Carnot, Lazare-Nicolas-Marguerite | Encyclopedia.com

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(*b*, Nolay, Côte-d'Or, France, 13 May 1753; *d*. Magdeburg, Germany, 2 August 1823),

mechanics, mathematics, engineering.

Known to French history as the "Organizer of Victory" in the wars of the Revolution and to engineering mechanics for the principle of continuity in the transmission of power, Carnot remains one of the very few men of science and of politics whose career in each domain deserves serious attention on its own merits. His father, Claude, lawyer and notary, was among the considerable bourgeois of the small Burgundian town of Nolay in the vicinity of Beaune though on the wrong side of the ridge for the vineyards. The family still owns the ancestral house. His most notable descendants have been his elder son Sadi, famous in thermodynamics, and a grandson, the latter's nephew, also called Sadi, president of the French Republic from 1887 until his assassination in 1894.

Carnot had his early education in the Oratorian *collège* at Autun. Thereafter, his father enrolled him in a tutoring school in Paris which specialized in preparing candidates for the entrance examinations to the service schools that trained cadets for the navy, the artillery, and the Royal Corps of Engineers. Strong in technique and weak in prestige, the Corps of Engineers was the only branch of military service in which a commoner might still hold a commission. Carnot graduated from its school at Mézières after the normal course of two years. Gaspard Monge was then at the height of his influence over the cadets as professor of mathematics and physics, but although Carnot's handling of problems always bore the mark of the engineer, it does not appear that he was one of Mongers favorite pupils. Carnot's approach to mathematics and mechanics was in a curious way more concerned with both fundamentals and operations than that of Monge. Its actual mathematical yield was consequently much less while its significance for the physics of work and energy and for the evolution of engineering mechanics was much greater.

Promotion was slower for engineers than for line officers, and Carnot's outlook on the world, like that of many able and industrious men in the last years of the old regime, was compounded of frustrated talent and civic affirmation, some of which he expressed in tolerable verse. After routine assignments at Calais, Cherbourg, and Béthune, he was posted to Arras, where society was livelier. There in 1787 he became acquainted with Maximilien de Robes-pierre, a fellow member of the literary and philosophic society of the Rosati. During these years of garrison duty, Carnot sought reputation by writing of mechanics, mathematics, and military strategy in essays prepared for competitions of the kind regularly set by learned societies in the eighteenth century. In 1777 the Académic des Sciences in Paris proposed the "theory of simple machines with regard to friction and the stiffness of cordage" for a prize. Carnot entered a memoir in that contest and revised it in 1780 for resubmission in a second round opened by the Academy when none of the entries in the first proved worthy of an award. It then received an honorable mention following a memoir on friction by Coulomb which won the prize. Carnot developed its theoretical portion into his first publication, *Essai sur les machines en général*, which appeared in 1783. In retrospect it is evident that that modest work inaugurated the peculiarly French literature of engineering mechanics, but it commanded little attention until after its author had become great and famous in the politics and military affairs of the Revolution.

The fate of an early mathematics memoir was similar. In 1784 the Academy of Berlin invited entries in a competition for a "clear and precise" justification of the infinitesimal calculus. The essay that Carnot submitted forms the basis of the *Réflexions sur lamétaphysique du calcul infinitésimal* that he published many years later, in 1797. Once again Carnot's entry received an honorable mention but no other notice.

It was, indeed, only through his writings on military strategy that Carnot won a certain minor recognition prior to the Revolution. In 1784 the Academy of Dijon set as subject for its annual prize the career of the founder of the Royal Corps of Engineers, the Maréchal de Vauban (like Carnot a Burgundian), whose theory and practice of fortification and siege-craft had guided the strategy of Louis XIV and become standard doctrine in the limited warfare of the Enlightenment. Carnot's *Éloge de Vauban* carried off that award. Its publication brought him into the crossfire of a skirmish of the books that had broken out among members of the French armed forces where in the political interests and social prestige of the several arms and services were entangled with opposing theories of warfare. Writing out of the specialist tradition of the engineering corps, Carnot upheld the purportedly humane view of warfare that made its purpose the defense of civilization in the wise employment of prepared positions rather than the barbarous destruction of an enemy in conflict to the death. His argument criticized the emphasis of the combat arms on gallantry, movement, and command under fire, aspects recommended anew to elements among the line officers by the recent example of Frederick the Great and the resurgence of aristocratic pretensions in French society. It is ironic, therefore, that the latter was the type of warfare that Carnot mastered and waged when charged with direction of the Revolutionary war effort. Not that he had any choice—in the emergency of 1793–1794 Carnot disposed not of the disciplined, professional, and cautious forces presupposed by conservative strategy, but of untrained levies, some under arms because of patriotism and some because of conscription. Patriotism produced dash and conscription mass, and in both respects the armies of the Republic were quite unlike those of the eighteenth century.

Carnot entered politics in 1791 when he was elected a deputy to the Legislative Assembly from the Pasde-Calais. No flaming radical, his idea of social justice was the career open to talent. As the monarchy proved its untrustworthiness in 1791 and 1792, he became a republican out of a kind of civic commitment natural to his class and family background. Following the outbreak of war in April 1792, the services of a patriotic deputy competent in military matters were at a premium. There was an integrity in Carnot that made itself felt and trusted in dangerous times. He combined it with the engineer's ability to improvise arrangements and organize procedures. His mission to the Army of the Rhine immediately after the overthrow of the monarchy in August 1792 imposed the sovereign authority of the Republic upon the officers and local agents whose allegiance had been to the Crown. Amid the military disasters in Belgium in the spring of 1793, it was Carnot who, as representative of the people incarnating its revolutionary will, had to override the demoralized generals and organize first the defense and then the attack to his own presciption. Never an ideologist, never really a democrat, he had the reputation of a tough and reliable patriot when, in August 1793, he was called by the more politically minded men already constituting the emergency Committee of Public Safety to serve among its membership of twelve men who, ruling France throughout the Jacobin Terror, converted anarchy into authority and defeat into victory.

His main responsibility was for the war, and he alone of his erstwhile colleagues continued in office after the fall of Robespierre in July 1794 through the ensuing reaction and on into the regime of the Directory that followed in 1795. For two more years Carnot was the leading member of that body, in which office he, together with four incompatible colleagues, exercised the executive power of the French Republic. In 1797 the leftist coup d'état of 18 Fructidor (4 September) displaced Carnot from government. He took refuge in Switzerland and Germany, returning in 1800 soon after Napoleon's seizure of power.

Carnot had given Napoleon command of the Army of Italy in 1797, and now Napoleon named his sometime patron minister of war. The Bonapartist dispensation proved uncongenial to an independent spirit, however, and after a few months Carnot resigned.

Thereupon he devoted his older years to the technical and scientific interests of his younger days, having qualified for membership in the Institut de France in 1796 by virtue of his prominence if not of any wide comprehension of his youthful work in mechanics, his only scientific work then in print. Throughout the Napoleonic period he served on numerous commissions appointed by the Institute to examine the merits of many of the mechanical inventions that testify to the fertility of French technical imagination in those years of conquest and warfare. He was never too old for patriotism, however. Amid the crumbling of the Napoleonic system, he offered his services when the retreat from Moscow reached the Rhine. In those desperate circumstances Napoleon appointed him governor of Antwerp. Carnot commanded the defense. He rallied to the emperor again during the Hundred Days and served as his last minister of the interior. That act bespoke the consistency of the old revolutionary yet more decisively than Carnot's having voted death to Louis XVI some twenty-two years before. He was not for given by a monarchy that had had to be restored twice, and he fled into exile once again, leaving in France his elder son, Sadi, an 1814 graduate of the É cole Polytechnique, and taking the younger, Hippolyte, to bring up in Magdeburg, in which tranquil city he lived out his days corresponding with old associates and publishing occasional verse.

Scientists often become public figures, but public figures seldom produce science, and a glance at the Bibliography below will suggest that the range and originality of his published work in mechanics, geomentry, and the foundations of the calculus in the ten years following his fall from power in 1797 must be unique in the annals of statesmen in political eclipse. The phenomenon is the more interesting in that its beginnings go back to his days as a young engineering officer who had failed to win a hearing for the approach that began to be appreciated only in the days of his later prominence. The *Essai sur les machines en général* of 1783 contains all the elements of his engineering mechanics, in which subject it was the first truly theoretical treatise. It attracted no detectable attention prior to the revision he published in 1803 under the title *Principes fondametauxde l'équilibre et du mouvement*. Even the analysis given the problem of machine motion in the latter work began to affect the actual treatment of problems only in the 1820's through the theoretical practice of his son Sadi and his contemporary polytechnicians, notably Navier, Coriolis, and Poncelet. The lag is not to be explained on the grounds either of some mathematical sophistication in the analysis or of the unexpectedness of some signal discoveries. On the contrary, the style and argument seem naïve and literal compared to the mathematical mode of treatment that Lagrange set as the standard of elegance for rational mechanics in *Mécanique analytique* (1788), a work to which Carnot never aspired to compare his own. The unfamiliarity of Carnot's approach derived from his purpose, not his content, from what he thought to do with the science of mechanics rather than what he added to it.

His purpose was to specify in a completely general way the optimal conditions for the operation of machines of every sort. Instead of adapting the laws of statics and dynamics to the properties of the standard classes of simple machine—i.e., the lever, the wedge, the pulley, the screw, and so forth—as did conventional manuals of applied mechanics, he began with a completely abstract definition. A machine is an intermediary body serving to transmit motion between two or more primary bodies that do not act directly on one another. The problem of a truly general mechanics of machines could then be stated:

Given the virtual motion of any system of bodies (i.e., that which each of the bodies would describe if it were free), find the real motion that will ensue in the next instant in consequence of the mutual interactions of the bodies considered as they exist in nature, i.e., endowed with the inertia that is common to all the parts of matter.

In developing the reasoning, Carnot gave important impetus to what was to become the physics of energy, not by building out from the mathematical frontier of analytical mechanics, but by starting behind those front lines with the elementary principles of me-chanics itself. From among them he selected the con-servation of live force or *vis viva*, the quantity MV^2 or the product of the mass of a body multiplied by the square of its velocity, as the basis from which propositions adaptable to the problem of machine motion might most naturally be derived. The principal explicit finding of the *Essai sur les machines en général* was that it is a condition of maximum effi-ciency in the operation of machines that power betransmitted without percussion or turbulence (in thecase of hydraulic machines). That principle of conti-nuity was usually known as Carnot's until after the middle of the nineteenth century, when its parentage became obscured in the generality of conservation of energy, of which felt but unstated law it was one of many early partial instances.

In the course of deriving his own principle of continuity from that of live force, Carnot recognized its equivalent, the product of the dimensions of force multiplied by distance—*MGH* in the gravitational case, for he usually preferred to reason on the example of weights serving as loads or motive forces—to be the quantity that mechanics might most conveniently employ to estimate the efficacy of machines. In 1829 Coriolis proposed to designate by the word "work" any quantity thus involving force times distance. Carnot had called it "moment of activity." Although he did not explicitly state the measure of power to be the quantity of live force (energy) expended or moment of activity (work) produced in a given time, that usage is implicit in the argument, of which the central thrust in effect transformed the discussion of force and motion into an analysis of the transmission of power. Dimensionally, of course, there was nothing new about it: the equivalence of $MV^2/2$ to *MGH* derived from Galileo's law of fall as the means of equating the velocity that a body would generate in falling a certain height with the force required to carry it back on rebound.

It was the engineer in Carnot that saw the advantage of winning from that trivial dimensional equivalence an application of the science of mechanics to analysis of the operation of machines. In the ideal case, live force reappears as moment of activity; or, in much later words, input equals output. It was simply a condition of perfect conversion that nothing be lost in impacts and that motion be communicated smoothly—hence his principle. His approach carried over the employment of live-force conservation from hydrodynamics into engineering mechanics generally. Following publication in 1738 of Daniel Bernoulli's *Hydrodynamica*, that subject had been the only one that regularly invoked the principle of live force in the solution of engineering problems; and except for certain areas of celestial mechanics, it was the only sector of the science in which it had survived thediscredit of the metaphysical disputes between parti-sans of the Newtonian and Leibnizian definitions of force early in the century. The evidence runs throughall of Carnot's writings on mechanics and into thoseof his son Sadi that the hydraulic application of principles and findings, although ostensibly a special case, actually occupied a major if not a primary place in their thinking.

Carnot did not reach his conclusions so easily as this, however, and certain features of his reasoning point much further than its mere results, namely in the direction of dimensional and vector analysis and also toward the concept of reversible processes. Wishing to attribute to machines no properties except those common to all parts of matter, Carnot envisaged them as intrinsically nothing more than systems composed of corpuscles. He began his analysis with the action of one corpuscle upon another in machine motions, and obtained an equation stating that in such a system of "hard" (inelastic) bodies, the net effect of mutual interaction among the corpuscles constituting the system is zero:

where the integral sign means summation; *m* is the mass of each corpuscle; *V* its actual velocity; *U* the velocity that it "loses", i.e., the resultant of *V* and its virtual velocity; and Z the angle between the directions of *V* and *U*. The notion of balancing the forces and motions "lost" and "gained" from the constraints of the system was central to his analysis of the manner in which he imagined forces transmitted by shafts, cords, and pulleys constraining and moving points within systems composed of rigid members. In his way of seeing the problem, he had necessarily to find constructs that would incorporate the direction as well as the intensity of forces in expressions of their quantity; and in combining and resolving such quantities, he habitually adopted simple trigonometric relations in a kind of proto-vector analysis that permitted him to represent the projection of the quantity of a force or velocity upon a direction other than its own. In the *Principes fondamentaux de l'équilibre et du mouvement*, he proposed that the projection of one force or velocity upon an intersecting straight line be represented by the notation

where the last term represents the angle, a conventionthat would have been obvious although cumbersome had it been much adopted (Fig. 1). Not that Carnot had given his reader the assistance of such a diagram in the *Essai sur les machines*, but adapting it to the equation above makes it easier to see at a glance what was then in his mind, and also to appreciate his strategy: since

$U \cos Z = V$,

the relation reduces to

which is to say the principle of live-force conservation.

Given the generality of his statement of the problem of machine motion, Carnot could not simply proceed to a direct application of that principle, for along with the legacy of eighteenth-century matter theory he inherited that of seventeenth-century collision theory. According to the former, solid matter consists in impenetrable, indeformable corpuscles connected by rods and shafts: rigid ones in "hard" or inelastic bodies and springs in elastic bodies. Micromachines Carnot imagined the former to be, and took them for the term of comparison to which real machines ideally reduced in nature. As for the other states of matter, liquids were fluids congruent with hardness in mechanical properties since they were incompressible although deformable (a circumstance reinforcing the primacy of hydrodynamics in his thought) and gases were fluids mechanically congruent with elasticity. His difficulty was that in classical collision theory live force was conserved only in the interaction of elastic bodies; in the supposedly more fundamental case of "hard" body, live force was conserved only when motion was communicated smoothly—by "insensible degrees" in his favorite phrase, to which processes was restricted the application of the fundamental equation stated above.

He had, therefore, to convert that equation into an expression applicable to all interactions in which motion was communicated, whatever the nature of the body or the contact. To that end Carnot introduced the notion that he always regarded as his most significant contribution to mathematics and to mechanics: the idea of geometric motions, which he defined as displacements depending for their possibility only on the geometry of a system quite independently of the rules of dynamics. The concept was that which in later mechanics was called virtual displacement. In the *Principes fondamentaux de l'é quilibre et du mouvement* of 1803, Carnot simplified his definition so that it amounted to specifying geometric motions as those that involve no work done on or by the system. But historically his first elucidation is the more interesting for it exhibits that what suggested to Sadi the idea of a reversible process must almost certainly have been his father's concept of geometric motion.

Imagine, Carnot charged the reader of the *Essai sur les machines en général*, that any system of hard bodies be struck by an impact, and further that its actual motion be stilled at just this instant, and it be made instead to describe two successive movements, arbitrary in character, but subject to the condition that they be equal in velocity and opposite in direction. Such an effect could be accomplished in infinitely many ways and (this was the essential matter) by purely geometric operations. An important, though not exhaustive class of such motions would be those that involved the constituent bodies of a system in no displacement relative to one another. In such a generalized system, the velocities of the neighboring corpuscles relative to each other would be zero, and Carnot could derive the further fundamental equation

which differs from the former in that the actual physical velocity *V* has been replaced by an idealized, geometric velocity *u*. It would apply, therefore, to interactions of elastic or of inelastic bodies whether gradual or sudden since by definition such motions were independent of the dynamical considerations that excluded inelastic collision. In later terms, what Carnot had done was to derive conservation of moment of momentum or torque from conservation of energy or work by considering the ideal system within which no energy or work was lost. What he himself claimed to have done was to derive a generalized indeterminate solution to the problem of machines from which could be deduced such established though partial principles as d'Alembert's and Maupertuis's least action. In actual cases, among all the motions of which a system was geometrically capable, that which would physically occur would be the geometric motion for which sum of the products of each of the masses by the square of the velocity "lost" was a minimum, i.e., for which

His conception of the science of machines as a subject, however, rather than his somewhat jejune solution to its generalized problem, was what made Carnot an important influence upon the science of mechanics. Through that and similar influence it became in fact and not just in precept the basis of the profession of mechanical engineering. His analysis, which balanced accounts between "moment of activity produced" (work done on) and "moment of activity consumed" (work done by) the system, was the kind that the physics of work and energy have found useful ever since those topics became explicit in the 1820's, 1830's, and 1840's. Its most recognizable offspring was the heat cycle of Sadi Carnot, which considered a system in view of what had been done to it or by it in shifting from an initial to a final state. The family resemblance was marked in the abstractedness of the systems imagined; in the discussion of force in terms of what it can do, taken usually over distance when it was a question of its measure, and over time when it was a question of its realization in mechanical processes; in the notion that process consisted in the transition between successive "states" of a system; in the requirement that for purposes of theory this transition occur in infinitesimal and reversible changes (which for Lazare Carnot was to say that all motions be geometric); in the indifference (given these conditions) to the details of rate, route, or order of displacements; in the restrictive mode of reasoning by which, perpetual motion being excluded as physically unthinkable, the maximum possibilities in operations were thereby determined in the ideal case; in the relevance of this extreme schematization to the actuality of tools, engines, and machinery operating discontinuously and irreversibly in physical fact; and, finally, in thus making theoretical physics out of the engineering practice and industrial reality of the age.

The operationalism of the engineer distinguishes Carnot's mathematical writings in a similar manner. They may be summarized more briefly because, although more voluminous and evidently more important to him in his middle years, his work in mathematics did not enter into the texture of the subject as it did in mechanics. Not that it lacked for a public: his *Reflexions sur la métaphysique du at lad infinité simal*, first published in 1797 while he was still in political office as director, was quickly translated into Portuguese, German, English, Italian, and Russian. Carnot revised and enlarged a second edition for publication in 1813; and that version was republished from time to time in Paris, most recently in 1921. His book frankly acknowledged the difficulties that infinitesimal analysis raises for common sense, and although it was reserved to the reforms initiated by Cauchy, Bolzano, and Gauss to put the calculus on a rigorous footing, Carnot's justification evidently answered for well over a century to the needs of a public that wished to understand its own use of the calculus.

The genius of the infinitesimal calculus, in Carnot's account, lay in its capacity to compensate in its own procedures for errors that it deliberately admitted into the process of computation for the purpose of facilitating a solution. The sort of error in question is that which arises from supposing that a curve can be considered equivalent to a polygon composed of a very large number of very short sides, but the compensation of error that justified the calculus meant neither the approximation of rectilinear elements to a curve by a method of exhaustion nor the cancellation of error through some balancing of excess against defect. Such procedures would merely reduce error to the tolerable or insignificant, whereas what Carnot meant by compensation actually eliminated error and made the procedures of analysis as rigorous as those of synthetic demonstration. What the calculus really involved, properly understood, was the auxiliary use of infinitesimal quantities in order to find relations between given quantities, and its results contained no errors at all, not even infinitesimal errors.

In explaining what he meant, Carnot proposed substituting for the conventional division of quantities into determinate and indeterminate a tripartite classification into quantities (1) that were invariant and given by the conditions of the problem, (2) that although variable by nature acquired determinate values because of the conditions of the problem, and (3) that were always indeterminate. To the last class belonged all infinitely great or small quantities, and also all those involving the addition of a finite and an infinitesimal quantity. Quantities of the first two classes Carnot called designated and those of the third nondesignated. What characterized these last was not that they were minute or negligible, but that they could be made arbitrarily small at the will of the calculator. Yet at the same time they were not merely arbitrary. On the contrary, they were related to quantities of the second class by one system of equations just as the latter were to quantities of the first class by another, not unrelated, system of equations. The systems containing only designated quantities Carnot called complete or perfect. Those containing terms in nondesignated quantities he called incomplete or imperfect. The art of the infinitesimal calculus, therefore, consisted in transforming insoluble or difficult complete equations into manageable incomplete equations and then managing the calculation so as to eliminate all nondesignated quantities from the result. Their absence proved its correctness.

The congruence between Carnot's point of view in calculus and in mechanics appears to excellent advantage in his development of these comparisons. Consider, he asked the reader, any general system of quantities, some constant and some variable, and suppose the problem is to find the relations between them. Let any specified state of the general system be taken for datum. Its quantities and the variables depending exclusively on them would be the designated ones. If now the system were to be considered in some different state invoked for the purpose of determining more readily the relations between the designated quantities of the fixed system, this latter state would serve an auxiliary role and its quantities would be auxiliary quantities. If further the auxiliary state be approached to the fixed state so that all the auxiliary quantities approach more and more closely to their designated analogues, and if it is in our power to reduce these remaining differences as far as we please, then the differences would be what is meant by infinitesimal quantities. Since they were merely auxiliary, these arbitrary quantities might not figure in the solution to the problem, which was to determine the relation between the designated quantities. Reciprocally, that no arbitrary quantities occur in the result was a proof of its validity. Thus had the error willfully introduced in order to solve the problem been eliminated. If it did persist, it could only be infinitesimal. But that was impossible since the result contains no infinitesimals. Hence the procedures of the calculus had eliminated it, and that was the secret of their success.

In effect, as Carnot made clear in a historical excursion, his doctrine of compensation of errors was an attempt to combine the analytical advantages of the Leibnizian method of infinitesimals with the rigor of the Newtonian method of limits or first and last ratios of vanishing quantities.

Carnot's main geometric writings were also motivated largely by the attempt to make reasonable the employment of unreasonable quantity in analysis, although with the focus on negative rather than infinitesimal quantity. Indeed, it is clear from the manuscript remains of his mature years that he regarded the unthinking manipulation of quantities that could have no literal physical meaning as an impropriety, not to say a scandal, in mathematics that made the obscurity attending infinitesimal quantity relatively venial. There was no doubt that infinitesimals existed. The problem was only how to understand and manage them. On the other hand the difficulty with negative and imaginary numbers concerned finite analysis, went deeper, and occurred in almost every algebraic operation. In Carnot's own view his resolution of that anomaly in *De la correlation des figures de géométrie* of 1801 and its extension, the *Géomé trie de position* of 1803, constituted his most significant clarification of the procedures of mathematics.

Carnot found absurd the notion that a quantity itself could be less than zero. "Every quantity is a real object such that the mind can be seized of it, or at least its representation in calculation, in an absolute manner," he held robustly in *Géométrie de position* (p. 7), and insisted in *Corrélation des figures* on distinguishing between a quantity properly speaking and the algebraic value of a function. It was equally unacceptable to interpret the minus sign as meaning simply that a quantity was to be taken in a direction opposite to a positive one. A secant to the arc of a circle in the third quadrant was indistinguishable from one in the first in magnitude and direction. According to this latter interpretation, it should be positive in sign. In fact it is negative. Neither could this view explain why it is impossible to take the square root of a negative quantity: There is no difficulty in taking the square root of a left ordinate, after all. In short, none of the usual conventions could stand scrutiny.

There were only two senses, in Carnot's view, in which negative quantity could be rightly understood. One was the trivial sense in which it was the magnitude of a value governed by a minus sign, and this usage could be correct only when it was preceded by a positive value of greater magnitude. The deeper and more revealing sense was that a negative quantity was a magnitude governed by a sign that was wrong.

Consider, for example, the formula

Now, in the first or literal sense, the last term here is negative. In the second sense it is positive, however, so long as *a, b*. and (*a* + *b*) are each less than 90°, because since the equation is exact in the first quadrant, the sign is correct. But if (*a* + *b*) were greater than 90°, the equation would be incorrect. For then it would turn out that

If formula (1) were to apply to this case, the first term, cos (*a* + *b*), would have to be regarded as governed by a sign contrary to what it ought to have been.

For changing the sign would give

which reduces to equation (2). In effect, therefore, applying equation (1) to the case in which (*a* + *b*) was greater than 90° subjected the term, cos (*a* + *b*), to the wrong sign and made it a negative quantity in the second sense.

For quantities of this sort Carnot proposed the term "inverse" in contradistinction to "direct quantities" that bear their proper sign. How, then, did these inversions get into the process of calculation? Carnot said by error, but not quite the kind of error that introduced infinitesimals into the calculus. The occasion of error in finite analysis was rather a mistaken assumption about the basic conditions of the problem. A credit might be mistaken for a debt in actuarial work. The algebraic expression of what was to be paid or received would then be reversed, and we would recognize the error in the solution through knowledge of whether money was owed or owing. But the question was not merely one of correcting trivial mis apprehensions. Often it was necessary to introduce quantities governed by a false sign into a calculation in order to formulate a problem at all. The ordinate of a curve might be required without the geometer's knowing in which quadrant it lay. Making an assumption would permit him to get an absolute value, and if he had been wrong about the sign, the error would show up as an absurdity in the result, which he could correct without redoing the computation. Indeed, what distinguished analysis from synthesis in Carnot's view was uniquely its capacity to employ negative and imaginary forms and to eliminate the unreal entities in the course of calculation after they had served their purpose of auxiliaries. In the end whatever was unintelligible was made to disappear, and there remained a result that in principle could have been discovered synthetically. But it had been obtained more easily and directly, almost mechanically. The difference between infinitesimal and finite analysis is that errors in the calculus are eliminated through the very process of computation, whereas in algebra they remain in the solution, where we recognize them by comparison with a rational or metrical reality, and that is where geometry comes in.

The service that geometry might render to analysis lay mainly through the study of the correlation of figures, and that in turn exhibits a certain congruence with his point of view in the mechanics of machine processes. Its subject matter is the comparison of geometric systems in various states that can evolve into each other. The state taken for term of comparison he called the primitive system and any other state a transformed system. An elementary example from the 1801 treatise *Corrélation des figures* will exhibit the nature of the problems and their relevance to resolving the anomaly of inverse quantity. In the triangle *ABC* (Fig. 2), the foot of the perpendicular *AD* falls between *B* and *C* in the primitive system.

It is possible to transform the system by moving *C* to the left, however. In effect, the figure constitute a system in which is a variable and a constant. In both primitive and transformed states > until *C* has passed to the left of *D*. Their difference, is a direct quantity, and and are in "direct order" in the two systems of which, therefore, the correlation was direct. Let the point *C* pass to the left of *D*, however, and then in the transformed system , their difference is inverse, and the quantities and are in inverse order in the two systems, the correlation of which thereby becomes indirect.

By correlative systems Carnot meant all those that could be considered as different states of a single variable system undergoing gradual transformation. It was not necessary that all correlative systems should actually have been evolved out of the primitive system. It sufficed that they might be assimilated to it by changes involving no discontinuous mutations. The whole topic may be taken as the geometric operation of Carnot's favorite reasoning device—a comparison of systems between which the nexus of change is a continuum. When the correlation was direct, any train of reasoning that was valid for the primitive system would hold for the correlative system. When it was indirect, the formulas of the primitive system were applicable to the other by changing the signs of all the variables that had become inverted. Reciprocally, the procedure might be used in solving problems. Suppose, in the example it was given that the three sides were in the proportion

and the segment was to be found. Trying the hypothesis that *D* was between *B* and *C*, we would have

Which works out to give a negative value

The minus sign signals that the hypothesis had been wrong, and that in fact *C* fell between *B* and *D*, for which assumption the problem yields a positive value in the solution.

In *Géométrie de position* Carnot developed what he had at first intended as a somewhat fuller edition of the *Corrélation des figures* into a vastly more extensive exploration of the problem-solving reaches of geometry. The method bore a marked resemblance to the porisms of Euclid, an affinity that Carnot recognized, for he was well versed in the history of geometry. He frequently expressed appreciation for eighteenth-century British geometry, and particularly the work of Colin Maclaurin, Robert Simson, John Landen, and Matthew Stewart—the last three names now largely forgotten in the history of mathematics.

He no longer limited the scope of his work to correlations of particular geometric systems, but proposed to associate in a single treatment relations of magnitude as studied by the ancients with relations of position as studied more characteristically by the moderns, and thus to compare and unify the two main types of geometric relation. The *Géométrie de position* wears the appearance of a sort of engineering handbook of geometric systems that, were it ever to be completed, would permit resolving problems by considering unknown systems as correlatives of the set of primitive systems of which the properties were known. The formulas were to contain only real and intelligible expressions—no imaginary and no inverse quantities.

The entire subject Carnot imagined as preliminary to that which was closest to his heart and on which he promised a further book: the science of geometric motion. He redeemed the promise that same year by publishing the *Principes fondamentaux de l'é quilibre et du mouvement*, which treats the science of machines in terms of geometric motions. For it is extremely interesting that it was in this that he thought to unify his geometric and mechanical interests. In *Géomé trie de position*, he defines such motions as he had in the *Essai sur les machines en général*, motions depending on the geometry and not the dynamics of a system such that "the contrary motion is always possible." Such a science would be intermediary between geometry and mechanics and would rid mechanics and hydrodynamics of their analytical difficulties because it would then be possible to base both sciences entirely on the most general principle in the communication of motion, the equality and opposition of action and reaction.

BIBLIOGRAPHY

I. Original Works. Carnot's scientific works are as follows:

(1) *Essai sur les machines en général, par un officier du Corps Royal du Génie* (Dijon, 1783). Although absent from the first edition, the name of the author did appear on the second, otherwise identical printing (Dijon-Paris, 1786).

(2) *Réflexions sur la métaphysique du calcul infinitésimal* (Paris, 1797). The identical text was also published, together with (1) above, in *Oeuvres mathématiques du citoyenCarnot* (Basel, 1797). In 1813 Carnot published a revised and enlarged edition, which has been reprinted without change in all later French editions.

(3) *Lettre du citoyen Carnot au citoyen Bossut, contenant quelques vues nouvelles sur la trigonometric*, in CharlesBossut. *Cours de mathématiques*, new ed., vol. II (Paris,1800).

(4) *De la corrélation des figures de géométrie* (Paris.1801).

(5) *Géométrie de position* (Paris, 1803).

(6) *Principes fondamentaux de l'équilihre et du mouvement* (Paris, 1803).

(7) *Mémoire sur la relation qui existe entre les distances respectives de cinq points quelonques pris dans l'espace, suivi d'un essai sur la théorie des transversales* (Paris, 1806), to which is appended a summary of the theory of negative quantity, "Digression sur la nature des quantités dites négatives."

Between 1800 and 1815 Carnot served on many commissions of the Institute to which numerous mechanical inventions and mathematical writings were submitted. His reports on these subjects will be found in Institut de France, Académie des Sciences, *Provés-verbaux des séances del'Acadéemie tenues depuis la fondation de l'Institut jusqu'au mois d'août 1835*, vols. I-V (Paris, 1910–1914). Each volume contains a full index of its contents. Carnot's most important reports were those on the Niepce and the Cagniard engines, III. 465–467, and IV. 200–202. The archives of the Academy in Paris contain the text of an unpublished "Lettre sur les aerostats" that Carnot submitted to the Academy on 17 January 1784 following the flight engineered by the brothers Montgolfier on 5 June 1783. The problem was that of propulsion.

II. Secondary Literature. There is a recent and largely reliable political biography, Marcel Reinhard, *Le grand Carnot*, 2 vols. (Paris, 1950–1952).

The memoir by Carnot's younger son, Hippolyte Carnot, *Mémoires sur Carnot*, 2 vols. (Paris, 1861–1863), is a work of anecdote animated by family piety but remains indispensable.

Carnot's scientific work forms the subject of a monograph, *Lazare Carnot savant*, by the undersigned (in press—scheduled for publication by Princeton University Press in 1971). The appendix of this work reprints in facsimile the theoretical portions of the two manuscripts on mechanics that Carnot submitted to the *Academic dessciences* in 1778 and 1780, and the entire text of the "Dissedation" on the infinitesimal calculus that he submitted to the Académic Royale des Sciences, Arts et Belles-Lettres de Berlin in 1785. The originals are contained in the archives of the respective academies. The forthcoming work also contains a commentary on this latter manuscript by A. P. Youschkevitch, who published an extensive introduction and commentary to the Russian translation of the *Réflexions sur la métaphysique du calcul infinitésimal: LazarKarno. Razmyshlenia o metafizike ischislenia beskonechno-malykh*, N. M. Solovev, trans., with a critical introduction by A. P. Youschkevitch and a biographical sketch by M. E. Podgorny (Moscow, 1933).

Persons who consult accounts of Carnot's work in mechanics in the secondary literature— e.g., René Dugas. *Histoire de la mécanique* (Neuchâtel. 1950), or Émile Jouguet, *Lectures de mécanique*, 2 vols. (Paris, 1908–1909), should be warned that their authors have sometimes confused item I, 6, with item I, above, and attributed passages from the later, more widely read work to the earlier one. There is a brief notice of Carnot's geometric work in Michel Chasles, *Apercu historique sur l'origine et le développement des méthades en géoméirie* (2nd ed., Paris, 1875), and Felix Klein, *Vorlesungen über die Entwicklung der Mathernatikim 19, Jahrhundert, 2* vols. (Berlin, 1926) I, 79–80.

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