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(*b*. Odessa, Russia, 5 March 1880; *d*. Moscow, U.S.S.R., 26 October 1968),

mathematics.

Bernstein was the son of Natan Osipovich Bernstein, lecturer in anatomy and physiology at the Novorossysky University in Odessa. After graduating from high school in 1898, he studied at the Sorbonne and the École d'Électrotechnique Supériure in Paris; in 1902-1903 he also studied ta Göttingen. He defended his master's thesis (1908) and his doctoral dissertation at Kharkov (1913) because scientific degrees awarded abroad did not entitle one to a university post in Russia. From 1907 to 1933 Bernstein taught at Kharkov University, first as a lecturer and then as a professor after 1917, laying the foundations of a mathematical school that included N.I. Akhiezer and V. L. Goncharov. During this period Bernstein frequently gave series of lectures and presented reports abroad; in 1915 he participated in the Second All-Russian Congress of High School Teachers; and in 1930 he organized the First All-Union Mathematical Congress, held in Kharkov. He was director of the Mathematical Research Institute in 1928-1931 and was one of the leaders of the Kharkov Mathematical Society from 1911. In 1925 he was elected member of the Academy of Sciences of the Ukrainian S.S.R.

In 1933 Bernstein began lecturing at the University of Leningrad and the Polytechnical Institute, while he worked also in the Mathematical Institute of the U.S.S.R. Academy of Sciences, of which he had been elected corresponding member in 1924 and member in 1929. He moved to Moscow in 1943, continuing his work at the Mathematical Institute. He edited Chebyshev's *Polnoe Sobranie sochieny* ("Complete Works"; 1944-1951) and in his later years prepared a four-volume edition of his own writings.

Bernstein was a member of the Paris Académie des Sciences (elected corresponding member in 1928 and foreign member in 1955), doctor *honoris causa* of the universities of Algiers (1944) and Paris (1945), and honorary member of the Moscow (1940) and Leningrad (1966) mathematical societies. In 1942 his scientific achievement was recognized with the State Award, first class; earlier he had received awards from the Belgian Academy of Sciences and from the Paris Academy of Sciences for the book based on his lectures at the Sorbonne in 1923.

In his work Bernstein united traditions of Chebyshev's St. Petersburg mathematical school with western European mathematical thought, particularly that of France (Picard, Vallée-Poussin) and Germany (Weierstrass, Hilbert), Three fields dominated his work: partial differential equations, theory of best approximation of functions, and probability theory.

Bernstein's doctoral dissertation at the Sorbonne held the solution of Hilbert's nineteenth problem (1900), particular cases of which had been treated shortly before by Picard and others. Bernstein's result read: If a solution Z (x,y) of an analytical differential equation F(x,y,p,q,r,s,t) = 0 of elliptical type , where , , , , possesses continuous derivatives up to the third order (inclusive), the solution is analytical. In his master's thesis Bern-stein solved Hilbert's twentieth problem, demonstrating the possibility of an analytical solution of Dirichlet's problem for a wide class of nonlinear elliptical equations, A number of theorems on the differential geometry of surfaces followed, particularly on the theory of minimal surfaces. The above-mentioned studies were further advanced by Bernstein and many others, and are still being developed.

Another area of Bernstein's work concerned the theory of the best approximation of functions, to which he contributed new ideas and that he called the constructive theory of functions (1938). In this theory, calculation and investigation of functions are carried out with the notion introduced by Chebyshev in 1854 of the best approximation of a given function *f(x)* by means of a polynomial $g_n(x)$ of a given degree *n* or by means of some other relatively simple function *g(x)* depending on the finite number of parameters. If parameters are selected so that on a segment (*a,b*) the value of max $|f(x)— g_n(x)|= E_nf(x)|$ is minimal, the function $g_n(x)$ is called the best approximation of a function *f(x)*.Chebyshev and his immediate successors were more interested in finding the polynomial of best approximation $g_n(x)$ when *n* is given, than in exploring the general functional properties of the quantity $E_n[f(x)]$. In 1885 Weierstrass demonstrated that any function *f(x)* continuous on a segment (*a,b*) can be expanded into a uniformly convergent series of polynomials so that , whatever the continuous function *f(x)* is.

The point of departure for Bernstein was the problem of estimating the order of the quantity E_n for *f(x)*= |*x*| on a segment (-1,+1), posed by Vallée-Poussin. In 1911 Bernstein demonstrated that the best approximation $E_{2n}(|x|)$ by means of a polynomial of degree$_{2n}$ lay between and that there existed with 0.278 < λ < 0.286, so that for sufficiently great values of *n* the inequality holds. Bernstein also studied asymptotic values of the best approximation for various classes of functions and established the closest relations between the law of decrease of the quantity $E_n[f(x)]$ and the analytical and differential properties of the function *f(x)*. These relations were the subject of his report at the Fifth International Congress of Mathematicians, held at

Cambridge in 1912. Bernstein later continued his studies in that area, solving problems relevant to the theory of interpolation, methods of mechanical quadratures, and the best approximation on an infinite axis. In 1914 he introduced an important new class of quasi-analytical functions.

Almost simultaneous with the theory of best approximation Bernstein approached probability theory. In 1917 he suggested the first system of axioms for probability theory; he later conducted a number of fundamental studies relevant to the generalization of the law of large numbers, the central limit theorem, Markov chains, the theory of stochastic processes, and applications of probability theory to genetics. All these works by Bernstein reatly influenced the advance of contemporary probability theory.

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