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(*b*. Overschie, Netherlands, 27 February 1881; *d*. Blaricum, Netherlands, 2 December 1966)

mathematics.

Brouwer first showed his unusual intellectual abilities by finishing high school in the North Holland town of Hoorn at the age of fourteen. In the next two years he mastered the Greek and Latin required for admission to the university, and passed the entrance examination at the municipal Gymnasium in Haarlem, where the family had moved in the meantime. In the same year, 1897, he entered the University of Amsterdam, where he studied mathematics until 1904. He quickly mastered the current mathematics, and, to the admiration of his professor, D. J. Korteweg, he obtained some results on continuous motions in four-dimensional space that were published in the reports of the Royal Academy of Science in Amsterdam in 1904. Through his own reading, as well as through the stimulating lectures of Gerrit Mannoury, he became acquainted with topology and the foundations of mathematics. His great interest in philosophy, especially in mysticism, led him to develop a personal view of human activity and society that he expounded in *Leven, Kunst, en Mystiek* ("Life, Art, and Mysticism"; 1905), where he considers as one of the important moving principles in human activity the transition from goal to means, which after some repetitions may result in activities opposed to the original goal.

Brouwer reacted vigorously to the debate between Russell and Poincaré on the logical foundations of mathematics. These reactions were expressed in his doctoral thesis, *Over de Grondslagen der Wiskunde* ("On the Foundations of Mathematics"; 1907). In general he sided with Poincaré in his opposition to Russell's and Hilbert's ideas about the foundations of mathematics. He strongly disagreed with Poincaré, however, in his opinion on mathematical existence. To Brouwer, mathematical existence did not mean freedom from contradiction, as Poincaré maintained, but intuitive constructibility.

Brouwer conceived of mathematics as a free activity of the mind constructing mathematical objects, starting from self-evident primitive notions (primordial intuition). Formal logic had its *raison d'être* as a means of describing regularities in the systems thus constructed. It had no value whatsoever for the foundation of mathematics, and the postulation of absolute validity of logical principles was questionable. This held in particular for the principle of the excluded third, briefly expressed by $A \vee \neg A$—that is, A or not A—which he identified with Hilbert's statement of the solvability of every mathematical problem. The axiomatic foundation of mathematics, whether or not supplemented by a consistency proof as envisaged by Hilbert, was mercilessly rejected; and he argued that Hilbert would not be able to prove the consistency of arithmetic while keeping to his finitary program. But even if Hilbert succeeded, Brouwer continued, this would not ensure the existence (in Brouwer's sense) of a mathematical system described by the axioms.

In 1908 Brouwer returned to the question in *Over de Onbetrouwbaarheid der logische Principes* ("On the Untrustworthiness of the Logical Principles") and—probably under the influence of Mannoury's review of his thesis—rejected the principle of the excluded third, even for his constructive conception of mathematics (afterward called intuitionistic mathematics).

Brouwer's mathematical activity was influenced by Hilbert's address on mathematical problems at the Second International Congress of Mathematicians in Paris (1900) and by Schoenflies' report on the development of set theory. From 1907 to 1912 Brouwer engaged in a great deal of research, much of it yielding fundamental results. In 1907 he attacked Hilbert's formidable fifth problem, to treat the theory of continuous groups independently of assumptions on differentiability, but with fragmentary results. Definitive results for compact groups were obtained much later by John von Neumann in 1934 and for locally compact groups in 1952 by A. M. Gleason and D. Montgomery and L. Zippin.

In connection with this problem—a natural consequence of Klein's Erlanger program—Brouwer discovered the plane translation theorem, which gives a homotopic characterization of the topological mappings of the Cartesian plane, and his first fixed point theorem, which states that any orientation preserving one-to-one continuous (topological) mapping of the two-dimensional sphere into itself leaves invariant at least one point (fixed point). He generalized this theorem to spheres of higher dimension. In particular, the theorem that any continuous mapping of the n-dimensional ball into itself has a fixed point, generalized by J. Schauder in 1930 to continuous operators on Banach spaces, has proved to be of great importance in numerical mathematics.

The existence of one-to-one correspondences between numerical spaces R_n for different n, shown by Cantor, together with Peano's subsequent example (1890) of a continuous mapping of the unit segment onto the square, had induced mathematicians to conjecture that topological mappings of numerical spaces R_n would preserve the number n (dimension). In 1910 Brouwer proved this conjecture for arbitrary n.

His method of simplicial approximation of continuous mappings (that is, approximation by piecewise linear mappings) and the notion of degree of a mapping, a number depending on the equivalence class of continuous deformations of a topological mapping (homotopy class), proved to be powerful enough to solve the most important invariance problems, such as that of the notion of n-dimensional domain (solved by Brouwer) and that of the invariance of Betti numbers (solved by J. W. Alexander).

Finally, mention may be made of his discovery of indecomposable continua in the plane (1910) as common boundary of denumerably many, simply connected domains; of his proof of the generalization to n–dimensional space of the Jordan curve theorem (1912); and of his definition of dimension of topological spaces (1913).

In 1912 Brouwer was appointed a professor of mathematics at the University of Amsterdam, and in the same year he was elected a member of the Royal Netherlands Academy of Science. His inaugural address was not on topology, as one might have expected, but on intuitionism and formalism.

He again took up the question of the foundations of mathematics. There was no progress, however, in the reconstruction of mathematics according to intuitionistic principles, the stumbling block apparently being a satisfactory notion of the constructive continuum. The first appearance of such a notion was in his review (1914) of the Schoenflies-Hahn report on the development of set theory. In the following years he scrutinized the problem of a constructive foundation of set theory and came fully to realize the role of the principle of the excluded third. In 1918 he published a set theory independent of this logical principle; it was followed in 1919 by a constructive theory of measure and in 1923 by a theory of functions. The difficulty involved in a constructive theory of sets is that in contrast with axiomatic set theory, the notion of set cannot be taken as primitive, but must be explained. In Brouwer's theory this is accomplished by the introduction of the notion of freechoice sequence, that is, an infinitely proceeding sequence of choices from a set of objects (e.g., natural numbers) for which the set of all possible choices is specified by a law. Moreover, after every choice, restrictions may be added for future possible choices. The specifying law is called a spread, and the everunfinished free-choice sequences it allows are called its elements. The spread is called finitary if it allows only choices from a finite number of possibilities. In particular, the intuitionistic continuum can be looked upon as given by a finitary spread. By interpreting the statement "All elements of a spread have property p" to mean "I have a construction that enables me to decide, after a finite number of choices of the choice sequence α, that it has property p," and by reflection on the nature of such a construction, Brouwer derived his so-called fundamental theorem on finitary spreads (the fan theorem). This theorem asserts that if an integer-valued function, f, has been defined on a finitary spread, S, then a natural number, n, can be computed such that, for any two free-choice sequences, α and β, of S that coincide in their first n choices, we have $f(\alpha) = f(\beta)$.

This theorem, whose proof is still not quite accepted, enabled Brouwer to derive results that diverge strongly from what is known from ordinary mathematics, e.g., the indecomposability of the intuitionistic continuum and the uniform continuity of real functions defined on it.

From 1923 on, Brouwer repeatedly elucidated the role of the principle of the excluded third in mathematics and tried to convince mathematicians that it must be rejected as a valid means of proof. In this connection, that the principle is noncontradictory, that is, that $\neg\neg(A \vee \neg A)$ holds, is a serious disadvantage. Using the fan theorem, however, he succeeded in showing that what he called the general principle of the excluded third is contradictory, that is, there are properties for which it is contradictory that for all elements of a finitary spread, the property either holds or does not hold—briefly, $\neg(\forall\, \alpha)\ (P\ (\alpha \vee \neg P\ (\alpha))$ holds.

In the late 1920's the attention of logicians was drawn to Brouwer's logic, and its relation to classical logic was investigated. The breakdown of Hilbert's foundational program through the decisive work of Kurt Gödel and the rise of the theory of recursive functions has ultimately led to a revival of the study of intuitionistic foundations of mathematics, mainly through the pioneering work of S. C. Kleene after World War II. It centers on a formal description of intuitionistic analysis, a major problem in today's foundational research.

Although Brouwer did not succeed in converting mathematicians, his work received international recognition. He held honorary degrees from various universities, including Oslo (1929) and Cambridge (1955). He was elected to membership in many scientific societies, such as the German Academy of Science, Berlin (1919); the American Philosophical Society, Philadelphia (1943); and the Royal Society of London (1948).

BIBLIOGRAPHY

I. Original Works. “Over een splitsing van de continue beweging om een vast punt 0 van R_4 in twee continue bewegingen om 0 van R_3’s,” in *Verslagen. Koninklijke akademie van wetenschappen te Amsterdam*, **12** (1904), 819–839; *Leven, Kunst en Mystiek* (Delft, 1905); *Over de Grondslagen der Wiskunde* (Amsterdam, 1907); “Over de onbetrouwbaarheid der logische principes,” in *Tijdschrift voor wijsbegeerte*, **2** (1908), 152–158; “Die Theorie der endlichen Kontinuierlichen Gruppen, unabhängig von den Axiomen von Lie (erste Mitteilung),” in *Mathematische Annalen*, **67** (1909), 246–267, and “… (zweite Mitteilung),” *ibid*., **69** (1910), 181–203; “Zur Analysis Situs,” *ibid*., **68** (1910), 422–434; “Beweis des Jordanschen Kurvensatzes,” *ibid*., **69** (1910), 169–175; “Beweis des Jordanschen Satzes für den n-dimensionalen Raum,” *ibid*., **71** (1912), 314–319; “Über eineindeutige, stetige Transformationen von Flächen in sich,” *ibid*., **69** (1910), 176–180; “Beweis der Invarianz der Dimensionenzahl,” *ibid*., **70** (1911), 161–165; “Über Abbildung von Mannigfaltigkeiten,” *ibid*., **71** (1912), 97–115, 598; “Beweis der Invarianz des n-dimensionalen Gebietes,” *ibid*., **71** (1912), 305–313; “Zur Invarianz des n-dimensionalen Gebiets,” *ibid*., **72** (1912), 55–56; “Beweis des ebenen Translationssatzes,” *ibid*., **72** (1912), 37–54; “Beweis der Invarianz der geschlossene Kurve,” *ibid*., **72** (1912), 422–425; “Überdennatürlichen Dimensionsbegriff,” in *Journal für die reine und angewandte Mathematik*, **142** (1913), 146–152; “Intuitionism and Formalism,” *in Bulletin of the American Mathematical Society*, **20** (1913), 81–96; review of A. Schoenflies and H. Hahn, *Die Entwicklung der Mengenlehre und ihrer Anwendungen. Erste Hälfte. Allgemeine Theorie der unendlichen Mengen und Theorie der Punktmengen* (Leipzig—Berlin, 1913), in *Jahresbericht der Deutschen Mathematikervereinigung*, **23** (1914), 78–83; “Begründung der Mengenlehre unabhängig vom logischen Satz vom ausgeschlossenen Dritten. Erster Teil: Allgemeine Mengenlehre,” in *Verhandelingen Koninklijke akademie van wetenschappen te Amsterdam*, **12** , no. 5 (1918), 1–43, and “… II. Theorie der Punktmengen,” *ibid*., **12** , no. 7 (1919), 1–33; “Begründung der Funktionenlehre unabhägig vom logischen Satz vom ausgeschlossenen Dritten. Erster Teil: Stetigkeit, Messbarkeit, Derivierbarkeit,” *ibid*., **13** , no. 2 (1923), 1–24; “Intuitionistische Einführung des Dimensionsbegriffes,” in *Proceedings. Koninklijke akademie van wetenschappen te Amsterdam*, **29** (1926), 855–863; “Über Definitionsbereiche von Funktionen,” in *Mathematische Annalen*, **97** (1927), 60–75; “Essentially Negative Properties.” in *Proceedings. Koninklijke akademie van wetenschappen te Amsterdam*, **51** (1948), 963–964; “Consciousness, Philosophy and Mathematics,” in *Proceedings of the Tenth International Congress of Philosophy*, I (Amsterdam, 1949), 1235–1249. For Brouwer’s topological work, consult the book by Alexandroff and Hopf listed below. Extensive bibliographies of his foundational work may be found in the books by Heyting and Van Heijenoort (see below). A complete edition of Brouwer’s work is planned by the Dutch Mathematical Society.

II. Secondary Literature. Brouwer or his work is discussed in P. Alexandroff and H. Hopf, *Topologie* (Berlin, 1935), *passim;* P. Benacerraf and H. Putnam, *Philosophy of Mathematics* (Englewood Cliffs, N.J., 1964), pp. 66–84; J. van Heijenoort, *From Frege to Gödel, a Source Book in Mathematical Logic*, 1879–1931 (Cambridge, Mass., 1967), pp. 334–345, 446–463, 490–492; A. Heyting, *Intuitionism, An Introduction* (Amsterdam, 1965), *Passim*; S. Lefschetz, *Introduction to Topology* (Princeton, N.J., 1949), pp. 1–26, 117–131; S. C. Kleene and R. E. Vesley, *The Foundations of Intuitionistic Mathematics* (Amsterdam, 1965); G. Kreisel, “Functions, Ordinals, Species,” in *Logic, Methodology and Philosophy of Science*, III, ed. B. van Rootselaar and J. F. Staal (Amsterdam, 1968), pp. 145–159; J. Myhill, “Formal Systems of Intuitionistic Analysis I,” *ibid*., pp. 161–178; A. S. Troelstra, “The Theory of Choice Sequences,” *ibid*., pp. 201–223.

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