Bolyai, János (Johann) | Encyclopedia.com

(*b*. 15 December 1802, Koloszvár [German, Klausenburg], Transylvania, Hungary [now Cluj, Rumania]; *d*. 27 January 1860, Marosvásárhely, Hungary [now Targu-Mures, Rumania])

mathematics.

The son of Farkas (Wolfang) Bolyai and Susanna von Árkos Bolyai, János Bolyai received his early education in Marosvásárhely, where his father was professor of mathematics, physics, and chemistry at Evangelical-Reformed College. The precocious lad was first taught by his father and showed early proficiency not only in mathematics but also in other fields, such as music. He mastered the violin at an early age. Form 1815 to 1818, he studied at the college where his father taught. The elder Bolyai had hopes that the son would go on to Göttingen to study with his friend Gauss, but he did not. In 1818 János entered the imperial engineering academy in Vienna, where he received a military education; he remained there until 1822.

From his father, János had inherited an interest in the theory of parallels: but in 1820 his father warned him against trying to prove the Euclidean axiom that there can be only one parallel to a line through a point outside of it:

You should not tempt the parallels in this way, I know this way until its end—I also have measured this bottomless night. 1 have lost in it every light, every joy of my life—… You should shy away from it as if from lewd intercourse, it can deprive you of all your leisure, your health, your peace of mind and your entire happiness.— This infinite darkness might perhaps absorb a thousand giant Newtonian towers, it will never be light on earth, and the miserable human race will never have something absolutely pure, not even geometry… [Stáckel, pp. 76–77].

In the same year, however, János began to think in a direction that led him ultimately to a non-Euclidean geometry. He profited by conversations with Karl Szász, governor in the house of Count Alexis Teleki, In 1823, after vain attempts to prove the Euclidean axiom, he found his way by assuming that a geometry can be constructed without the parallel axiom: and he began to construct such a geometry. “From nothing I have created another entirely new world,” he jubilantly wrote his lather in a letter of 3 November 1823. By this time János had finished his courses at the academy and had entered upon a military career, beginning as a sublieutenant. His duties took him first to Temesvar (now Timisoara, Rumania), in 1823–1826, then to Arad (Rumania), in 1826–1830, and finally to Lemberg (now Lvov. W. Ukraine), where in 1832 he was promoted to lieutenant second class. During his military service, he was often plagued with intermittent fever, but he built up a reputation as a dashing officer who dueled readily. In 1833 he was pensioned off as a semi-invalid, and he returned to his father’s home in Marosvásárhely.

While visiting his father in February 1825, János had shown him a manuscript that contained his theory of absolute space, that is a space in which, in a plane through a point *P* and a line *I* not through *P* there exists a pencil of lines through *P* which does not intersect *I*. When this pencil reduces to one line, the space satisfies the Euclidean axiom. Farkas Bolyai could not accept this geometry, mainly because it depended on an arbitrary constant, but he finally decided to send his son’s manuscript to Gauss. The first letter (20 June 1831) went unanswered, but Gauss did answer a second letter (16 January 1832). In this famous reply, dated 6 March 1832 and directed to his “old, unforgettable friend,” Gauss said:

Now something about the work of your son. You will probably be shocked for a moment when I begin by saying *that I cannot praise it*, but I cannot do anything else, since to praise it would be to praise myself. The whole content of the paper, the path that your son has taken, and the results to which he has been led, agree almost everywhere with my own meditations, which have occupied me in part already for 30–35 years. Indeed, I am extremely astonished….

Further on, after mentioning that there had been a time when he had been inclined to write such a paper himself, Gauss continued, “Hence I am quite amazed, that now I have been saved the trouble, and I am very glad indeed that it is exactly the son of my ancient friend who has preceded me in such a remarkable way.” Gauss ended with some minor remarks, among them a challenge to János to determine, in his geometry, the volume of a tetrahedron, and a critique of Kant’s theory of space.

It is now known from Gauss’s diaries and from some of his letters that he was not exaggerating; but for János the letter was a terrible blow, since it robbed him of the priority. Even after he became convinced that Gauss spoke the truth, he felt that Gauss had done wrong in remaining silent about his discovery. Nevertheless, he allowed his father to publish his manuscript, which appeared as an appendix to the elder Bolyai’s *Tentamen* (1832), under the title “Appendix scientiam spatii absolute veram exhibens” (“Appendix Explaining the Absolutely True Science of Space”). This classic essay of twenty-four pages, which contains János’ system of non-Euclidean geometry, is the only work of his published in his lifetime. Gauss’s letters had such a discouraging influence on him that he withdrew into himself more and more, and for long periods he did hardly any mathematics. Disappointment grew when his essay evoked no response from other mathematicians.

After his retirement from the army, János lived with his father, who was then a widower. This arrangement lasted only a short time, however. Tension grew between father and son, who were both disappointed at the poor reception given their work, and János withdrew to the small family estate at Domáld, visiting Marosvásárhely only occasionally. In 1834 he contracted an irregular marriage with Rosalie von Orban. The couple had three children, the first born in 1837.

In an attempt to reestablish themselves in mathematics, both father and son participated in the Jablonow Society prize contest in 1837. The subject was the rigorous geometric construction of imaginary quantities, at that time a subject to which many mathematicians (for example, Augustin Catchy, W.R. Hamilton, and Gauss) were paying attention. The Bolyais’ solutions were too involved to gain a prize, but János’ solution resembled that of Hamilton, which was published about the same time, although in simpler terms, and published about the same time, although in simpler terms, and which considered complex numbers as ordered Pairs of real numbers. Again the Bolyais had failed to obtain due recognition. János continued to do mathematical work, however, some of it strong and some, because of his isolation, very weak. His best work was that on his absolute geometry and spherical trigonometry and on the volume of the tetrahedron in absolute space. On the last subject, there are notes written as late as 1856. Nikolai Lobachevski’s *Geometrische Untersuchungen zur Theorie der parallinein* (1840), which reached him through his father in 1848, worked as a powerful challenge, for it established independently the same type of geometry that he had discovered. In his later days he occasionally worried about the possibility of contradictions in his absolute geometry— a real difficulty that was not overcome until Beltrami did so later in the nineteenth century. János also worked on a salvation theory, which stressed that no individual happiness can exist without a universal happiness and that no virtue is possible without knowledge.

János’ father died in 1856 and his relationship with Rosalie ended at about the same time, thus depriving him of two of his few intimate contacts. However, in the four years left to him, he did have his good moments. He could write enthusiastically about the ballet performances of the Vienna Opera and compose some beautiful lines to the memory of his mother. He died after a protracted illness, and was buried in the Evangelical-Reformed Cemetery in Marosvásárhely.

The “Appendix” was practically forgotten until Richard Baltzer discussed the work of Bolyai and Lobachevski in the second edition of his *Elemente der Mathematik* (1867). Jules Houel, a correspondent of Baltzer’s then translated Lobachevski’s book into French (1867) and did the same with Bolyai’ Appendix” (1868). Full recognition came with the work of Eugenio Beltrami (1868)and Felix Klein (1871).

BIBLIOGRAPHY

In addition to the works cited in the article on Farkas Bolyai, see the English translation of the “Appendix” with an introduction by G. B. Halstad (Austin, Texas, 1891; new ed., Chicago-London, 1914), reprinted in R. Bono la *Non-Euclidean Geometry* (reprinted New York 1995). There are accounts of Bolyai’ geometry in the many books on non-Euclidean geometry. See D. M. Y. Sommerville, *Bibliography of Non-Euclidean Geometry* (London, 1911). Further material may be found in I. Tóth, *Bolyai János élete és mive* (“Life and Work of Johann Bolyai” Bucharest 1953); *János Bolyai Appendix* (Bucharest 1954), in Rumanian.

E. Sarlóska “János Bolyai the Soldier” in *Magyar tudományos akadémia Matematikai eś fizikai osztályanak közlemenyei*, **15** (1965), 341–387, contains a documentary study of Bolyai’ army life.

D. J. Struik