Castigliano, (Carlo) Alberto | Encyclopedia.com

(*b*. Asti, Italy, 9 November 1847; *d*. Milan, Italy, 25 October 1884),

structural engineering.

During the second half of the nineteenth century there flourished in Italy a large group of structural engineers and elasticians in great measure responsible for establishing and popularizing the various methods of structural analysis based on the concepts of energy and work. This group included men of a variety of callings, and the following list of the principal names is remarkable as much for the versatility of the individuals it contains as for the evidence it presents of the intellectual and scientific vigor of the times: Alessandro Dorna (1825–1866, engineer and astronomer), Luigi Menabrea (1809–1896, general and statesman), Emilio Francesco Sabbia (1838–1914, general), Angelo Genocchi (1817–1889, mathematician), Enrico Betti (1823–1892, mathematician and engineer), Vincenzo Cerruti (1850–1909, engineer and mathematician), Francesco Crotti (1839–1896, engineer), Luigi Donati (1846–1932, physicist), and, of course, Castigliano. Beyond this point the list would not be useful in illustrating the background of Castigliano's world, partly because of his untimely death and partly because of the larger variety of problems which engaged the attention of the later Italian elasticians.

Castigliano left his native Piedmont at the age of nineteen to teach mechanics and machine design at the Technical Institute of Terni in Umbria for four years, returning only in 1870 to the Polytechnic of Turin. As a student there he began his work on the theory of structures, leading to his first publication, that of his celebrated dissertation, in 1873, He then found employment with the Northern Italian Railroads and soon became chief of the office responsible for artwork, maintenance, and service. He maintained that position until his death, all the while continuing to study and write.

The principal contributions of Castigliano are the two theorems known by his name. The first of these, contained in his dissertation, *Intorno ai sistemi elastici*, states that the partial derivative of the strain energy (*lavoro molecolare*), considered as a function of the applied forces (or moments) acting on a linearly elastic structure, with respect to one of these forces (or moments), is equal to the displacement (or rotation)in the direction of the force (or moment) of its point of application. He included the case of external reactions, not prescribed, noting that when the support corresponding to these reactions is unyielding, the partial derivative is zero and that his theorem then reduces to Menabrea's "principle of least work." In 1875 he published his second theorem, in which the strain energy is considered a function of the unprescribed displacements of discrete boundary points; its derivative with respect to one of these displacements gives the corresponding force acting there.

Earlier milestones in the history of energy principles of this type were the proof by Clapeyron, in 1827, of the principle of the conservation of work, equating the work performed by the applied external forces with the internal work performed by the stresses; Menabrea's development of his principle of least work; and Cotterill's independent proof (unknown to Castigliano) of Castigliano's theorems. It is clear that Menabrea's principle may be considered to be included in Castigliano's theorems; furthermore, Menabrea's proofs were not satisfactory and were in fact repeatedly modified by him as a result of considerable criticism. A new demonstration of Menabrea's principle was given by him in 1875 on the basis of some of the newly published results of Castigliano, who, however, was referred to only in a footnote. Castigliano strongly objected to this lack of sufficient recognition in a letter full of youthful indignation to the president of the Accademia dei Lincei. Menabrea replied in the reasoned and somewhat condescending tones befitting an elder statesman, pointing out the priority of his work. The mathematician and engineer Luigi Cremona, acting as chairman of a meeting of the Academy, gave a solomonic judgment on the controversy, stating that

he believes that Mr. Castigliano's complaint is not sufficiently well founded: the theorem in question precedes the work of both authors, and the proofs do not seem free of every objection. It is thus his opinion that there is no matter for dispute, and concludes: Mr. Castigliano can have the honor of having done a good piece of work: no one will be able to take away from Member Menabrea the merit of having made popular and of common use a general principle, which is certainly destined to receive ever more extensive application.

The entire correspondence and reply are found in *Atti della Reale Accademia Nazionale dei Lincei* (2nd ser., 2[1874/75], 59). The controversy, while not subsiding completely, then lost most of its virulence. The principal continuations of Castigliano's work were Crotti's extension to nonlinear elastic systems (principle of complementary energy) and Donati's work on the mathematical and conceptual basis of energy methods.

Among Castigliano's minor contributions, one might mention an engineer's handbook; works on the theory of leaf and torsion springs (published in a book in Vienna. 1884), on masonry arches, and on water hammers; and the invention of an extensometer.

Castigliano's principal work, while not free of conceptual shortcomings, represented a definite advance over that of his predecessors. To assess the importance of his contribution, however, it is important to note that, although there is some validity in Cremona's attribution of the popularization of energy methods to Menabrea, it is precisely in this respect that Castigliano excels. He solved an amazing number of important structural problems by his methods, pointing out by comparisons with previously known *ad hoc* solutions both their superiority and their correctness and establishing once and for all their convenience and versatility. As he states in the preface to his *Théorie de l'équilibre des systèmes élastiques, et sesapplications*, this was indeed one of his explicit goals, and the success with which he achieved it is remarkable both because of his short career (he died at thirty-seven) and his lack of formal academic or other strong "establishment" ties.

BIBLIOGRAPHY

I. Original Works. Castigliano's principal works are his diss., *Intorno ai sistemi elastici* (Turin, 1873), and the book *Théorie de l'équilibre des systèmes élastiques, et ses applications* (Turin, 1879). The diss, arid the main part of the book were repr. on the fiftieth anniversary of his death as *Alberto Castigliano. Selecta*, G. Colonnetti, ed. The book itself was published in English trans, by E. S. Andrews as *Elastic Stresses in Structures* (London 1919). Mention should also be made of three papers by Castigliano, all in the *Atti della Reale Accademia della scienze* (Turin), 2nd ser.; "Intorno all'equilibrio dei sistemi elastici," in **10** (1875), 10; "Nuova teoria intorno all'equilibrio dei sistemi elastici," in **11** (1875), 127; and "Intorno ad una proprietà dei sistemi elastici," in **17** (1882), 705.

II. Secondary Literature. Derivations and applications of Castigliano's theorems may be found in any standard text on strength of materials or theory of structures. It should be noted in this connection that the references to the first and second theorems given in the text correspond to the most usual modern practice but are opposite to those employed by Castigliano. Futhermore, Menabrea's principle of least work is occasionally referred to as Castigliano's second theorem. A comprehensive historical review of the *Historical Development of Energetical Principles in Elastomechanics* was given by G. Ae. Orawas and L. McLean in *Applied Mechanics Reviews*, **19** , no. 8 (Aug. 1966), 647–658, and no. 11 (Nov. 1966), 919–933. Clapeyron's proof of the principle of the conservation of work is in his "Mémoire sur le travail des forces élastiques dans un corps solideélastique déformé par l'action des forces extérieures," in *Comptes rendus de l'Académie des sciences*, **46** (1827), 208. A eulogy of Castigliano was given by F. Crotti, "Commemorazione di Alberto Castigliano," in *Politecnico*, **32** , nos. 11/12 (Nov./Dec. 1884), 597.

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