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mathematics. For the original article on Bourbaki see *DSB*, vol. 2.

Bourbaki was a pseudonym adopted in 1934 by a group of young French mathematicians for their joint activities, which led to the publication of a highly influential collection of books on several fields of mathematics, including analysis, algebra, and topology, among others. The group was active for several decades thereafter (exactly how long is unknown) and shaped mathematical activity in many countries across the world, especially in France, through research, teaching, publishing, career building, and resource allocation. The name of [Nicolas Bourbaki](#) and the kind of associations it typically raises in mathematicians' minds play a unique role in the history of twentieth-century mathematics. The very inclusion of an article about a group of mathematicians under the name of an individual in the first edition of the *Dictionary of Scientific Biography*, and the fact that the article spoke about Bourbaki mostly in the third-person singular (following a practice well-entrenched among mathematicians), indicate the force of the myth surrounding the group's activities; at the same time, this history illuminates the difficulties that have been faced in attempts to produce serious historical accounts of the group's actual contribution to mathematics.

Early Years. The would-be members of Bourbaki met for the first time to discuss the project at the end of 1934 in a Parisian café. They stated as the goal of their joint undertaking "to define for twenty-five years the syllabus for the certificate in differential and [integral calculus](#) by writing, collectively, a treatise on analysis. Of course, this treatise will be as modern as possible" (Beaulieu, 1993, p. 28). They were motivated by an increasing dissatisfaction with the texts then traditionally used in their country for analysis courses. These were based on the university lectures of the old French masters: Jacques Hadamard, Emile Picard, and Edouard Goursat. They also felt that French mathematical research was lagging far behind that of other countries, especially Germany, and they sought to provide a fresh perspective from which to reinvigorate local mathematical activity.

The founding members of the group included Henri Cartan, Claude Chevalley, Jean Coulomb, Jean Delsarte, Jean Dieudonné, Charles Ehresmann, Szolem Mandelbrojt, René de Possel, and André Weil. Cartan, Chevalley, Delsarte, Dieudonné, and Weil, all former students of the École Normale Supérieure, remained the most influential and active force within the group for decades. Jean Leray and Paul Dubreil had attended the Paris meeting but did not join the group. Over the years, many younger, prominent mathematicians joined the group, while the elder members were supposed to quit at the age of fifty. Among later-generation Bourbaki members, the most prominent include: Samuel Eilenberg (one of the few who were not French), Alexander Grothendieck, Pierre Samuel, Laurent Schwartz, Jean-Pierre Serre, Serge Lang, and Armand Borel. All members were among the most prominent mathematicians of their generation, actively pursuing separately their own individual research in different specialties, while the activities of Bourbaki absorbed a part of their time and effort.

Beginning in 1935, and except for a break during the war years, the group met three times per year in different places around France for one or two weeks. At each meeting, individual members were commissioned to produce drafts of the different chapters. The drafts were then subjected to harsh criticism by the other members and then were reassigned for revision. Only after several drafts had been written and criticized was the final document ready for publication. What was initially projected as a modern textbook for a course of analysis eventually evolved into a multivolume treatise entitled *Éléments de mathématique*, each volume of which was meant to contain a comprehensive exposition of a different mathematical subdiscipline. Every chapter and every volume of Bourbaki's treatise was the outcome of arduous collective work, and the spirit and point of view of the person or persons who had written it were hardly recognizable.

The first chapters of Bourbaki's book on topology, for instance, were published in 1940, following almost four years of the usual procedure of drafting and criticism. The book was meant to provide the conceptual basis needed for discussing convergence and continuity in real and complex analysis. Bourbaki's early debates on topology were gradually dominated by a tendency to define this conceptual basis in the most general framework possible, avoiding whenever possible the need to rely on traditional, intuitive concepts such as sequences and their limits. This effort helped the understanding of, among other concepts, the centrality of compactness in general topology. It also yielded a thorough analysis of the various alternative ways to define general topological spaces and their central characteristic concepts: open and closed sets, neighborhoods, and uniform spaces. Moreover, an important by-product of Bourbaki's discussions was the introduction of filters and ultrafilters as a basis for defining convergence while avoiding reliance on countable sequences. Bourbaki, however, rather than including these latter concepts in the treatise, encouraged Henri Cartan to publish them, while elaborating on their relation to topological concepts, under his own name.

Over the next years, alternative approaches to questions of continuity and convergence were developed by other mathematicians, based on concepts such as directed systems and nets. The equivalence of the various alternative systems and those of Bourbaki was proven in 1955. Thus, the history of the development of topology, at least from 1935 to 1955, cannot be told without considering in detail the role played in it by both Bourbaki as a group and its individual members.

Bourbaki's Influence. In the decades following the founding of the group, Bourbaki's books became classics in many areas of pure mathematics in which the concepts and main problems, the nomenclature, and the peculiar style introduced by Bourbaki were adopted as standard. The branches upon which Bourbaki exerted the deepest influence were algebra, topology, and functional analysis. Notations such as the symbol \emptyset (for the empty set, and terms such as *injective*, *surjective*, and *bijective* owe their widespread use to their adoption in the *Eléments de mathématique*.

Disciplines such as logic, probability, and most fields of applied mathematics were not within the scope of interests of Bourbaki, and they were therefore hardly represented in the many places in the world where Bourbaki's influence was more strongly felt. This was the case for many French and several American universities at various times between 1940 and 1970. In addition, disciplines such as group theory and [number theory](#), in spite of being strong points for some of the members (notably Weil for [number theory](#)) would not be treated in the *Eléments*, mainly because their character as mathematical disciplines was less amenable to the kind of systematic, comprehensive treatment typical of the other disciplines in the collection. As part of a basic estrangement of pictorial or intuitive elements in mathematics, geometry was completely left out of the Bourbakian picture of mathematics, except for what could be reduced to linear algebra.

Bourbaki's *Eléments* came to comprise a large collection of more than seven thousand pages. The first chapter appeared in 1935, and new ones continued to appear until the early 1980s. In its final form it comprised the following: I. Theory of Sets; II. Algebra; III. General Topology; IV. Functions of a Real Variable; V. Topological Vector Spaces; VI. Integration; Lie Groups and Lie Algebras; Commutative Algebra; Spectral Theories; Differential and Analytic Manifolds (which is essentially no more than a summary of results).

Bourbaki's austere and idiosyncratic presentation of the topics discussed in each of the chapters—from which diagrams and external motivations were expressly excluded—became a hallmark of the group's style and a manifestation of its thorough influence. Also the widespread adoption of approaches to specific questions, concepts, and nomenclature promoted in the books of the series indicate the breadth of this influence. Concepts and theories were presented in a thoroughly axiomatic way and were systematically discussed, always proceeding from the more general to the particular and never generalizing a particular result. A curious consequence of this approach was that the real numbers could only be introduced well into the treatise and not before a very heavy machinery of algebra and topology had been prepared in advance.

The Hierarchy of Structures. Underlying all of Bourbaki's presentation of mathematics is the conception of the discipline as a hierarchy of structures. In the more reduced framework of algebra alone, this idea had reached maturity in 1930 as it appeared in the famous book by Bartel L. van der Waerden, *Moderne Algebra*. The discipline appeared in this book as the investigation, from a unified point of view, of several concepts that were defined in similar, abstract terms: groups, rings, ideals, modules, fields, and hypercomplex systems. All of them comprised a set on which one or more operations were defined that satisfied certain properties defined in advance and prescribed in the form of abstract axioms. The kinds of questions asked about each of them were similar and so were the tools used to investigate them. Each of them was an individual manifestation of a more general idea, that of an algebraic structure.

In Bourbaki's presentation of mathematics, different mathematical branches, such as algebra, topology, and functional analysis, appeared as individual materializations of one and the same underlying, general idea, that is, the idea of a mathematical structure. Bourbaki attempted to present a unified and comprehensive picture of what they saw as the main core of mathematics, using a standard system of notation, addressing similar questions in the various fields investigated, and using similar conceptual tools and methods across apparently distant mathematical domains.

In 1950, Dieudonné published, under the name of Bourbaki, an article that came to be identified as the group's manifesto, "The Architecture of Mathematics." Dieudonné raised the question of the unity of mathematics, given the unprecedented growth and diversification of knowledge in this discipline over the preceding decades. Mathematics is a strongly unified branch of knowledge in spite of appearances, he claimed, and the basis of this unity is the use of the axiomatic method. Mathematics should be seen, he added, as a hierarchy of structures at the heart of which lie the so-called "mother structures":

At the center of our universe are found the great types of structures ... they might be called the mother structures. ... Beyond this first nucleus, appear the structures which might be called multiple structures. They involve two or more of the great mother-structures not in simple juxtaposition (which would not produce anything new) but combined organically by one or more axioms which set up a connection between them. ... Farther along we come finally to the theories properly called particular. In these the elements of the sets under consideration, which in the general structures have remained entirely indeterminate, obtain a more definitely characterized individuality. (Bourbaki 1948 [1950], pp. 228–229)

While van der Waerden had left the idea at the implicit level, the centrality of the hierarchy of structures became explicit and constitutive for Bourbaki. Moreover, Bourbaki wanted, in addition, a formally defined concept of *structure* that would provide a conceptual foundation on which the whole edifice of mathematics as presented in the *Eléments* could supposedly be built. This concept was introduced in the fourth chapter of the book on set theory. In the opening chapters of the books on specific

branches, for example, algebra and topology, some sections were devoted to show how the specific branch can be formally connected with the concept of *structure* that had been defined. This connection, however, was rather feeble and amounted to not much more than a formal exercise.

The central notion of structure, then, had a double meaning in Bourbaki's mathematical discourse. On the one hand, it suggested a general organizational scheme of the entire discipline, which turned out to be very influential. On the other hand, it comprised a formal concept that was meant to provide the underlying formal unity but was of no mathematical value whatsoever either within Bourbaki's own treatise or outside it.

In 1945, Saunders Mac Lane and Samuel Eilenberg introduced the concepts of category and functor that were to become central as a unifying tool and language for mathematical disciplines that followed the structural approach developed in the various books of Bourbaki. Grothendieck and Serre were among the mathematicians who made a most impressive use of this new theory in their own research starting in the early 1950s. Grothendieck attempted to introduce these concepts, together with some topics that were yet to be treated, into the Bourbakian agenda, but he succeeded only partially. A curious situation ensued in which a theory that could have fitted nicely into the overall picture of mathematics promoted by Bourbaki was not adopted, partly because that would have meant reformulating considerable parts of the already existing texts in order to make them fit the new approach. In Bourbaki's book on homological algebra, published in 1980, this tension reached a noticeable peak. While the categorical approach has become widespread and indeed the standard one in this mathematical discipline, Bourbaki's presentation could not rely on it, because category theory had not been developed in the treatise. Using it here would go against the most basic architectonic principles that had guided the enterprise since its inception. Thus, while Bourbaki's treatment of a field such as general topology had embodied in the 1940s a truly innovative approach that many others were to follow, this would hardly be the case with homological algebra in the 1980s.

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