Bowen, Rufus (Robert) | Encyclopedia.com

(*b*. Vallejo. California, 23 February 1947; d.Santa Rosa. California, 30 July 1978)

mathematics.

The son of Marie De Winter Bowen, a school-teacher, and Emery Bowen, a budget officer at Travis Air Base in California, Robert Bowen attended public schools in the small city of Fairfield. He published his first mathematical paper at the age of seventeen, and four more before he was twenty-one. The University of California at Berkeley awarded him a bachelor's degree, with prizes for scholarship, in 1967, and the doctorate in 1970. He was appointed assistant professor of mathematics at Berkeley in 1970 and was promoted to professor in 1977. On 6 March 1968 Bowen married Carol Twito of Hayward, California; they had no children. Deeply concerned with social problems, as were many of his generation, he was active in organizations devoted to preventing nuclear war, Bowen belonged to Phi Beta Kappa and the American Mathematical Society. In 1970 he changed his first name to Rufus. He died of a cerebral aneurysm.

The subject of Bowen's doctoral dissertation, and of all his later work, was dynamical systems theory. Originated by Henri Poincaré in the 1880's, this field was developed intensively in the 1960's, largely under the leadership of Bowen's dissertation supervisor, Stephen Smale, The latters seminal paper. "Differentiable Dynamical Systems" (*Bulletin of the American Mathematical Society*, **73** [1967], 747–817), was much admired by Bowen and strongly influenced the direction of his work.

As studied by Smale, a dynamical system comprises a manifold M and a smooth mapping $f{:}M \rightarrow M$ (usually a diffeomorphism); the goal is to describe the limiting behavior of the trajectories $f^n(x)$ of points $x \in$ M as n goes to infinity. As Poincaré emphasized, there is no general procedure for this, and therefore one must resort to describing average, typical, or most probable behavior. Bowen's work is an important part of the program of expressing these vague ideas in mathematically precise and useful ways.

Smale singled out what he called "axiom A systems" as being simple enough to study and complex enough to include many interesting examples. His "spectral decomposition theorem" states that there is a finite number of indecomposable subsystems, called "basic sets," to which all trajectories tend, and that the dynamics of the basic sets is not too wild. Most of Bowen's work is concerned with the dynamics in a basic set of an axiom A system.

Bowen's early papers give useful estimates for the topological entropy $h(f)$ of a dynamical system. This topological invariant, which had recently been discovered by others, measures the complexity of the system. For axiom A systems Bowen proved that

where $N_n(f)$ is the number of fixed points of f^n. Other results give lower bounds for $h(f)$ in terms of the automorphisms induced by f on the fundamental and homology groups of M.

Most of Bowen's subsequent work concerns invariant measures associated to dynamical systems. First used by Poincaré and J. Willard Gibbs, these measures are intimately related to statistical mechanics and other branches of mathematical physics. A major achievement was Bowen's construction of an invariant. ergodic probability measure μ_x for any basic set X, for which periodic points are uniformly distributed.

In this work Bowen developed new methods in the symbolic dynamics pioneered by Jacques Hadamard and Marston Morse. He constructed a certain covering of X by closed sets $R_1 \ldots R_n$ called a Markov partition. To any x in M one associates a doubly infinite sequence $y=(y_1)$ such that y {1, ...n} and $f^1 x \in R_j$ if $y_i = j$. The Markov partition has the property that each x corresponds to at most n2 sequences, and most x to exactly one sequence. The space σ of all such sequences is readily described: it carries the "shift" homeomorphism Ty = z defined by $z_i = y_i + 1$. There is a continuous map P from Σ onto X such that $P(T_y) = f(P_y)$. This means that T and f have practically the same dynamics. Bowen was able to infer much about f from the easily analyzed dynamics of T. In particular he showed that (f, X), considered as an automorphism of the measure space (X, μ_x), is a Markov chain. This means that most points of X have trajectories that are randomly distributed over X.

Bowen then turned to analogous but much more subtle problems for continuous-time systems, called flows: here one has a one-parameter group of maps indexed by the real numbers. An important technical achievement was the description of suitable analogues, for flows, of symbolic dynamics. An important theorem (proved with David Ruelle) states that every attracting basic set Z of a twice-differentiable, axiom A flow on M carries an invariant probability measure μ_0 with the following property: For any continuous function g on M and almost every point x in a neighborhood of Z (in the sense of Lebesgue measure), the time average of *e* over the forward orbit of *x* equals the μ_0 average *g* over *Z*. This measure is now called the Bowen-Ruelle measure. Bowen applied his results to the geodesic How on the unit tangent bundle of a compact manifold of constant negative curvature, showing that periodic geodesies are equidistributed in the Riemannian measure as the periods tend to infinity.

In his last, posthumous paper, Bowen applied his measure-theoretic methods in a novel way to classical problems in the geometry of Fuchsian groups. Besides its considerable intrinsic interest, this paperdemonstrates that Bowern's methods have application far beyond the axiom A systems for which they were invented. The Bowen-Ruelle measure, for example, has already proved significant in other dynamical settings. His papers, models of clarity, simplicity, and originality, have a permanent importance in dynamical systems theory.

BIBLIOGRAPHY

The best sources for an overview of much of Bowen's work are the following surveys written by him: "Symbolic Dynamics for Hyperbolic Flows," in *Proceedings of theInternational Congress of Mathematicians* (Vancouver. 1974), 299–302; *Equilibrium States and the Ergodic Theory of Anosov Diffeomorphisms*. Lecture Notes in Mathematics no. 470 (New York. 1975); and *On Axiom A Diffeomorphisms* (Providence. R.I., 1978). His work with David Ruelie is "The Ergodie Theory of Axiom A Flows," in *Inventiones Mathematicae* **29** (1975), 181–202. Mis last paper is "Hausdorff Dimension of Quasi-circles." in *Publications mathematiques de l'Institut des hautes études scientifiques*. no. 50 (1978), 259–273. Papers covering other aspects of his work are "Emtropy for Maps of the Interval," *in Topology*, **16** (1977), 465–467; and "A Model for Couette Flow-data." in P. Bernard and T. Ratin, eds., *Turbulence Seminar, Proceedings 1976/77* (New York, 1977), 117–133.

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