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(*b*. Milan, Italy, probably 1598; *d*. Bologna, Italy, 30 November 1647). *mathematics*.

Cavalieri's date of birth is uncertain; the dale given above is the one cited by Urbano d'Aviso, a disciple and biographer of Cavalieri. The name Bonaventura was not his baptismal name but rather that of his father. It is the name the mathematician adopted when, as a boy, he entered the Jesuati religious order, adherents of the rule of St. Augustine. Cavalieri was received in the minor orders in Milan in 1615 and in 1616 transferred to the Jesuati monastery in Pisa, where he had the good fortune of meeting the Benedictine monk Benedetto Castelli, who had studied with Galileo at Padua and was at the time a lecturer in mathematics at Pisa. Through him Cavalieri was initiated into the study of geometry. He quickly absorbed the classical works of Euclid, Archimedes. Apollonius, and Pappus, demonstrating such exceptional aptitude that he sometimes substituted for his teacher at the University of Pisa. He was introduced by Castelli to Galileo, whose disciple he always considered himself. He wrote Galileo at least 112 letters, which are included in the national edition of the *Opere di Galileo;* only two of Galileo's letters to Cavalieri have come down to us, however.

In 1620 Cavalieri returned to Rome under orders of his superiors, and in 1621 he was ordained a deacon to Cardinal Federigo Borromeo, who held Fra' Bonaventura in great esteem and gladly discussed mathematics with him; the cardinal subsequently wrote a letter commending him to Galileo. Cavalieri was hardly twenty-one when he taught theology at the monastery of San Girolamo in Milan, attracting attention by his profound knowledge of the subject.

During his Milan period (1620–1623) Cavalieri developed his first ideas on the method of indivisibles, his major contribution to mathematics. From 1623 to 1626 he was the prior of St. Peter's at Lodi. Later he was a guest in Rome of Monsignor Ciampoli, to whom he later dedicated his *Geometria*. From 1626 to 16291 he was the prior of the monastery of the Jesuati in Parma, hoping in vain to be appointed lecturer in mathematics at the university there. In the autumn of 1626, during a trip from Parma to Milan, he fell ill with the gout, from which he had suffered since childhood and which was to plague him to the end of his life. This illness kept him at Milan for a number of months. On 16 December 1627 he announced to Galileo and Cardinal Borromeo that he had completed his *Geometria*. In 1628, learning that a post of lecturer at Bologna had become vacant through the death of the astronomer G. A. Magini, he wrote Galileo for assistance in securing the appointment. Galileo, in 1629, wrote to Cesare Marsili, a gentleman of Bologna and member of the Accademia dei Lincei, who had been commissioned to find a new lecturer in mathematics. In his letter, Galileo said of Cavalieri, "few, if any, since Archimedes, have delved as far and as deep into the science of geometry." In support of his application to the Bologna position, Cavalieri sent Marsili his geometry manuscript and a small treatise on conic sections and their applications in optics. Galileo's testimonial, as Marsili wrote him. Induced the "Gentlemen of the Regiment" to entrust the first chair in mathematics to Cavalieri, who held it continuously from 1629 to his death.

At the same time he was appointed prior of a convent of his own order in Bologna, specifically, at the Church of Santa Maria della Mascarella, enabling him to pursue without any impediment both his work in mathematics and his university teaching. During the period in which Cavalieri taught at Bologna, he published eleven books in that city, including the *Geometria* (1635).

Cavalieri's theory, as developed in this work and in others subsequently published, relates to an inquiry in infinitesimals, stemming from revived interest in Archimedes' works, which during the Renaissance were translated from Greek into Latin, with commentaries. The translations of Tartaglia, Maurolico, and Commandino are cited since they served as a point of departure for new mathematical developments.

The only writings of Archimedes known to seventeenth-century mathematicians were those based upon the strict method of exhaustion, by which the ancient mathematicians dealt with questions of an infinitesimal character without recourse to the infinite or to the actual infinitesimal. Nevertheless, the great mathematicians of the seventeeth century were so thoroughly pervaded with the spirit of Archimedes as to appreciate that in addition to the "method of exhaustion" the ancient geometricians must have known a more manageable and effective method for research. On this point Torricelli wrote:

I should not dare affirm that this geometry of indivisibles is actually a new discovery. I should rather believe that the ancient geometricians availed themselves of this method in order to discover the more difficult theorems, although in their demonstration they may have preferred another way, either to conceal the secret of their art or to afford no occasion for criticism by invidious detractors. Whatever it was, it is certain that this geometry represents a marvelous economy of labor in the demonstrations and establishes innumerable, almost inscrutable, theorems by means of brief, direct, and affirmative demonstrations, which the doctrine of the ancients was incapable of. The geometry of indivisibles was indeed, in the mathematical briar bush, the so-called royal road, and one that Cavalieri first opened and laid out for the public as a device of marvelous invention [*Opere*, I, pt. I. 139–140].

In 1906 J. L. Heiberg found, in a palimpsest belonging to a Constantinople library, a small work by Archimedes in the form of a letter to Eratosthenes, which explained a method by which are as, volumes, and centers of gravity could be determined. This method, which in turn was related to the procedures of Democritus of Abdera, considered a plane surface as made up of chords parallel to a given straight line, and solids as made up of plane sections parallel to one another. In addition, according to Archimedes, principles of statics were applied, where by the figures, thought of as heavy bodies, were weighed in an ideal scale. "I do believe," said Archimedes, "that men of my time and of the future, and through this method, might find still other theorems which have not yet come to my mind" (Rufini, *II "Metodo" di Archimedee le origini del calcolo infinitesimale nell'antichità* [Milan, 1961], p. 103). The challenge that Archimedes extended was not taken up, as we know, by his contemporaries and fell into oblivion for many centuries.

The concept of indivisibles does sometimes show up fleetingly in the history of human thought: for example, in a passage by the eleventh-century Hebrew philosopher and mathematician Abraham bar Hiyya (Savasorda); in occasional speculations—more philosophical than mathematical—by the medieval Scholastics; in a passage by Leonardo da Vinci; in Kepler's *Nova stereometria doliorum* (Linz, 1615). By a conception differing from Cavalieri's, indivisibles are treated by Galileo in his *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*.

In Cavalieri we come to a rational systematization of the method of indivisibles, a method that not only is deemed useful in the search for new results but also, contrary to what Archimedes assumed, is regarded as valid, when appropriately modified, for purposes of demonstrating theorems.

At this point a primary question arises: What significance did Cavalieri attribute to his indivisibles? This mathematician, while perfectly familiar with the subtle philosophical questions connected with the problem of the possibility of constituting continuous magnitudes by indivisibles, seeks to establish a method independent of the subject's hypotheses, which would be valid whatever the concept formed in this regard. While Galileo asserted, "The highest and the ultimate, although primary components of the continuous, are infinite indivisibles" (*Opere*, VII, 745–750),

Cavalieri did not dare to assert that the continuous is composed of indivisible elements, about which he did not give an explicit definition, nor did he clarify whether they were actual or potential infinitesimals. It is also probable that Cavalieri's conception of his indivisibles underwent a change and that these were born as actual infinitesimals (like those of Galileo) and grew to become potential infinitesimals (see G. Cellini). It must be further pointed out, according to L. Lombardo Radice, that the Cavalieri view of the indivisibles has given us a deeper conception of the sets: it is not necessary that the elements of the set be assigned or assignable; rather it suffices that a precise criterion exist for determining whether or not an element belongs to the set.

Quite aside from any philosophical considerations on the nature of indivisibles, the determinations of area and volumes made by Cavalieri are based on the principle bearing his name, which may be formulated as follows:

If two plane figures cut by a set of parallel straight lines intersect, on each of these straight lines, equal chords, the two figures are equivalent; if the chords pertaining to a single straight line of the set have a constant ratio, the same ratio obtains between the two figures.

Similarly, in space: if the sections of two solids obtained by means of planes that are parallel to each other are equivalent two by two, the two solids are equivalent; if the two sections obtained with a given plane have a constant ratio when the plane is varied, the two solids have a ratio that is equal to that of two of their sections obtained with one same plane.

From the viewpoint of modern infinitesimal analysis, the Cavalieri principle affirms in substance that two integrals are equal if the integrands are equal and the integration limits are also equal. Furthermore, a constant that appears as a multiplier in the integrand may be carried out of the sign of integration without causing the value of the integral to vary.

However, the concept of the integral, according to the definition of A. Cauchy, was not precisely in the mathematical thought of Cavalieri, but rather was looked into by P. Mengoli, his disciple and successor in the chair at Bologna. Cavalieri pursued many paths to demonstrate his principle, and they are to be found in Book VII of his *Geometry*.

Let us consider the case in plane geometry, where, on the hypotheses of the stated principle, the corresponding chords of the given figures are equal in pairs (see Fig. 1). Cavalieri then, through a translation in the direction of the parallel straight lines in question, superimposes two equal chords. The parts of the figure which thus are superimposed are therefore equivalent or, rather, equal, because they are congruent. The remaining parts, or residuals, which are not superimposed, will still satisfy the conditions relative to the chords that were satisfied in the original figure. In this way, one can proceed with successive superpositions by translation, and it is impossible at a given point in the successive operations that one figure be exhausted unless the other is also. Cavalieri concludes that the given figures are therefore equivalent. The argument is ingenious and intuitive, but it contains a weak point in that it is not proved that the residuals, in the described operations,become exhausted ;nor is it established that the sum of such residuals can be made less than a given surface. Nevertheless,Cavalieri,in replying to the objections raised by Guldin, claims that elimination of the residuals in one of the figures,hence in the other, can be performed by means of infinite operations. The other demonstration

of the Cavalieri principle is made by the ancients' method of exhaustion and is a rigorous one for the figures that satisfy certain conditions: that is, the demonstration is valid for figures which, in addition to satisfying the hypothesis of the principle, fall into one of the following classes:

(1) Generalized parallelograms, namely, figures included between straight parallel lines *p* and *l* which intersect chords of constant length on straight lines running in the same direction as *p* and *l* (see Fig. 2).

(2)The *figurae in alteram partem deficientes* ("figures deficient in another part") are included between two parallel lines *p* and *l* and, in addition, the chords intercepted by a transverse line parallel to *p* diminish as the distance of the transverse from straight line *p* increases (see Fig. 3).

(3) Figures capable of being broken down into a finite number of parts belonging to either of the aforementioned two classes (see Fig. 4).

Notwithstanding the demonstrations mentioned and the success of the method of indivisibles, contemporary mathematicians, who were more attached to the traditions of classical mathematics, entered into a polemic with Cavalieri, unaware that Archimedes himself had already used methods similar to those that they were opposing. Such is the case of Guldin, who had an interesting discussion with Cavalieri that is summed up in exercise III of the *Exercitationes geomeiricae sex*.

Many results that were laboriously obtained by the method of exhaustion were obtained simply and rapidly through the Cavalieri principle: for example, the area of an ellipse and the volume of a sphere. Through his methods, Cavalieri had found the result which in today's symbols would be expressed as:

for any natural number *n* (*n* = 1,2,3,...). Cavalieri was unaware that this result, which appears in the *Centuria di varii problemi* (1639), had already been found as early as 1636 by Fermat and Roberval, who had arrived at it by other means.

By means of the method of indivisibles and based upon a lemma established by his pupil G. A. Rocca, Cavalieri proved the Guldin theorem on the area of a surface and the volume of rotating solids. This theorem, which also appears in certain editions of Pappus' works, although held to be an interpolation, was enunciated in the *Centrobaryca* of Guldin, who proved its correctness in certain particular cases, without, however, providing the general proof.

The most significant progress in the field of infinitesimal analysis along the lines set forth by Cavalieri was made by Evangelista Torricelli. In his *Arithmetica infinitorum* (1655), John Wallis also makes use of indivisibles.

Especially interesting is the opinion of the Cavalieri method expressed by Pascal in his *Letires de Dettonville* (1658): "Everything that is demonstrated by the true rules of indivisibles will also and necessarily be demonstrated in the manner of the ancients. For which reason, in what follows, I shall not hesitate to use the very language of indivisibles." Although in the following years in the field of analysis of the infinitesimal, new ideas replaced the old on the indivisibles, the methods of Cavalieri and Torricelli exerted a profound influence, as Leibniz acknowledged in a letter to G. Manfredi: "... in the sublimest of geometry, the initiators and promotors who performed a yeoman's task in that field were Cavalieri and Torricelli; later, others progressed even further by availing themselves of the work of Cavalieri and Torricelli." Moreover, Newton, while assuming in his *Principia* a critical attitude in the matter of indivisibles, did nevertheless in his *Tractatus de quadratura curvarum*, use the term *fluens* to indicate a variable magnitude—a term previously used by Cavalieri in his *Exerciiationes geomeiricae sex*.

In proposition I of Book I of the *Geometria*, we find in geometric form the theorem of mean value, also known as the Cavalieri theorem. The theorem is presented as the solution of the following problem: Given a plane curve, provided with a tangent at every point and passing through two points *A* and *B*, to find a straight line parallel to *AB* and tangent to the curve at some point on the curve between *A* and *B*. Analytically we have: If the real function *f(x)* of the real variable *x* is continuous in the interval (*a*, *b*) and at every point within this interval it is differentiable, at least one point of exists such that $a< \xi <b$, so that

Logarithms were introduced into mathematics in the work of Napier in 1614. In Italy such valuable auxiliaries to numerical calculation were introduced by Cavalieri, together with noteworthy developments in trigonometry and applications to astronomy. In this connection we might mention *Directorium generale uranometricum* (1632), *Compendio delle regole dei triangoli* (1638), *Centuria di varii problemi* (1639), *Nuova pratica astrologica* (1639), and *Trigonometria plana, et sphaerica, linearis et logarithmica* (1643). The *Directorium*, the *Pratica*, and the *Trigonometria* contain, moreover, excellent logarithmictrig-onometric tables.

In the *Centuria*, Cavalieri dealt with such topics as the general definition of cylindrical and conical surfaces, formulas to determine the volume of a barrel and the capacity of a vault with pointed arches, and the means of obtaining from the logarithms of two numbers the logarithm of the sum or of the difference, a problem that was subsequently taken up by various mathematicians. Gauss among others. *Lo specchio ustorio* ("The Burning Glass") contains some interesting historical data on the origin of the theory of the conics among the Greeks; according to Cavalieri, the origins are to be found in the gnomonic requirements. In this work, we find a theory of conics with applications to optics and acoustics. Among the former, we note the idea of the reflecting telescope, of which—according to Piola and Favaro—Cavalieri was the first inventor, preceding Gregory and Newton; determination of the focal length of a lens of uneven sphericity and explications of the burning glass of Archimedes, In the field of acoustics, Cavalieri attempted the archaeological reconstruction of the resonant vases mentioned by Vitruvius and used in theaters for amplifying sound.

In this work, various pointwise constructions of conies appear. More interesting still are the constructions given in the *Geometria* and in the *Exercitationes*, obtained by means of projective pencils which antedated the work of Steiner.

A delicate question relates to the astrological activities that Cavalieri engaged in by virtue of his office, but, as pointed out by D'Aviso, he was opposed to predictions based upon the position of the stars and states so at the end of his *Pratica astrologica*.

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