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(*b*. Montjoie [now Monschau], near Aachen, Germany, 10 November 1829; *d*. Strasbourg, Frange [then Germany], 15 March 1900),

mathematics

Christoffel studied at the University of Berlin, where he received his doctorate in 1856 with a dissertation on the motion of electricity in homogeneous bodies. He continued his studies in Montjoie. In 1859 he became lecturer at the University of Berlin, in 1862 professor at the Polytechnicum in Zurich, and in 1869 professor at the Gewerbsakademie in Berlin. In 1872 he accepted the position of professor at the University of Strasbourg, newly founded after its acquisition by the Germans. Here he lectured until 1892, when his health began to deteriorate.

Christoffel has been praised not only as a very conscientious mathematician but also as a conscientious teacher. Politically he represented the traditional Prussian academician loyal to emperor and army. This may have contributed to his choice of Strasbourg and his endeavor to create a great German university in that city.

Scientifically, Christoffel was primarily a follower of Dirichlet, his teacher, and of Riemann, especially of the latter. Their ideas inspired his early publications (1867, 1870) on the conformal mapping of a simply connected area bounded by polygons on the area of a circle, as well as the paper of 1880 in which he showed algebraically that the number of linearly independent integrals of the first kind on a Riemann surface is equal to the genus p. The posthumous "Vollständige Theorie der Riemannschen θ-Function" also shows how, rethinking Riemann's work, Christoffel came to an independent approach characteristic of his own way of thinking. Also in the spirit of Riemann is Christoffel's paper of 1877 on the propagation of plane waves in media with a surface of discontinuity, an early contribution to the theory of shock waves.

Another interest of Christoffel's was the theory of invariants. After a first attempt in 1868, he succeeded in 1882 in giving necessary and sufficient conditions for two algebraic forms of order p in n variables to be equivalent. Christoffel transferred these investigations to the problem of the equivalence of two quadratic differential forms, again entering the Riemannian orbit. In what well may be his best-known paper, "Über die Transformation der homogenen Differentialausdrücke zweiten Grades," he introduced the three index symbols

now called Christoffel symbols of the first and second order, and a series of symbols of more than three indices, of which the four index symbols already introduced by Riemann, are now known as the Riemann-Christoffel symbols, or coordinates of the Riemann-Christoffel curvature tensor. The symbols of an order higher than four are obtained from those of a lower order by a process now known as covariant differentiation. Christoffel's reduction theorem states (in modern terminology) that the differential invariants of order $m \geq 2$ of a quadratic differential form

$$\Sigma a_{ij}(x)\ dx^i dx^j$$

are the projective invariants of the tensors a_{ij}, its Riemann-Christoffel tensor, and its covariant derivatives up to order $(m - 2)$. The results of this paper, together with two papers by R. Lipschitz, were later incorporated into the tensor calculus by G. Ricci and T. Levi-Cività.

Christoffel also contributed to the differential geometry of surfaces. In his "Allgemeine Theorie der geodätischen Dreiecke" he presented a trigonometry of triangles formed by geodesics on an arbitrary surface, using the concept of reduced length of a geodesic arc. When the linear element of the surface is $ds^2 = dr^2 + m^2 dx^2$, m is the reduced length of arc r. In this paper Christoffel already uses the symbols , but only for the case $n = 2$.

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II. Secondary Literature. Christoffel and his work are discussed in C. F. Geiser and L. Maurer, "E. B. Christoffel," in *Mathematische Annalen*, **54** (1901), 328–341; and W. Windelband, *ibid*., 341–344; there is a bibliography on 344–346. These articles are excerpted in *Gesammelte Abhandlungen*, I, v-xv. The papers by Libschitz that were incorporated into the tensor calculus are "Untersuchungen in Betreff der ganzen homogenen Functionen von *n* Differentialen," in *Journal für die reine and angewandte Mathematik* **70** (1869), 71–102; **72** (1870), 1–56; and "Entwickelung einiger Eigenschaften der quadratischen Formen von *n* Differentialen," *ibid*., **71** (1870), 274–287, 288–295. Beltrami's comment on Christoffel's Allgemeine Theorie der geodätischen Dreiecke" is in his *Opere matematiche*, 4 vols. (Milan, 1904), II, 63–73.

Mlle. L. Greiner of the Bibliothèque Nationale et Universitaire, Strasbourg, informs me that the *Handschriftlicher Nachlass* mentioned in the *Verzeichniss der hinterlassenen Büchersammlung des Herrn Dr. E. B. Christoffel* (n.p., 1900) is not in this library, contrary to what might be expected from a statement in the Geiser-Maurer article.

D. J. Struik