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(worked Paris, France, *ca*. 1800-at least 1853)

mathematics.

Almost nothing is known of Budan's life except for the information he provided on the title pages of his published works. He was a doctor of medicine and an amateur mathematician. He was educated in the classics and occasionally quoted Virgil and Horace in his works. A royalist, he published a Latin ode on the birth of the posthumous son of the duke of Burgundy. Budan was named *chevalier* of the Legion of Honor in 1814. He held the post of inspector general of studies at the University of Paris for over twenty years; this post may have been responsible for his interest in finding mathematical methods that would be easy for beginning students to use.

Budan is known in the theory of equations as one of the independent discoverers of the rule of Budan and Fourier, which gives necessary conditions for a polynomial equation to have n real roots between two given real numbers. He announced his discovery of the rule and described its use in a paper read to the Institut de France in 1803 and published the paper, with explanatory notes, as *Nouvelle mériques d'un degré quelconque*, in 1807.

Budan's definitive formulation of his rule was the following: "If an equation in x has n roots between zero and some positive number p, the transformed equation in $(x - p)$ must have at least n fewer variations [in sign] than the original" ("Appendice", p.89). The "transformed equation in $(x - p)$" is the original polynomial equation developed in powers of $(x - p)$. In modern notation: let $P(x) = 0$ be the given polynomial equation, and let $G(x - p) = P(x)$. Then $G(x - p) = 0$ is what Budan called the "transformed equation in $(x - p)$." The term "variation in sign" is borrowed from Descartes's rule of signs: No polynomial can have more positive roots than there are variations in sign in the successive terms of that polynomial. Indeed, Budan appears to have been led to his rule by Descartes's rule of signs.

Budan's first formulation of his rule assumed that all the roots of the original equation were real. In this case, Budan's rule tells exactly how many roots there are between zero and p, just as Descartes's rule gives the exact number of positive roots in the same case. Budan stated that, for the case of all real roots, his rule could be derived from Descartes's rule. It is not difficult to reconstruct such a derivation, even though Budan did not give it, once one has observed that when x is between zero and p, $(x - p)$ is negative.

The need for a rule such as his was suggested to Budan by Lagrange's *Traité de la résolution des équations numériques* (1767). This seems to have been almost the only nonelementary work Budan had read, and it influenced him greatly. He quoted Lagrange to show that it would be useful to give the rules for solving numerical equations entirely by means of arithmetic, referring to algebra only if absolutely necessary. Budan's goal was to solve Lagrange's problem—between which real numbers do real roots lie?—purely by methods of elementary arithmetic. Accordingly, the chief concern of Budan's *Nouvelle méthode* was to give the reader a mechanical process for calculating the coefficients of the transformed equation in $(x - p)$. He did not appeal to the theory of finite differences or to the calculus for these coefficients, preferring to give them "by means of simple additions and subtractions".1. His *Nouvelle méthode* includes many specific numerical examples in which the cofficients are calculated and the number of sign changes in the polynomials P and G are compared; he intended this to be a simple and practical procedure.

In 1811 Budan presented a proof for his rule to the Institute; he published the proof, along with a reprint of his original article, as *Appendice à la nouvelle méthode*, in 1822. A. M. Legendre, reporting to the Institute in 1811 on Budan's rule and its proof, recognized the utility of being able to know that there could be no real roots between two given real numbers. Apparently unaware of the prior work of Joseph Fourier, he stated that the result was new. 2. Legendre added that the proof given by Budan was valid only after certain gaps were filled, notably the assumption without proof of Segner's theorem (1756): If $P(x)$ is multiplied by $(x - a)$, the number of variations in sign in the product polynomial is at least one greater than that in $P(x)$. Budan himself did not appreciate the force of this objection; he protested that there was nothing wrong with using a known result, although in fact he assumed it without stating it and, until Legendre's remark, did not seem to realize that the proof needed it.

Budan’s success in discovering a correct rule and giving a reasonably satisfactory proof of it shows that, at the beginning of the nineteenth century, it was still possible for one without systematic training in mathematics to contribute to its progress; but mathematics was giving increasing attention to rigor and precision of statement, qualities slighted in Budan’s work. The professionals were about to take over. Fourier’s simultaneous and independent discovery, using derivatives, exemplifies the powerful methods available to one thoroughly schooled in mathematics. J. C. F. Sturm (1836) gave a necessary *and sufficient* condition for a root to lie between two bounds, thus completely solving the theoretical problem of how many roots lie between given limits. Yet Budan’s rule remains the most convenient for computation, although finding bounds on roots is no longer the major business of the algebraist.

NOTES

1. This method is fairly efficient. It is equivalent to the use of successive synthetic divisions, a method often discussed in works on the theory of equations. See, e.g., W. S. Burnside and A. W. Panton, *The Theory of Equations* (Dublin-London, 1928), I, 10 ff., 64 ff.

2. Fourier taught his version of the rule before 1797, although it was not published until after his death, in 1831. Fourier’s version is: If $f(x)$ is a polynomial of degree n, the number of real roots of $f(x) = 0$ lying between a and b cannot exceed the difference in the number of changes in sign in the sequence $f(b), f'(b), f''(b),\ldots, f^n(b)$ and that of the sequence $f(a), f'(a), f''(a),\ldots, f^n(a)$, See J. Fourier, *Analyse des équations déterminées*, pp. 98–100; on his priority, see the “Avertissement” to that work by C. L. M. H. Navier, p. xxi. Although the formulations of Budan and Fourier are equivalent, the great difference in conception argues for independence of discovery.

BIBLIOGRAPHY

I. Original Works. Budan’s writings are Nouvelle méthode pour la résolution des équation numériques d’un degré quelconque (Paris, 1807), and “Appendice á la nouvelle méthode”, in Nouvelle méthode pur la résolution des équations numériques d’un degré quelconque, revue, augmentée d’un appendice, et suivie d’un apperçu concernant les suites syntagmatiques (Paris, 1822).

II. Secondary Literature. Additional information may be found in J. Fourier, *Analyse des équations déterminées* (Paris, 1830[*sic*]), which includes C. L. M. H. Navier, “Avertissement de I’éditeru”, pp. i-xxiv, dated 1 July 1831; and F.N.W. Moigno, “Note sur la détermination du nombre des racines réelles ou imaginaires d’une équation numérique, comprises entre des limites données. Theorèmes de Rolle, de Budan ou de Fourier, de Descartes, de Sturm et de Cauchy”, in *Journal de mathématiques pureset appliquées*, 5 (1840), 75–94.

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