Burali-Forti, Cesare | Encyclopedia.com

(*b*. Arezzo, Italy, 13 August 1861; *d*. Turin, Italy, 21 January 1931)

mathematics.

After obtaining his degree from the University of Pisa in December 1884, Burali-Forti taught at the Scuola Tecnica in Augusta, Sicily. In 1887 he moved to Turin after winning a competition for extraordinary professor at the Accademia Militare di Artiglieria e Genio. In Turin he also taught at the Scuola Tecnica Sommeiller until 1914. He remained at the Accademia Militare, teaching analytical projective geometry, until his death. He was named ordinary professor in 1906 and held a prominent position on the faculty; in 1927 he was the only ordinary among twenty-five civilian professors.

After an early attempt to obtain the *libera docenza* failed because of the antagonism to the new methods of vector analysis on the part of some members of the examining committee, he never again attempted to obtain it and thus never held a permanent universty position. (The *libera docenza* gave official permission to teach at a university and was required before entering a competition for a university chair.) He was assistant to Giuseppe Peano at the University of Turin during the years 1894–1896, but he had come under Peano’s influence earlier, however, and had given a series of informal lectures at the university on mathematical logic (1893–1894). These were published in 1894. Many of Burali-Forti’s publications were highly polemical, but to his family and his friends he was kind and gentle. He loved music; Bach and Beethoven were his favorite composers. He was not a member of any academy. Always an independent thinker, he asked that he not be given a religious funeral.

The name Burali-Forti has remained famous for the antinomy he discovered in 1897 in his critique of Georg Cantor’s theory of transfinite ordinal numbers. The critique begins: “The principal purpose of this note is to show that there exist *transfinite numbers* (or *ordinal types*) *a, b*, such that *a* is neither equal to, nor less than, nor greaer than *b*.” Essentially, the antinomy may be formulated as follows: To every class of ordinal numbers there corresponds an ordinal number which is greater than any element of the class. Consider the class of all ordinal numbers. It follows that it possesses an ordinal number that is greater than every ordinal number. This result went almost unnoticed until Bertrand Russell published a similar antinomy in 1903. It should be noted, however, that Cantor was already aware of the Burali-Forti antinomy in 1895 and had written of it to David Hilbert in 1896.

Burali-Forti was one of the earliest popularizers of Peano’s discoveries in mathematical logic. In 1919 he published a greatly enlarged edition of the *Logica mathematica*, which contained many original contributions. He also contributed much to Peano’s famous *Formulaire de mathématiques* project, especially with his study of the foundations of mathematics (1893).

Burali-Forti’s most valuable mathematical contributions were his studies devoted to the foundations of vector analysis and to linear transformations and their various applications, especially in differential geometry. A long collaboration with Roberto Marcolongo was very productive. They published a series of articles in the *Rendiconti del Circolo matematico di Palermo* on the unification of vectorial notation that included a full analysis, along critical and historical lines, of all the notations that had been proposed for a minimal system. There followed a book treating the fundamentals of vector analysis (1909), which was almost immediately translated into French. Their proposals for a unified system of vectorial notation, published in *L'enseignement mathématique* in 1909, gave rise to a polemic with various followers of Josiah Gibbs and Sir William Hamilton that lasted into the following year and consisted of letters, responses, and opinions contributed by Burali-Forti and Marcolongo, Peano, G. Comberiac, H.C. F. Timerding, Felix Klein, E. B. Wilson, C. G. Knott, Alexander Macfarlane, E. Carvallo, and E. Jahnke. The differences in notation continued, however, and the Italian school, while quite productive, tended to remain somewhat isolated from developments elsewhere. Also in 1909 Burali-Forti and Marcolongo began their collaboration in the study of linear transformations of vectors.

Burali-Forti's introduction of the notion of the derivative of a vector with respect to a point allowed him to unify and greatly simplify the foundations of vector analysis. The use of one simple linear operator led to new applications of the theory of vector analysis, as well as to improved treatment of operators previously introduced, such as Lorentz transformations, gradients, and rotors, and resulted in the publication (1912–1913) of two volumes treating linear transformations and their applications. Burali-Forti was able to apply the theory to the mechanics of continuous bodies, optics, hydrodynamics, static, and various problems of mechanics, always refining methods, simplifying proofs, and discovering new and useful properties. He did not live to see the completion of his dream, a small encyclopedia of vector analysis and its applications. The part dealing with differential projective geometry (1930) was Burali-Forti's last work.

The long collaboration with Marcolongo—their friends called them "the vectorial binomial"—was partly broken by their divergent views on the theory of relativity, the importance of which Burali-Forti never understood. With Tommaso Boggio he published a critique (1924) in which he meant "to consider Relativity under its mathematical aspect, wishing to point out how arbitrary and irrational are its foundations." "We wish," he wrote in the preface, "to shake Relativity in all its apparent foundations, and we have reason for hoping that we have succeeded in doing it." At the end he stated: "Here then is our conclusion. Philosophy may be able to justify the space-time of Relativity, but mathematics, experimental science, and common sense can justify it NOT AT ALL."

Burali-Forti had a strong dislike for coordinates. In 1929, in the second edition of the *Analisi vettoriale general*, written with Marcolongo, we find: "The criteria of this work… are not different from those with which we began our study in 1909, namely, an absolute treatment of all physical, mechanical, and geometrical problems, independent of any system of coordinates whatsoever."

BIBLIOGRAPHY

Besides his scientific publications, Burali-Forti wrote many school texts. In all, his publications total more than two hundred.

No complete list of the works of Burali-Forti has been published, but the following may be considered representative: *Teoriadelle grandezze* (Turin, 1893); *Logica matematica* (Milan, 1894; 2nd ed., rev., Milan, 1919); "Una questione sui numeri transfiniti," in *Rendiconti del Circolo matematico di Palermo*, **11** (1897), 154–164; *Lezioni di geometria metrico-proiettiva* (Turin, 1904); *Elementi di calcolo vettoriale, con numerose applicazioni alla geometria, alla meccanica e alla fisica-matematica*, written with R. Marcolongo (Turin, 1909), translated into French by S. Lattès as *Éléments de calcul vectoriel, avec de nombreuses applications à la géometrie, à la mécanique et è la physique mathématique* (Paris, 1910); "Notations rationelles pour le système vectoriel mininum," in *L'enseignement mathématique*, **11** (1909), 41–45, written with Marcolongo; *Omografie vettoriali con applicazioni alle derivate rispetto ad un punto ed alla fisica-matematica* (Turin, 1909), written with Marcolongo; *Analyse vectorielle générale*, 2 vols. (Pavia, 1912–1913), written with Marcolongo; *Espaces courbes. Critique de la relativité* (Turin, 1924), written with Tommaso Boggio; and *Analisi vettoriale generale e applicazioni*, Vol. II, *Geometria differenziale* (Bologna, 1930), written with P. Burgatti and Tommaso Boggio.

A work dealing with Burali-Forti is Roberto Marcolongo, “Cesare Burali-Forti,” in *Bollettino dell’Unione Matematica italiana*, **10** (1931), 182–185.

Hubert C. Kennedy