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(*b*. Königsberg, Germany [now Kaliningrad, U. S. S. R.], 19 January 1833; *d*. Göttingen, Germany, 7 November 1872),

*mathematics*.

In 1850 Clebsch entered the University of Königsberg, where the school of mathematics founded by Jacobi was then flourishing. His teachers included the mathematical physicist Franz Neumann and the mathematicians Friedrich Richelot and Ludwig Otto Hesse, both pupils of Jacobi. After graduation, in 1854, he went to Berlin, where he was taught under the direction of Karl Schellbach at various schools. Clebsch's academic career began in 1858, when he became *Privatdozent* at the University of Berlin. Soon afterward he moved to Karlsruhe, where he was a professor at the Technische Hochschule from 1858 to 1863. From 1863 to 1868 he was professor at the University of Giessen, collaborating with Paul Gordan. From 1868 until his death, he was professor at the University of Göttingen and in the forefront of contemporary German mathematics. In 1868 he and his friend Carl Neumann, son of Franz Neumann, founded the *Mathematische Annalen*.

Clebsch's doctoral dissertation at the University of Königsberg concerned a problem of hydrodynamics, and the main problems considered in the first period of his scientific career were in mathematical physics, especially hydrodynamics and the theory of elasticity. His book on elasticity (1862) may be regarded as marking the end of this period. In it he treated and extended problems of elastic vibrations of rods and plates. His interests concerned more the mathematical than the experimental side of the physical problems. He soon moved on to pure mathematics, where he achieved a dominant place.

Clebsch's first researches in pure mathematics were suggested by Jacobi's papers concerning problems in the theory of variation and of partial differential equations. He had not known Jacobi personally but collaborated in the edition of his *Gesammelte Werke*. For general problems in the calculus of variations, Clebsch calculated the second variation and promoted the integration theory of Pfaffian systems, surpassing results that Jacobi had obtained in these fields.

Although in these analytical papers Clebsch already proved himself to be highly skilled in calculus, his fame as a leader of contemporary scientists was first gained through his contributions to the theory of projective invariants and algebraic geometry. In the nineteenth century these fields were called modern geometry and modern algebra. The term "modern" or "new" geometry was applied to the projective geometry developed in synthetic form by Poncelet, Steiner, and Staudt, and in analytical form by Plücker and Hesse. Clebsch wrote a biographical sketch of Plücker, giving evidence of the author's deep insight into mathematical currents of the nineteenth century. In numerical geometry we still speak of the Plücker-Clebsch principle of the resolubility of several algebraic equations.

The name "Modern algebra" was applied to the algebra of invariants, founded by the English mathematicians Cayley, Salmon, and Sylvester. One of the first German contributors to this discipline was Aronhold. It was especially the papers of Aronhold that incited Clebsch to his own researches in the theory of invariants, or "algebra of quantics," as it was called by the English. The results in the theory of invariants are to be interpreted by geometric properties of algebraic curves, surfaces, and so on. This connection between algebra and geometry attracted Clebsch in a special way. He was soon a master of the difficult calculations with forms and determinants occurring in the theory of invariants. In this he surpassed his teacher Hesse, whose ability and elegance in analytical geometry were praised at the time.

Clebsch completed the symbolic calculus for forms and invariants created by Aronhold, and henceforth one spoke of the Clebsch-Aronhold symbolic notation. Clebsch's own contributions in this field of algebraic geometry include the following. With the help of suitable eliminations he determined a surface of order $11n - 24$ intersecting a given surface of order $n$ in points where there is a tangent that touches the surface at more than three coinciding points. For a given cubic surface he calculated the tenth-degree equation on the resolution of which the determination of the Sylvester pentahedron of that surface depends. For a plane quartic curve Clebsch found a remarkable invariant that, when it vanishes, makes it possible to write the curve equation as a sum of five fourth-degree powers. At the end of his life Clebsch inaugurated the notion of a "connex," a geometrical object in the plane obtained by setting a form containing both point and line coordinates equal to zero.

The general interest in the theory of invariants began to abate somewhat in 1890, when Hilbert succeeded in proving that the system of invariants for a given set of forms has a finite basis. In 1868 Gordan had already proved the precursory theorem on the finiteness of binary invariants. The theory of binary invariants thus being more complete, Clebsch published a book on this part of the theory (1872), giving a summary of the results obtained.

In the last year of his life Clebsch planned the publication of his lectures on geometry, perhaps to include those on *n* dimensions, results by no means as self-evident then as now. After Clebsch's death his pupil Karl Lindemann published two volumes of these lectures (1876–1891), completed with his additions but confined to plane and three-dimensional geometry. Between 1906 and 1932 a second edition of volume I, part I of this work appeared under the name of both Clebsch and Lindemann. The first volume contained almost all of the known material on plane algebraic curves and on Abelian integrals and invariants connected with them.

In 1863 Clebsch began his very productive collaboration with Gordan by inviting him to Göttingen. In 1866 they published a book on the theory of Abelian functions. Papers by Clebsch alone on the geometry of rational elliptic curves and the application of Abelian functions in geometry may be regarded as ancillary to the book. All these works are based on Riemann's fundamental paper (1857) on Abelian functions. In this celebrated work Riemann based the algebraic functions on the Abelian integrals defined on the corresponding Riemann surface, making essential use of topological ideas and of the so-called Dirichlet principle taken from potential theory.

Riemann's ideas were difficult for most contemporary mathematicians, Clebsch included. In the following years mathematicians sought gradually to eliminate the transcendental elements from the Riemann theory of algebraic functions and to establish the theory on pure algebraic geometry. Clebsch's papers were an essential step in this direction. The application of Abelian functions to geometry in his principal paper (1865) is to be understood as the resolution of contact problems by means of Abel's theorem, i.e., the determination of systems of curves or surfaces touching a curve in a plane or in space in given orders, such as the double tangents of a plane quartic curve.

Clebsch and Gordan's book had the following special features: (*a*) use of homogeneous coordinates for the points of an algebraic curve and for the Abelian integrals defined on it and (*b*) definition of the genus *p* for a plane algebraic curve of order *n* possessing as singularities only *d* double points and *s* cusps. They were the first to define the genus *p* by the expression

whereas Riemann had defined this expression as the topological genus of the corresponding Riemann surface. Also, as the title indicates, the transcendental point of view is prevalent in the book. It treats the Jacobian problem of inversion, introduces the theta functions, and so on. On the whole, to a modern reader a century later, the book may seem old fashioned; but it must be remembered that it appeared long before Weierstrass' more elegant lectures on the same object.

As successors to Clebsch there arose the German school of algebraic geometry, led by Brill and M. Noether, both regarded as his pupils. At the end of the nineteenth century, algebraic geometry moved to Italy, where particular attention was paid to the difficult theory of algebraic surfaces. But the beginnings of the theory of algebraic surfaces go back to Cayley, Clebsch, and Noether, for Clebsch described the plane representations of various rational surfaces, especially that of the general cubic surface. Clebsch must also be credited with the first birational invariant of an algebraic surface, the geometric genus that he introduced as the maximal number of double integrals of the first kind existing on it.

## BIBLIOGRAPHY

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