Nicolas Leonard Sadi Carnot | Encyclopedia.com

(*b* Paris, France, 1 June 1796; *d*. Paris, 24 August 1832),

thermodynamics.

The eldest son of Lazare Carnot, Sadi was born in the Palais du Petit-Luxembourg, where his father lived as a member of the Directory. The powerful and often turbulent worlds of French politics and science were an integral part of the environment in which Sadi and his younger brother Hippolyte spent their youth. Withdrawing from public life in 1807, Lazare Carnot concentrated on science and the education of his sons. Through his studies Sadi acquired not only his taste and aptitude for mathematics but also a solid training in physics, the natural sciences, languages, and music.

Because of his rapid progress it was decided that Sadi should attend the elite École Polytechnique upon attaining the age of sixteen, the minimum for admission. Following a few months' preparation at the Lycée Charlemagne in Paris, he passed the entrance examination and was admitted to the Polytechnique. His studies there from 1812 to 1814 stressed analysis, mechanics, descriptive geometry, and chemistry, taught by a distinguished faculty including Poisson, Gay-Lussac, Ampère, and Arago.

In 1813 Sadi addressed a letter to Napoleon on behalf of his fellow students, asking permission to join the fight against the invading Allies, and in March 1814 he was among the students who fought bravely, though in vain, at Vincennes. Ranking sixth in his class, he finished his studies at the Polytechnique in October 1814 and was immediately sent to the École du Génie at Metz as a student second lieutenant. During the two-year course in military engineering Sadi wrote several scientific papers, now lost but which his brother said were well received. During the Hundred Days, Lazare Carnot was Napoleon's minister of the interior, and Sadi became an object of special attention from his superiors. This ended in October 1815, when Lazare was exiled by the Restoration.

In late 1816 Sadi finished his studies and began serving as a second lieutenant in the Metz engineering regiment. For the next two years he was shifted about from garrison to garrison, inspecting fortifications and drawing up plans and reports doomed to bureaucratic oblivion.[1] In spite of some connections with high officials, his father's name and reputation became a burden to him in the first years of the Restoration, and his intellectual development was frustrated by the tedium of military garrisons. In 1819 he seized an opportunity to escape by passing a competitive examination for appointment to the army general staff corps in Paris. He immediately obtained a permanent leave of absence and took up residence in his father's former Paris apartment.

Relieved of the constraints of military life, Carnot began the wide range of study and research that continued, despite numerous interruptions, until his death. In addition to private study he followed courses at the Sorbonne, the Collège de France, the École des Mines, and the Conservatoire des Arts et Métiers. At the latter he became a friend of Nicolas Clément, who taught the course in applied chemistry and was then doing important research on steam engines and the theory of gases. One of Carnot's particular interests was industrial development, which he studied in all of its ramifications. He made frequent visits to factories and workshops, studied the latest theories of political economy, and left in his notes detailed proposals on such current problems as tax reform. Beyond this, his activity and ability embraced mathematics and the fine arts.

In 1821 Carnot interrupted his studies to spend a few weeks with his exiled father and brother in Magdeburg. It was apparently after this visit that, once again in Paris, he began to concentrate on the problems of the steam engine. After Lazare's death in August 1823, Hippolyte returned to Paris to find his brother at work on the manuscript of the *Réflexions*. In an attempt to make his work comprehensible to a wide audience, Sadi forced Hippolyte to read and criticize portions of the manuscript. On 12 June 1824 the *Réflexions sur la puissance motrice du feu et sur les machines propres à dévetopper cette puissance* was published by Bachelier, the leading scientific publisher in France. By all reasonable standards the book was well received. On 14 June it was formally presented to the Académie des Sciences, and on 26 July P.-S. Girard read a lengthy and very favorable review to the Academy. This review, printed in the August issue of the *Revue encyclopédique*, emphasized the book's conclusions and its applications for steam-engine construction. Although the major theorems were cited, there was no discussion of the highly original reasoning that Carnot had employed.

Following the publication of his book, Carnot continued his research, fragments of which are preserved in his manuscript notes. A reorganization of the general staff corps, however, forced Carnot to return to active service in 1827 with the rank of captain. After less than a year of routine duty as a military engineer in Lyons and Auxonne, Carnot resigned permanently and returned to Paris. He again focused his attention on the problems of engine design and the theory of heat. In 1828 a contemporary referred to Carnot as a "builder of steam engines,"[2] although there is no record of his formal connection with any firm. Except for his informal contact with Clément, Carnot always worked independently and rarely discussed his research.

Although sensitive and perceptive, he appeared extremely introverted, even aloof, to all but a few close friends, most of whom, like Claude Robelin and the geometer Michel Chasles, were his classmates at the Polytechnique.

True to his father's republican principles, Carnot welcomed the July Revolution, but was soon disappointed with the new government. Nonetheless, he was highly regarded in some political circles, for shortly after the Revolution he was mentioned as a possible member of the Chambre des Pairs. He objected to the hereditary nature of this position, however, and refused to be nominated.

In 1831 Carnot began to investigate the physical properties of gases and vapors, especially the relationship between temperature and pressure. In June 1832, however, he contracted scarlet fever. This was followed by "brain fever," which so undermined his fragile health that on 24 August 1832 he fell victim to a cholera epidemic and died within the day, at the age of thirty-six. In accordance with the custom, his personal effects, including nearly all of his papers, were burned. Although for eight years his work had been almost completely ignored, he was not for gotten. The news of his death received a front-page article in the 27 August *Moniteur*, and a note describing his book as "remarkable for its original views" appeared in the February 1833 issue of the *Annales de chimie et de physique*. The only full obituary was published in the August 1832 issue of the *Revue encyclopédique*, of which Hippolyte Carnot was then the editor.

The extant scientific work of Sadi Carnot includes three major items: the "Recherche" manuscript, the *Réflexions*, and the manuscript notes. Fragments of mathematics, his translations of two of Watt's papers, and lecture notes from various mathematics and physics courses have also survived.

The earliest of the major works is a twenty-one-page manuscript written probably in 1823 and entitled "Recherche d'une formule propre à represénter la puissance motrice de la vapeur d'eau."[3] As the title indicates, the paper was an attempt to find a mathematical expression for the motive power (work, in modern terms) produced by one kilogram of steam. Explicitly seeking a general solution covering all types of steam engines, Carnot reduced their operation to three basic stages: an isothermal expansion as the steam is introduced into the cylinder, an adiabatic expansion, and an isothermal compression in the condenser. Although Carnot claimed the cycle was complete, this was true only with respect to the motion of the piston and not to the working substance, which was not returned to the temperature of the boiler. Employing Clément's law for saturated vapors and devising an approximation function for Dalton's table of vapor pressures, Carnot neatly derived the motive power as a function of the initial and final temperatures and pressures of the steam. Since Dalton's table relates pressure to temperature, Carnot in effect expressed the motive power of a unit quantity of steam as a function of temperature alone.

The essày, both in methods and in objectives, is similar to the many papers published between 1818 and 1824 by such "power engineers" as Hachette, Navier, Petit, and Combes. Carnot's work, however, is distinguished for his careful, clear analysis of the units and concepts employed and for his use of both an adiabatic working stage and an isothermal stage in which work is consumed. The polished nature of the paper, in contrast with his rough notes, makes it appear intended for publication, although it remained unknown, in manuscript, until 1966.

The *Réflexions*, the only work that Carnot published in his lifetime, appeared in 1824 as a modestly priced essay of only 118 pages. After a concise review of the industrial, political, and economic importance of the steam engine, Carnot raised two problems that he felt prevented further development of both the utility and the theory of steam engines. Does there exist an assignable limit to the motive power of heat, and hence to the improvement of steam engines? Are there agents preferable to steam in producing this motive power? As Carnot conceived it, the *Réflexions* was nothing more, nor less, than a "deliberate examination" of these questions. Both were timely problems and, although French engineers had investigated them for a decade, no generally accepted solutions had been reached. In the absence of a clear concept of efficiency, proposed steam-engine designs were judged largely on practicality, safety, and fuel economy.[4] Air, carbonic acid, and alcohol had all been advocated by some engineers as a better working substance than steam. The usual approach to these problems was either an empirical study of the fuel input and the work output of individual engines or the application of the mathematical theory of gases to the abstract operations of a specific type of engine. In his choice of problems Carnot was firmly in this engineering tradition; his method of attacking them, however, was radically new and is the essence of his contribution to the science of heat.

Previous work on steam engines, as Carnot saw it, had failed for want of a sufficiently general theory, applicable to all imaginable heat engines and based on established principles. As the foundations for his study Carnot carefully set out three premises. The first was the impossibility of perpetual motion, a principle that had long been assumed in mechanics and had recently played an important role in the work of Lazare Carnot. As his second premise Carnot used the caloric theory of heat, which, in spite of some opposition, was the most accepted and most developed theory of heat available. In the *Réflexions*, heat (*calorique*) was always treated as a weightless fluid that could neither be created nor be destroyed in any process.[5] As an element in Carnot's demonstrations this assumption asserted that the quantity of heat absorbed or released by a body in any process depends only on the initial and final states of the body. The final premise was that motive power can be produced whenever there exists a temperature difference. The production of motive power was due "not to an actual consumption of caloric, but to its transportation from a warm body to a cold body." [6] Making the analogy with a waterwheel, Carnot observed that this motive power must depend on both the amount of caloric employed and the size of the temperature interval through which it falls. In his concept of reversibility Carnot also implicitly assumed the converse of this premise, that the expenditure of motive power will return caloric from the cold body to the warm body.

The analysis of heat engines began where the "Recherche" ended, with an abstract, three-stage steam-engine cycle. The incompleteness of this cycle proved troublesome, and Carnot pushed the abstraction one step further, producing the ideal heat engine and the cycle that now bear his name. The "Carnot engine" consisted simply of a cylinder and piston, a working substance that he assumed to be a perfect gas, and two heat reservoirs maintained at different temperatures. The new cycle incorporated the isothermal and adiabatic expansions and the isothermal compression of the steam engine, but Carnot added a final adiabatic compression in which motive power was consumed to heat the gas to its original, boiler temperature. In describing the engine's properties, Carnot introduced two fundamental thermodynamic concepts, completeness and reversibility. At the end of each cycle the engine and the working substance returned to their original conditions. This complete cycle not only provided an unambiguous definition of the input and output of the engine, but also rendered superfluous the detailed examination of each stage of the cycle. With each cycle the engine transferred a certain quantity of caloric from the high-temperature reservoir to the low-temperature reservoir and thereby produced a certain amount of motive power. Since each stage of the cycle could be reversed, the entire engine was reversible. Running backward, the engine consumed as much motive power as it produced running forward and returned an equal amount of caloric to the high-temperature reservoir. Joined together but operating in opposite directions, two engines would therefore produce no net effect.

Carnot then postulated the existence of an engine that, by virtue of design or working substance, would produce more motive power than a "Carnot engine" operating over the same temperature interval and with the same amount of caloric. A reversed "Carnot engine" would be able to return to the boiler all of the caloric transported to the condenser by the hypothetical engine. Yet the reversed "Carnot engine" would consume only a portion of the motive power produced by the hypothetical engine, leaving the remainder available for external work. Together these two engines would form a larger engine whose only net effect was the production of motive power in unlimited quantities. Since such a perpetual motion machine violated his first premise, Carnot concluded that no engine whatsoever produced more motive power than a "Carnot engine." Formulating the result now known as "Carnot's theorem", he stated that "the motive power of heat is independent of the agents employed to realize it; its quantity is fixed solely by the temperatures of the bodies between which is effected, finally, the transfer of caloric."[7]

To elucidate further the motive power of heat, Carnot turned his attention to the physical properties of gases, a subject in which there had been considerable activity for over a decade. By 1823 a sizable body of experimental data on adiabatic and isothermal processes and on specific heats had been assimilated into the caloric theory of heat and mathematized by Laplace and Poisson. Combining the results of this activity with the concepts involved in his fundamental theorem, Carnot derived a series of seven theorems. With the exception of a long footnote in which he attempted to cast his results in algebraic form, Carnot developed his theorems in a synthetic, geometric manner that, although clear and logically rigorous, was in sharp contrast with the mathematical analysis dominant in the scientific community. Nonetheless, at least three of the theorems represented major advances.[8] The first, that the quantity of heat absorbed or released in isothermal changes is the same for all gases, was experimentally established by Dulong in 1828, but without any reference to Carnot. In a very subtle verbal argument, Carnot also demonstrated that "the difference between specific heat under constant pressure and specific heat under constant volume is the same for all gases."[9] The final theorem proved that the fall of caloric produces more motive power when the temperature interval is located lower rather than higher on the temperature scale. Although aware of the uncertainties introduced by some assumptions and experimental data for specific heat changes, Carnot was able to calculate motive power values and to verify the theorem. The works of Clapeyron and William Thomson were in part motivated by the desire to derive an algebraic expression ("Carnot's function") for this theorem and to verify it with more accurate data.

In the final section of the *Réflexions*, Carnot returned to his original questions on steam engines. With experimental data taken from the current literature he verified that all gases produce the same amount of motive power and was able to estimate the ideal limit for its production. In a review of the most common types of steam engines, Carnot sought to apply his findings to the practical questions of steam-engine design and operation. His contributions, however, fell short of his original goal. His conclusions—that steam ought to be used expansively (adiabatically), over a large temperature interval, and without conduction losses—were already widely recognized by engineers of his time. Because of difficulties in engine construction even the problem of the best working substance was not conclusively answered.

Although the *Réflexions* was regarded by contemporaries as primarily an essay on steam engines, Carnot's most important innovations lay in a new approach to the study of heat. While he accepted, and in some theorems furthered, the theory of heat developed by Laplace and Poisson, Carnot also shifted the emphasis from the microscopic to the macroscopic. Rather than build upon the notion of gas particles surrounded by atmospheres of caloric, he began with the directly measurable entities of volume, pressure, temperature, and work.

Of his concepts of an ideal engine, completeness, and reversibility, there were some vague anticipations. The notion of an abstract heat engine was approximated in the work of Hachette and was more clearly present in the studies by Cagniard de La Tour and Clément of the motive power produced by a bubble of gas rising adiabatically in water. Jacob Perkins's team engine, widely discussed in 1823, represented an attempt to design a closed, complete system, and engineers were aware that certain types of hydraulic engines were reversible. The most striking parallels, however, are found in two ideas stressed by Lazare Carnot: the concept of geometrical (reversible) motions in mechanical machines and the necessity of computing the work done by a machine only after a complete cycle of operation. In addition, the style of Sadi Carnot's work, the synthetic, abstract generality that made his work distinctively different from that of both engineers and physicists, was in large measure due to the example set by the scientific work of Carnot *père*. Thus, in applying the concepts of ideal engines, completeness, and reversibility to the study of heat, Sadi Carnot gave them an unprecedented precision and generality and placed them in a highly original and fruitful combination.

Although the exact reasons are impossible to determine, the *Réflexions* had almost no influence on contemporary science. The original edition had not sold out by 1835; by 1845 booksellers had forgotten it completely. Aside from the reviews in 1824 and the references in obituaries, Carnot's work was mentioned only twice between 1824 and 1834. Clément recommended the book in his 1824–1825 lectures and Poncelet, writing sometime before 1830, cited it in his *Introduction à la mécanique industrielle* (Paris, 1839). In Carnot's obituary Robelin attributed the neglect of the *Réflexions* to its difficulty, an explanation that would have applied only to engineers and craftsmen unfamiliar with contemporary physics and mathematics. Another explanation points to the failure of the *Réflexions* to reach conclusions of real value to steam engineers. The silence on the part of physicists like Dulong, who later retraced portions of Carnot's work, is more difficult to explain. One probable factor, however, was Carnot's use of the caloric theory and of experimental results such as Clément's law of saturated vapors. His work was thus especially vulnerable, as he realized himself, when Clément's law was disproved in 1827 and when problems of radiant heat initiated a period of "agnosticism" concerning the nature of heat.

In 1834 Clapeyron, with whom Carnot may have been acquainted in 1832, published an analytical reformulation of the *Réflexions*. While Clapeyron preserved the premises, the theorems, and some of the specific arguments, the emphasis and style were considerably altered. He related the Carnot cycle to the pressure-volume indicator diagram and, emphasizing Carnot's function, translated Carnot's synthetic work from the world of heat engines to the realm of the mathematical theory of gases. Carnot's work attracted no further attention until C. H. A. Holtzman in 1845 and William Thomson in 1848 began working on special aspects of Clapeyron's paper. Between 1848 and 1850 Thomson, working directly from the *Réflexions*, published a series of papers that both extended and confirmed Carnot's results. These papers constituted a strong defense for Carnot's work, including his use of the caloric theory, at a time when Joule, Julius Mayer, and Helmholtz were establishing the convertibility of heat and work and the principle of energy conservation. In 1850 Clausius showed that Carnot's theorem was correct as stated but that Carnot's proof, which assumed no heat was lost, needed modification. Clausius added the statements that in the Carnot engine a certain quantity of heat is destroyed, another quantity is transferred to the colder body, and both quantities stand in a definite relation to the work done. With these additions, which Thomson also adopted in 1851, Carnot's theorem became the second law of thermodynamics.

The third major item of Carnot's scientific work is a bundle of twenty-three loose sheets of manuscript notes that escaped destruction after his death. Rough and disjointed, they contain notes from journal articles, outlines of experiments, and drafts of research results. Most concern one of three issues: adiabatic processes, the source of heat generated by friction, and the nature of radiant heat. From the development of these themes and some internal evidence the notes have been chronologically ordered, and many appear to have been written between 1824 and 1826. Several passages in the *Réflexions* indicate that Carnot had some serious reservations about the validity of the caloric theory. In the notes he sharpened these doubts and gathered relevant evidence. The difficulty of explaining certain adiabatic phenomena, Fresnel's vibrational theory of light, and the widespread speculation on the similarity of light and radiant heat all appear in the early notes. Carnot accepted Fresnel's theory and employed it against the caloric theory with the argument that motion (radiant heat) could never produce matter (caloric). Shortly after this he conducted an extensive literature search in which the works of Rumford and Davy figure prominently. Also among these notes are plans for experiments to test the temperature effects of friction in liquids and gases, several of which are nearly identical with those Joule performed almost twenty years later.

Finally convinced that "heat is nothing else than motive power, or rather motion which has changed its form,"[10] Carnot began to work out the details of a kinetic theory. He was aware that this new theory of heat negated the arguments of the *Réflexions*, but the notes contain no specific attempts to reformulate his earlier work. Although his procedure is missing from the notes, he calculated a conversion coefficient for heat and work and went on to assert that the total quantity of motive power in the universe was constant. These notes anticipated nearly all of the groundwork for the first law of thermodynamics. They remained undiscovered and unpublished until 1878, however.

NOTES

1. Two of his later military papers have survived in MS: "Route de Coulommiers à Couilly" (13 pp., dated 25 Oct. 1824) and "Essai sur I'artillerie de campagne" (82 pp., dated Mar. 1828).

2. A. Fourcy, *Histoire de l'École polytechnique* (Paris, 1828), p. 445.

3. Gabbey and Herivel give the period 1816–1824 as the date of composition. The lower bound is set by Carnot's citation of Biot's *Traitéde physique* (Paris, 1816), and the incomplete cycle and the lack of any explicit comparison between heat employed and work produced would suggest a date prior to the *Réflexions*. A reference to Clément as a professor at the Conservatoire des Arts et Métiers, however, indicates the "Recherche" was written sometime after 1819. Carnot's use of "dynames" as the unit of motive power (one kilogram raised one meter) suggests 1823 as the most probable date of composition, since Dupin coined that term in a report to the Academy in Apr. 1823.

4. Since heat and work were regarded as separate entities, there was no way to measure engine output in terms of input and, hence, no indication of what 100 percent efficiency would be. Carnot's search for a limit to the motive power of heat was essentially an attempt to establish such a criterion.

5. Carnot's use of the two words "chaleur" and "calorique" has led to several misinter pretations of his work. In 1910 H. L. Callendar, in the article "Heat" for the 11th and subsequent eds. of the *Encyclopaedia Britannica*, denied that Carnot regarded heat as a substance conserved in all processes. A more extreme position was advanced by V. K. La Mer (*American Journal of Physics*, **22** [1954], 20–27), in which he argued that by "calorique" Carnot meant "entropy" and by "chaleur", "heat." This interpretation, however, has no historical foundation. Several theorems in the *Réflexions* clearly used "calorique" as a material fluid that is conserved, and Carnot explicitly stated that "quantitéde chaleur" and "quantité de calorique" were interchangeable expressions. Cf. T. S. Kuhn, "Carnot's Version of 'Carnot's Cycle,'" in *American Journal of Physics*, **23** (1955), 91–95.

6.*Reflections*, E. Mendoza, ed., p. 7.

7.*Ibid*., p. 20.

8. In their original form all but one of Carnot's theorems are regarded as valid today. The incorrect theorem, in modern notation, asserts that $c = c_0 + k \log v/v_0$ where c is the specific heat of a gas at constant volume. This increase in specific heat with expansion was universally accepted on the basis of experiments performed in 1812 by Bérard and Delaroche.

9.*Reflections*, E. Mendoza, ed., p. 24.

10.*Sadi Carnot, biographie et manuscrit*, p. 81.

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II. Secondary Literature. The main biographical source is Hippolyte Carnot, "Notice biographique sur Sadi Carnot", first published in 1878 (see above). This is an expanded version of a letter written by Hippolyte's son, also named Sadi, and published by the Conte di St. Robert in "Notice biographique sur Sadi Carnot", in *Atti delta R. Accademia delle scienze di Torino*, **4** (1868), 151–170. Other sources are Hippolyte Carnot, *Mémoires sur Carnot par son fils*(Paris, 1863); a note by Michel Chasles in *Comptes rendusde l'Académie des sciences*, **68** (1869), 115–117: and the obituary by Robelin, "Notice sur Sadi Carnot", in *Revue encyclopédique*, **55** (1832), 528–530.

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Also of interest is Robert Fox, "Watt's Expansive Principle in the Work of Sadi Carnot and Nicholas Clément", in *Notes and Records of the Royal Society*, **24** (Apr. 1970), 233–243, which, however, appeared too late to be utilized in writing this biography.