Condorcet, Marie-Jean-Antoine-Nicolas Caritat, Mearquis De | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
24-30 minutes

(*b*. Ribemont, France, 17 September 1743; *d*. Bourg-la-Reine, France, 27[?] March 1794),

mathematics, applied mathematics.

Condorcet's family came originally from the Midi. Although converted at the beginning of the Reformation, the Caritat family renounced Protestantism during the seventeenth century; most of the young men of the family led lives typical of provincial noblemen, becoming either clergymen or soldiers. One of Condorcet's uncles was the bishop of Auxerre and later of Lisieux; his father, Antoine, was a cavalry captain stationed in the tiny Picardy village of Ribemont when he married Mme. de Saint-Félix, a young widow of the local bourgeoisie. It was there that the future marquis was born a few days before his father was killed during the siege of Neuf-Brisach. Raised by an extremely pious mother and tutored in his studies by his uncle the bishop, he was sent to the Jesuits of Rheims and sub sequently to the Collège de Navarre in Paris. from which he graduated with a degree in philosophy in 1759, having written a thesis in Latin on mathematics;[1] d'Alembert was a member of the board of examiners.

Condorcet's mathematical ability asserted itself even though his family would have preferred that he pursue a military career. He took up residence in Paris, where he lived on a modest sum provided by his mother. He worked a great deal and became better acquainted with d'Alembert, who introduced him into the salons of Mlle. de Lespinasse and Mme. Helvétius; at the first salon he met Turgot, who subsequently became his close friend. In 1765 Condorcet published a work on integral calculus and followed it with various mathematical *mémoires* that earned him the reputation of a scientist. He was elected to the Academie des Sciences as *adjoint-mécanicien* (adjunct in mechanics), succeeding Bezout (1769), and later as an associate in the same section. He thereafter became closely involved with scientific life. As assistant secretary (1773) and then permanent secretary of the Académie des Sciences (1776), Condorcet published a great many mathematical *mémoires* and, in 1785, the voluminous *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*. Simultaneously he wrote his *eloges* of deceased academicians, essays that are often remarkable for his fairness of judgment and broadness of view; he also tried, but in vain, to organize scientific activity in France along rational lines.[2]

In the 1770's, however, another of Condorcet's interests came to the surface, possibly as a result of a meeting with Voltaire at Ferney. The notion of a political, economic, and social reform to be undertaken on scientific bases was its most recurrent subject. From it stemmed, in the theoretical domain, his calculus of probabilities and, in the practical domain, the numerous applied-research projects he had set up or carried out as inspector of the mint (1776) and director of navigation under the ministry of Turgot. To that end he also participated, beginning in 1775, in the movement of political and social dissent from the regime through the publication of various pamphlets. On the eve of the Revolution, Condorcet was inspector of the mint, permanent secretary of the Academie des Sciences, and a member of the Académie Française (1782); in 1786 he married Sophie de Grouchy, whose salon at the Hôtel des Monnaies (the mint) had taken the place of the salons of Mlle. de Lespinasse and Mme. Helvetius. In 1776 he was entrusted with the articles on analysis for Panckoucke's *Supplément to the Encyclopédie*, and in 1784 he revised and rewrote, with Lalande and Bossut, the mathematical part of that work, recast as the *Encyclopédie méthodique*. Thus he can be considered one of the most representative personages of the Enlightenment and, so far as France is concerned, one of the most influential.

After 1787 Condorcet's life is scarcely of interest to the historian of science. He then stood forth as an advocate of calling a national assembly that would reform the regime according to the views of the liberal bourgeoisie. After twice failing to be elected to the States-General (from Mantes-Gassicourt, where he had landholdings, and then from Paris), he was elected representative from his quarter of Paris to the Municipal Council, sitting in the Hôtel de Ville, and founded, in association with Emmanuel Sieyès, the Society of 1789. In September 1791 he succeeded in becoming a delegate to the Legislative Assembly and later to the Convention of 1792. In the Assembly he concentrated his efforts mainly on the work of the Commission of Public Instruction, for which he was suited by virtue of the *mémoires* that he had published in the *Bibliothèque de l'homme public* (1791–1792). At the convention he drew up the draft of a constitution (1793), but it was not adopted. A friend of Jacques Brissot and closely linked to the political battle waged by the Girondists, Condorcet came under suspicion following their expulsion from the convention on 2 June 1793; when the draft constitution that had been substituted for his own by the Committee of Public Safety was voted on, he published *Avis aux français*. This pamphlet was the cause of his downfall. An order was issued for his arrest on 8 July, but he managed to escape and found refuge in a house in the present Rue Servandoni. He hid there until 25 March 1794, and there he composed the work that constitutes his philosophical masterpiece, the *Esquisse d'un tableau des progrès de l'esprit humain*. On the latter date, fearful of being discovered, he left on foot for Fontenay-aux-

Roses, where he had friends. However, they managed not to be at home when he arrived. On 27 March he was arrested in Clamart under a false name and was taken to the prison of Bourg-la-Reine. The next morning he was found dead in his cell. It has never been possible to verify the rumor, originating as early as 1795, that he committed suicide by poison.

Condorcet seems to have had the character of a systematic and passionate intellectual. Described by d'Alembert as a "volcano covered with snow," he was praised by Mlle. de Lespinasse for his great kindness; and accusations of avarice and social climbing sometimes directed at him have never been corroborated by documentary evidence. He was the typical Encyclopedist and perhaps the last of them. All fields of knowledge fascinated him, as is shown by the equal care and competence that he devoted to his *éloges* of Euler as well as Buffon, d'Alembert as well as Jean de Witt, Frénicle as well as Pascal. He was thoroughly convinced of the value of science and of the importance of its diffusion as a determining factor in the general progress and well-being of mankind. That is why his interest never flagged in the applications of science,[3] or in the organization of scientific education,[4] or in the establishment of a universal scientific language.[5] His entire concept of scientific knowledge is essentially probabilist: "We give the name of mathematical certitude to probability when it is based on the content of the laws of our understanding. We call physical certitude the probability that further implies the same constancy in an order of phenomena independent of ourselves, and we shall reserve the term probability for judgments that are exposed to other sources of uncertainty beyond that" (*Essai sur l'application de l'analyse*, p. xiv). The concept of a science based on human actions, intrinsically probabilist, therefore seemed to him to be just as natural as a science of nature, and he attempted to establish a portion of such a science by proposing a theory of votes based on the calculus of probability. Thus, despite his reputation as a mathematician, much exaggerated in his lifetime, it is the novelty, the boldness, and the importance of this attempt that today seem to constitute his just claim on the attention of the historian of scientific knowledge.

Let us, nevertheless, briefly examine his work in pure mathematics before summarizing his work on probability and his application of analysis to the theory of votes.

Differential equations. Condorcet's scientific *mémoires* of the type contained in the principal periodicals of the academies have not been collected in book form. To those he did write may be added a few works that have not been republished and several unpublished manuscripts. It must be acknowledged that reading Condorcet's mathematical works is a thankless task and often a disappointing one. The notation is inconsistent, the expression of ideas often imprecise and obscure, and the proofs labored. This is certainly what Lagrange was complaining about when he wrote to d'Alembert on 25 March 1792: "I would like to see our friend Condorcet, who assuredly has great talent and wisdom, express himself in another manner; I have told him this several times, but apparently the nature of his mind compels him to work in this way: we shall have to let him do so…"[6]

Nevertheless, the esteem expressed by good judges, such as Lagrange himself, leaves the contemporary reader perplexed; and one suspects that sometimes friendship and sometimes the respect due to the influential secretary of the Académie des Sciences may have somewhat dulled the critical sense of the mathematicians of his day. Indeed, it now seems that the mathematical part of Condorcet's work contributed nothing original and that it deserves to be considered by the historian merely as evidence of the manner in which a man highly educated in that science could comprehend it and keep abreast of its progress. Wishing to introduce into the theory of differential equations general concepts that would be capable of systematizing it, he prematurely outlined a philosophy of mathematical notions that failed to issue in any coherent or practical organization.

Although he seemed ahead of his time when in an unpublished and incomplete *traité*[7] he defined a function as any relation whatever of corresponding values, he nevertheless thought it possible to propose a systematic and exhaustive classification of all functions. The main idea of the construction of classes of functions was, however, interesting in itself: it was the idea of a procedure of iteration that allows for definitions by recurring algorithms. What he did emphasize was the supposed closed-system character of all analytical entities. He exhibited this intention in an even more dogmatic manner in his *Lettre à d'Alembert* appended to the *Essais d'analyse* of 1768, in which he affirmed that all transcendental functions could be constructed by means of a circle and hyperbola; and he actually expressed himself in many of his *mémoires* as if any nonalgebraic function were of a trigonometric, logarithmic, or exponential nature.

The manner in which Condorcet conceived of the problem of the integration of differential equations or partial differential equations arose from the same tendency to generalization, which appeared a bit hasty even to his contemporaries.[8] Most of his *mémoires* and analytical works deal with this problem: to find conditional equations by relating the coefficients of a differential equation in such a manner as to render it integrable or at least such that its order may be lowered by one degree. Once the existence of solutions was proved, he hoped to be able to determine their form a priori, as well as the nature of the transcendental functions included; a calculation identifying the parameters would thus complete the integration. (See, e.g., "Histoire de l'Académie des sciences de Paris," "Mémoires" [1770], pp. 177 ff.). At least, therefore, he must be credited with having clearly conceived and stated that it is normal for an arbitrary differential equation not to be integrable.

Let us limit ourselves to indicating the approach by which Condorcet proceeded, as set forth in the 1765 text *Du calcul intégral* and resumed in the *traité* of 1786. Given the differential expression of any order

$V(x, y, dx, dy, d^2x, d^2y, \ldots, d^nx, d^ny)$, the question is under what conditions it might be the differential of an expression B, such that

Let us say $dx = p$, $dy = q$, $d^2y = dq = r$, $d^3y = dr = s$, x being taken as an independent variable, $d^2x = 0 = dp$. The differential of V, dV, has the form $dV = Mdx + Ndy + Pd^2y + Qd^3y + \ldots$, or

The coefficients M, N, P, Q are functions calculable from V.

Condorcet next differentiated the two sides of equation (1):

The terms of the right-hand side of equations (2) and (3) are then

Obtaining the differential of $P, Q, \ldots$

by successive addition and subtraction of (5), (8), (9),…, we obtain

which is the condition of existence of an integral B.

Condorcet then generalized his example by abandoning the hypothesis $d^2x = 0$, i.e., by supposing that there is no constant difference. He noted that the equations of these conditions are the same ones that determine the extreme values of the integral of V, as established by Euler and Lagrange.

Nowhere in Condorcet's various *mémoires* and other works, whether they deal with differential equations or equations with partial derivatives or equations with finite differences, can any results or methods be found that are truly creative or original relative to the works of Fontaine, d'Alembert, Euler, or Lagrange.

Calculus of probability. In the calculus of probability also, Condorcet did not bring any significant perfection to the resources of mathematics; he did, however, discuss and explain in depth an interpretation of probability that had far-reaching consequences in the applications of the calculus. First of all, he made a very clear distinction between an abstract or "absolute" probability and a subjective probability serving merely as grounds for belief. An example of the former is probability, in throwing an ideal die, that a given side will appear. Condorcet attempted to justify the passage from the first to the second by invoking three axiomatic principles that in effect reduced to the simple proposition "A very great absolute probability gives 'grounds for belief' that are close to certainty." (See "Probabilités," in *Encycloédie méthodique; Éléments du calcul des probabilités*, art 4.) As for passage from observed frequencies to "grounds for belief," this is effected through an estimated abstract probability; and the instrument for this estimation was Bayes's theorem, which had recently been reformulated by Laplace (*Mémoires par divers savants*, **6** , 1774). Condorcet made a very shrewd study of this. In the fourth "Mémoire sur le calcul des probabilités" (published in 1786), he noted that Bayes assumed the a priori law of probability to be constant, whereas it was actually possible to exhibit experimental variations of that law whether or not it depended on the time factor. Let us examine the latter case by means of an example. In a series of urns everything happens as if the drawings were made each time from an urn selected at random from a group, the numerical order of the drawing having no influence upon the choice. Thus, let $N = m + n + p + q$ decks of mixed cards. The first draw produces m red careds and n black ones from $(m + n)$ decks. We are asked the probability of drawing p red ones and q black ones from the $(p + q)$ remaining decks. Bayes's simple scheme (from a single deck of n cards, $m + n$ drawings have already been made, yielding m red and n black) furnishes the value calculated by Laplace for the probability wanted:

The hypothesis of a variable law led Condorcet to the alternative value

the multiple integrals being taken, for each variable, between 0 and 1 and x_i being the a priori probability assigned to the first deck of cards.

It is not correct to say, as Todhunter does (p. 404), that such an improvement on Laplace's formula was purely "arbitrary"; it must, nevertheless, be admitted that even though the main idea presupposes a thorough analysis of the requirements of a probabilistic model, it is no less true that the result is a complication offering no real utility.

"*Social mathematics*." Condorcet's most significant and fruitful endeavor was in a field entirely new at the time. The subject was one that departed from the natural sciences and mathematics but nevertheless showed the way toward a scientific comprehension of human phenomena, taking the empirical approach of natural science as its inspiration and employing mathematics as its tool. Condorcet called this new science "social mathematics." It was apparently intended to comprise, according to the "Tableau general de la science qui a pour objet l'application du calcul aux sciences physiques et morales," *Journal d'instruction sociale* (22 June, 6 July 1795; *Oewvres*, I, 539–573), a statistical description of society, a theory of political economy inspired by the Physiocrats, and a combinatorial theory of intellectual processes. The great work on the voting process, published in 1785, is related to the latter.

Condorcet there sought to construct a scheme for an electoral body the purpose of which would be to determine the truth about a given subject by the process of voting and in which each elector would have the same chance of voicing the truth. Such a scheme was presented exactly like what is today called a model. Its parameters were the number of voters, the majority required, and the probability that any particular vote voices a correct judgment. Condorcet's entire analysis consisted, then, of calculating different variable functions of these structural parameters. Such, for example, was the probability that a decision reached by majority vote might be correct. An interesting complication of the model is introduced by the assumption that individual votes are not mutually independent. For example, the influence of a leader might intervene; or when several successive polls are taken, the electors' opinions may change during the voting process. On the other hand, the problem of estimating the various parameters on a statistical basis was brought out by Condorcet, whose treatment foreshadowed very closely that employed by modern users of mathematical models in the social sciences. The mathematical apparatus may be reduced to simple theorems of addition and multiplication of probabilities, to binomial distribution, and to the Bayes-Laplace rule.

Here is an example of this analysis. Let v be the individual probability of a correct judgment. The probability that a collective decision having obtained q votes might be correct is

If one requires a majority of q with n voters, the probability that it will be attained and will furnish a correct decision is given by the sum

In the case of a leader's influence, if a voters out of n shared his opinion in a prior vote, the a posteriori probability of a voter's following the leader is, according to Bayes's rule, $a + 1/n + 2$; if v' represents the probability of any individual's holding a correct opinion and v'' is the same probability for the leader, then the probability of a voter's fortuitously holding the same opinion as the leader is

$v' v'' + (1 - v')(1 - v'')$.

Condorcet then proposed to measure the magnitude of the leader's influence by the difference between the probabilities:

Along the way he encountered a completely different problem, the decomposition and composition of electoral decisions in the form of elementary propositions on which voters pronounce either "Yes" or "No." He then anticipated, without being aware of it, the logical import of this problem, which was the theory of the sixteen binary sentence connectives, among which he emphasized the conditional. He showed that a complex questionnaire could be reduced to a sequence of dichotomies and that constraints implicitly contained in the complex questionnaire are equivalent to the rejection of certain combinations of "Yes" and "No" in the elementary propositions. This is literally the reduction into normal disjunctive forms as practiced by contemporary logicians (*Essai*, pp. xiv ff.). He therefore brought to light, more completely and more systematically than his predecessor Borda,[9] the possible incoherence of collective judgment in the relative ordering of several candidates.

If there are three candidates to be ranked and sixty voters, the voting may be done on the three elementary propositions *p, q, r*:

p: A is preferable to *B*
q: A is preferable to *C*
r: B is preferable to *C*.

Since the choice of each voter is assumed to be a coherent one—namely, determining a noncyclical order of the three candidates *A, B*, and *C*—an individual choice such as "*p* and not-*q* and *r*" is excluded.

However, let us assume that the results of the vote were as follows:

"*p* and *q* and *r*"	order *A-B-C*	23 votes
"not-*p* and not-*q* and *r*"	*B-C-A*	17 votes
"not-*p* and *q* and *r*"	*B-A-C*	2 votes
"not-*p* and not-*q* and not-*r*"	*C-B-A*	8 votes
"*p* and not-*q* and not-*r*"	*C-A-B*	10 votes

If we calculate the votes by opinion, then it is the order *A-B-C* that wins. But if we calculate the votes according to elementary propositions, the following is obtained: "*p*," 33 votes; "not-*q*," 35 votes; "*r*," 42 votes. These are the majority propositions and define a cyclical order, and therefore an incoherent collective opinion. This is the paradox that Condorcet pointed out, one that poses the problem, taken up again only in modern times, of the conditions of coherence in a collective opinion.

No doubt the results obtained in the *Essai d'application de l'analyse* were modest ones. "In almost all cases," Condorcet said, "the results are in conformity with what simple reason would have dictated; but it is so easy to obscure reason by sophistry and vain subtleties that I should feel rewarded if I had only founded a single useful truth on a mathematical demonstration" (*Essai*, p. ii). One must nevertheless recognize, in this work dated 1785, the first large-scale attempt to apply mathematics to knowledge of human phenomena.

NOTES

1. We believe we can identify a fragment of it with the MS at the Institut de France, fol. 222–223, carton 873.

2. See Baker's excellent "Les débuts de Condorcet…."

3. He became interested in hydraulics together with the Abbé Bossut ("Nouvelles expériences sur la résistance des fluides," in *Mémoires de l'Académie des sciences*, 1778); he became interested in demographic statistics together with Dionis du Séjour and Laplace ("Essai pour connaître la population du royaume," *ibid*., 1783–1788). The *Mémoires* (1782) contains a curious report on a proposal for a rational distribution of taxes using geographical, economic, and demographic data as a basis; it was formulated in collaboration with two physicists (Bossut and Desmarest), an agronomist (Tillet), and an astronomer (Dionis du Séjour).

4. In the five *mémoires* on public instruction (1791–1792) and in the draft decree of 20 April 1792, he proposed to replace the old literary instruction at the *collèges* with truly modern humanistic subjects including four categories of study: the physical and mathematical sciences, the moral and political sciences, the applications of science, and letters and fine arts.

5. See the fragment of the MSS published in Granger, "Langue universelle…"; also *Esquisse*, O. H. Prio, ed. (Paris, 1937), p. 232.

6.*Oeuvres de Lagrange*, XIII (Paris, 1882), 232.

7. Pt. I, sec. I. The handwritten MS is at the Institut de France (cartons 877–879) with the first 152 pages in printed form. Printing was suspended in 1786; the MS was composed in 1778.

8. For example, Lagrange wrote to d'Alembert, on 6 June 1765, that he wanted Condorcet "to explain in more detail the manner in which he arrived at the various integrals to which a single differential equation was susceptible." The reference here is to Condorcet's first work, *Du calcut intégral*. Likewise, on 30 September 1771, Lagrange pointed out to Condorcet that he had tried to apply one of his methods of approximation to an equation already studied, and that the result obtained was inaccurate.

9. Charles de Borda's "Mémoire sur les élections au scrutin" was published in the *Mémoires de l'Académie des sciences* (1781), 657–665, but it had been presented to the Academy in 1770 and Condorcet knew of it.

BIBLIOGRAPHY

The *Oeuvres*, 12 vols., pub. by Mme. Condorcet-O'Connor and François Arago (Paris, 1847–1849), do not contain the scientific writings. A bibliography and a chronology of the latter will be found in Henry, "Sur la vie et les écrits…" and in Granger (see below).

Condorcet's MSS are at the library of the Institut de France; the scientific writings are for the most part found in cartons 873–879 and 883–885.

On Condorcet see K. M. Baker, "An Unpublished Essay of Condorcet on Mechanical Methods of Classification," in *Annals of Science*, **18** (1962), 99–123; "Les débuts de Condorcet au Secrétariat de l'Académie royale des sciences," in *Revue d'histoire des sciences*, **20** (1967), 229–280; "Un 'éloge' officieux de Condorcet: Sa notice historique et critique sur Condillac," in *Revue de synthèse*, **88** (1967), 227–251; and "Scientism, Elitism and Liberalism: The Case of Condorcet," in *Studies on Voltaire and the 18th Century* (Geneva, 1967), pp. 129–165; L. Cahen, *Condorcet et la Révolution française* (Paris, 1904), with bibliography; Gilles Granger, "Langue universelle et formalisation des sciences," in *Revue d'histoire des sciences*, **7** , no. 4 (1954), 197–219; and *La mathématique sociale du Marquis de Condorcet* (Paris, 1956), with bibliography; Charles Henry, "Sur la vie et les écrits mathématiques de J. A. N. Caritat Marquis de Condorcet," in *Bollettino di bibliografia e storia delle scienze matematiche*, **16** (1883), 271 ff.; and his ed. of "Des méthodes d'approximation pour les équations différentielles lorsqu'on connaîtune première valeur approchée," *ibid*., 292–324; F. E. Manuel, *The Prophets of Paris* (Cambridge, Mass., 1962); René Taton, "Condorcet et Sylvestre-François Lacroix," in *Revue d'histoire des sciences*, **12** (1959), 127–158, 243–262; and I. Todhunter, *History of the Mathematical Theory of Probability From Pascal to Laplace* (Cambridge, 1865), pp. 351–410.

Gilles Granger