Jakob's procedure (1715) was he induced to accept Jakob's point of view. In the 1718 series of the *Mémoires* of the Paris Académic des Sciences, of which the brothers had been corresponding members since 1699, Johann gave a presentation, based on Jakob's basic ideas but improved in style and organization. It was not superseded until Leonhard Euler's treatment of the problems of variations (1744).

The antagonism between the brothers Soon led tougly critical remarks. In 1695 Jakob failed to appreciate the significance of Johann's extraordinarily effective series expansion (1694), which is based on iterated integration by parts and leads to a remainder in integral form. On the other hand, Johann, who, in 1697 had challenged the criterion for geodetic lines on convex surfaces, complained in the following year that his brother knew how to solve the problem "only" on rotation surfaces. Other items of disagreement were the determination of elementary quadrable segments of the common cycloid and related questions. The brothers argued over this in print in 1699 and 1700. The formulas for the multisection of angles are connected with these problems. In his ingenious use of Wallis' incomplete induction (1656), Jakob presented $2 \cdot \cos n\alpha$ and $2 \cdot \sin n\alpha$ as functions of $2 \cdot \sin \alpha$. This is related to his notes from the winter of 1690–1691, in which, furthermore, the exponential series was derived from the binomial series in a bold but formally unsatisfactory manner.

Jakob Bernoulli's decisive scientific achievement lay not in the formulation of extensive theories but in the clever and preeminently analytical treatment of individual problems. Behind his particular accomplishments there were, of course, notions of which Bernoulli was deeply convinced, primarily concerning continuity of all processes, of nature (*natura non facit saltum*). Although Bernoulli assigned great significance to experimental research, he limited himself—for example, in investigations of mechanics—to a few basic facts to which he tried to cling and an which he sought to base full theories. For this reason his final results were intellectually interesting, and as points of departure they were significant for further investigation by his contemporaries and subsequent generations. Naturally enough, they usually do not conform to more modern conclusions, which rest on far wider foundations. It is to be regretted, however, that Bernoulli's contributions to mechanics are hardly ever mentioned in the standard works.

The theory that seeks to explain natural phenomena by assuming collisions between particles of the ether, developed in the *Dissertatio de gravitate aetheris* (1683), of course does not mean much to a later generation. There are extensive discussions about the center point of oscillation, which had been determined correctly for the first time by Huygens in his *Horologium oscillatorium* (1673), but this was strongly debated by some of the members of the Cartesian school. On this subject Bernoulli expressed his opinions first in 1684, and then in more detail in 1686 and 1691; finally he succeeded in developing a proof from the properties of the lever (1703–1704). Important also is his last work, on the resistance of elastic bodies (1705). Supplementary material from his scientific diary is contained in the appendix to his *Opera*. Additional, but unpublished, material deals with the center of gravity of two uniformly moved bodies, the shape of a cord under the influence of several stretching forces, centrally accelerated motion (in connection with the statements of Newton in his *Principia*[1687]), and the line of action and the collective impulse of infinitely many shocks exerted on a rigid arc in the plane.

In the field of engineering belongs the 1695 treatment of the drawbridge problem (the curve of a sliding weight hanging on a cable that always holds the drawbridge in balance), stemming from Joseph Sauveur and investigated in the same year by L'Hospitaland Johann Bernoulli. Leibniz was also interested in the problem. In Bernoulli's published remains, the contour is determined upon which a watch spring is to be developed so that the tension always remains the same for the movements of the watch.

The five dissertations in the *Theory of Series* (1682–1704) contain sixty consecutively numbered propositions. These dissertations show how Bernoulli (at first in close cooperation with his brother) had thoroughly familiarized himself with the appropriate formulations of questions to which he had been led by the conclusions of Leibniz in 1682 (series for $\pi/4$ and log 2) and 1683 (questions dealing with compound interest). Out of this there also came the treatise in which Bernoulli took into account short-term compound interest and was thus led to the exponential series. He thought that there had been nothing printed concerning the theory of series up until that time, but he was mistaken: most conclusions of the first two dissertations (1689, 1692) were already to be found in Pietro Mengoli (1650), as were the divergence of the harmonic series (Prop. 16) and the sum of the reciprocals of infinitely many figurate numbers (Props. 17–20).

The so-called Bernoullian inequality (Prop. 4), $(1 + x)^n > 1 + nx$, is intended for $x > 0$, n as a whole number > 1. It is taken from Barrow's seventh lecture in the *Lectiones geometricae* (1670). Bernoulli would have been able to find algebraic iteration processes for the solution of equations (Props. 27–35) in James Gregory's *Vera… quadratura* (1667). The procedure of proof is still partially incomplete because of inadmissible use of divergent series. At the end of the first dissertation Bernoulli acknowledged that he could not yet sum up in closed form (Euler succeeded in doing so first in 1737); but he did know about the majorant , which can be summed in elementary terms. In Proposition 24 it is written that equals $(2^m - 1)/1$ (m integer > 1), and that diverges more rapidly than . Informative theses, based on Bernoulli's earlier studies, were added to the dissertations: and theses 2 and 3 of the second dissertation are based on the still incomplete classification of curves of the third degree according to their shapes into thirty-three different types.

The third dissertation was defended by Jakob Hermann, who wrote Bernoulli's obituary notice in *Acta eruditorum* (1706). In the introduction L'Hospital's *Analyse* is praised. After some introductory propositions, there appear the logarithmic series for the hyperbola quadrature (Prop. 42), the exponential series as the inverse of the logarithmic series (Prop. 43), the geometrical interpretation of

(Prop. 44), and the series for the arc of the circle and the sector of conic sections (Props. 45, 46). All of these are carefully and completely presented with reference to the pertinent results of Leibniz (1682; 1691). In 1698 previous work was supplemented by Bernoulli's reflections on the catenary (Prop. 49) and related problems, on the rectification of the parabola (Prop. 41), and on the rectification of the logarithmic curve (Prop. 52).

The last dissertation (1704) was defended by Bernoulli's nephew, Nikolaus I, who helped in the publication of the *Ars conjectandi* (1713) and the reprint of the dissertation on series (1713) and became a prominent authority in the theory of series. In the dissertation Bernoulli first (Prop. 53) praises Wallis' interpolation through incomplete induction. In Proposition 54 the binomial theorem is presented, with examples of fractional exponents, as an already generally known theorem. Probably for this reason there is no reference to Newton's presentation in his letters to Leibniz of 23 June and 3 November 1676, which were made accessible to Bernoulli when they were published in Wallis' *Opera* (Vol. III, 1699). In proposition 55 the method of indeterminate coefficients appears, without reference to Leibniz (1699). Propositions 56–58 and 60 deal with questions related to the *elastica*.

In Proposition 59 it is stated that the series for log 2 should be replaced by

which converges more rapidly. From the letter to Leibniz of 2 August 1704, we know that in Proposition 59 Bernoulli used an idea of Jean-Christophe Fatio-de-Duillier (1656–1720), an engineer from Geneva, for the improvement of convergence. The procedure was expanded by Euler in the *Institutiones calculi differential is* (1755) to his so-called series transfonnation. In the dissertations on series Bernoulli apparently wished to reproduce everything he knew about the subject. In this he was primarily concerned with the careful rendering of the results and not so much with originality.

The *Ars conjectandi* is Bernoulli's most original work, but unfortunately it is incomplete. The first part is basically a first-rate commentary on Huygens' *De ratiociniis in aleae ludo*, which was published as an appendix to van Schooten's *Exercitationes mahematicae* (1657). In the second part Bernoulli deals with the theory of combinations, based on the pertinent contributions of van Schooten (1657), Leibniz (1666), Wallis (1685), and Jean Prestet's *Élémens de mathématiques* (1675; 2nd ed., 1689). The chief result here is the rigid derivation of the exponential series through complete induction by means of the so-called Bernoullian numbers. In the third part Bernoulli gives twentyfour examples, some simple, some very complicated, on the expectation of profit in various games.

The fourth part contains the philosophical thoughts on probability that are especially characteristic of Bernoulli: probability as a measurable degree of certainty; necessity and chance; moral versus mathematical expectation; a priori and a posterioti probability; expectation of winning when the players are divided according to dexterity; regard of all available arguments, their valuation, and their calculable evaluation; law of large numbers, and reference to the *Art de penser (Logique de* Port Royal, Antoine Arnauld and Pierre Nicole, eds., 1662). The last section contains a penetrating discussion of *jeu de paume*, a complicated predecessor of tennis that was very popular. This part is Bernoulli's answer to the anonymous gibes occasioned by his debate of 1686 on scholarly logic.

Bernoulli's ideas on the theory of probability have contributed decisively to the further development of the field. They were incorporated in the second edition of Rémond de Montmort's *Essai* (1713) and were considered by Abraham de Moivre in his *Doctrine of Chances* (1718).

Bernoulli greatly advanced algebra, the infinitesimal calculus, the calculus of variations, mechanics, the theory of series, and the theory of probability. He was self-willed, obstinate, aggressive, vindictive, beset by feelings of inferiority, and yet firmly convinced of his own abilities. With these characteristics, he necessarily had to collide with his similarly disposed brother. He nevertheless exerted the most lasting influence on the latter.

Bernoulli was one of the most significant promoters of the formal methods of higher analysis. Astuteness and elegance are seldom found in his method of presentation and expression, but there is a maximum of integrity. The following lines taken from the *Arsconjectandi* (published posthumously in 1713) are not without a certain grace, however, and represent a nearly statement, made with wit and clarity, of the boundaries of an infinite series.

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Ut non-finitam Seriem tinita cöercet,
  Summula, & in nullo limite limes adest:
Sic modico immensi vestigia Numinis haerent
  Corpore, & angusto limite limes abest.
Cernere in immenso parvum, dic, quanta voluptas!
  In parvo immensum cernere, quanta, Deum!
Even as the finite encloses an infinite series
  And in the unlimited limits appear,
So the soul of immensity dwells in minutia
  And in narrowest limits no limits inhere.
What joy to discern the minute in infinity!
  The vast to perceive in the small, what divinity!
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The collected works are in preparation. Included are Bernoulli's correspondence with Nicolas Fatio-de-Duillier (1700–1701) and with Otto Mencke (1686, 1689). The most important correspondence with L'Hospital and Pierre Varignon seems to have been lost.

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The poem "On Infinite Series," Helen M. Walker, trans., appears in D. E. Smith. *A Source Book in Mathematics*; repr. by permission Harvard University Press.

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