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(*b*. Pavia, Italy, 7 December 1830; *d*. Rome, Italy, 10 June 1903)

mathematics.

Cremona was the eldest child of Gaudenzio Cremona and his second wife, Teresa Andereoli. One of his brothers, Tranquillo, attained some fame as an artist.

Luigi was educated at the *ginnasio* in Pavia. When he was eleven, the death of his father threatened to interrupt his schooling; his stepbrothers came to his support, however, and he was enabled to continue his education. He was graduated first in his class with special recognition for his work in Latin and Greek, and entered the University of Pavia.

In 1848 Cremona joined the “Free Italy” battalion in the revolt against Austrian rule and attained the rank of sergeant. He took part in the unsuccessful defense of Venice, which capitulated on 24 August 1849. Because of the gallantry of the defenders, they were permitted to leave the city as a unit, with honors.

When Cremona returned to Pavia, he discovered that his mother had died. With the help of the family he reentered the university. On 27 November 1849 he was granted permission to study civil engineering. Here he came under the influence of A. Bordoni, A. Gabba, and especially Francesco Brioschi. Cremona was always grateful to Brioschi, later writing: “The years that I passed with Brioschi as pupil and later as colleague are a grand part of my life; in the first portion of those years I learned to love science and in the other how to transfer it to a large circle of auditors” (Gino Loria, “Luigi Cremona et son oeuvre mathématique,” p.129).

On 9 May 1853 Cremona received the doctorate in civil engineering and architecture, and on 4 August of the following year he was married. His record of military service against Austrian rule prevented him from obtaining an official teaching post in the educational system, so his first employment was as a private tutor to several families in Pavia.

On 22 November 1855 Cremona was granted permission to teach on a provisional basis at the *ginnasio* of Pavia, with special emphasis on physics. He was already engaged in the mathematical research for which he is so well known. His first paper, “Sulle tangenti sfero-conjugate,” had appeared in March 1855.

On 17 December 1856 Cremona was appointed associate teacher at the *ginnasio* in recognition of his good work and his mathematical activity: his second paper, “Intorno al un teorema del Abel” had appeared in May 1856. On 17 January 1857 he was appointed full teacher at the *ginnasio* in Cremona.

He remained in Cremona for nearly three years, during which time he wrote a number of articles. Some were merely answers to problems proposed in the *Nouvelles annales de mathématique*, but at least four contained original results, including his method of examining curves by projective methods: “Sulle linee del terz’ ordine a doppia curvatura—nota” (1858); “Sulle linee del terz’ ordine a doppia curvatura—teoremi” (1858); “Intorno alle superficie della seconda classe inscritte in una stessa superficie sviluppabile della quarta classe—nota” (1858); and “Intorno alle coniche inscritte in una stessa superficie del quart’ ordine e terza classe—nota” (1859).

On 28 November 1859 the Italian government of newly liberated Lombardy appointed Cremona a teacher at the Lycée St. Alexandre in Milan, and on 10 June 1860 he received his first college appointment. A royal decree appointed him ordinary professor at the University of Bologna, where he remained until October 1867.

Cremona’s most important original research into transformations in the plane and in space was published while he was at Bologna. His first paper, “Introduzione ad un teoria geometrica della curve piane,” appeared in December 1861. The first statement of his general theory for transformations involving plane curves was “Sulle trasformazione geometriche della figure piane” (1863). In March 1866 he published his second paper on transformations, “Mémoire de géométrie pure sur les surfaces du troisième ordre,” This earned Cremona half of the Steiner Prize for 1866, the other half going to Richard Sturm. Both papers on transformations were translated into German by Curtze and published as *Grundzüge der allgemeinen Theorie der Oberflāchen in synthetischer Behandlung* (1870). It was during this period at Bologna that Cremona developed the theory of birational transformations (Cremona transformations). Besides being a creative mathematician, Cremona was an excellent lecturer: calm, rigorous, yet interesting and even exciting.

In October 1867 Cremona was transferred by royal decree, and on the recommendation of Brioschi, to the Technical Institute of Milan, to be in charge of the courses in higher geometry. He received the title of ordinary professor in 1872. During this period Cremona continued to produce mathematical articles that appeared in many Italian and French journals. His paper "Sulle trasformazione razionale… nello spazio…" (1871), which extended his transformation theory to space curves, supplemented and completed the main outlines of the theory of birational transformations.

The period at Milan, where he remained until 1873, was the time of Cremona's greatest creativity. He wrote articles on such diverse topics as twisted cubics, developable surfaces, the theory of conics, the theory of plane curves, third- and fourth-degree surfaces, statics, and projective geometry. He also turned out a number of excellent texts, including *Le figure reciproche nella statica grafica* (1872), *Elementi di geometria projettiva* (1873), and *Elementi de calcolo grafico* (1874).

In 1873 Cremona was offered a political post as secretary-general of the new Italian Republic by the minister of agriculture, but he refused. On 9 October of that year Cremona was appointed, by royal decree, director of the newly established Polytechnic School of Engineering in Rome. He was also to be professor of graphic statics. Administrative and supervisory work took up so much of Cremona's time during this period that it effectively ended his creative work in mathematics.

In November 1877 Cremona was appointed to the chair of higher mathematics at the University of Rome, and on 16 March 1879 he was appointed a senator. The duties entailed by this position put a complete stop to his research activities. On 10 June 1903, after leaving a sickbed to act on some legislation, Cremona succumbed to a heart attack.

Cremona's main contributions to mathematics lie in the areas of birational transformations, graphic statics, and projective geometry.

The earliest modern use of one-to-one transformations appears to have been that of Poncelet in 1822. Bobillier used them in 1827–1828, and in 1828 Dandelin used double stereographic projections. The algebraic approach seems to have been used first by Plücker in 1830, and in 1832 Magnus used noninvoluntory transformations. Cremona combined all these developments and added much of his own material to create a unified theory. The clarity and polish of his presentation did much to publicize and popularize birational transformations.

The theory of birational transformations is basically as follows: Given a plane curve, $f(x,y) = 0$, which is irreducible and of degree m in x and n in y. Suppose also that $x' = \phi_1(x,y), y' = \phi_2(x,y)$ are rational functions in x and y. The eliminant of the three equations yields a new equation, $F(x',y') = 0$, which may be easier to examine and more revealing than the original, $f(x,y) = 0$.

If we solve the transformation equations $x' = \phi_1(x,y)$ and $y' = \phi_2(x,y)$ for x and y thus

$$x = \theta_1(x',y'), y = \theta_2(x',y'),$$

and if these are rational functions in x', y', then we say that the transformation $x' = \phi_1(x,y), y' = \phi^2(x,y)$ is birational.

Transformations of this nature are called Cremona transformations when there is a one-to-one reciprocal relation between the sets (x,y) and (x',y'). Geometrically, suppose the curve $f(x,y) = 0$ to be on the plane P and the curve $F(x',y') = 0$. Then we seek a one-to-one correspondence between the sets of points. A very simple example would be that of a curve $f(x,y) = 0$ on P and its projection by a central perspectivity onto the plane P'.

One of the simplest examples of a Cremona transformation is the homographic transformation

If homogeneous coordinates are used, we may write

$$x':y':z' = (ax+by+c):(a'x+b'y+c'):(px+qy+r).$$

Another example of a birational transformation is the Bertini transformation: x':y':z' = xy:xz:yz.

Cremona transformations have been used for studying rational surfaces, for the resolution of singularities of plane and space curves, and for the study of elliptic integrals and Riemann surfaces. They are effective in the reduction of singularities of curves to double points with distinct tangents.

Cremona's main contribution to graphic statics seems to have been the skillful use of the funicular diagram, or the reciprocal figure. This is defined as follows: Let P be a planar polygon with vertices $A, B, C,\ldots, K$, and let V be a point in the plane not on any side. Let $VA, VB, VC,\ldots, VK$ be drawn. Construct a polygon whose sides are parallel to $VA, VB, VC,\ldots, VK$. This polygon, P', is called the polygon reciprocal to P, or the funicular diagram.

By a theorem of Maxwell, "If forces represented in magnitude by the lines of a figure be made to act between the corresponding lines of the reciprocal figure, then the points of the reciprocal figure will all be in equilibrium under the action of these forces." Geometers will probably recognize Maxwell's theorem more readily in the following simplified form.

Given ΔABC, and V any point in the plane of the triangle; let lines VA, VB, BC, be drawn. In $\Delta A'B'C'$, $B'C'$ is parallel to VA, $C'A'$ is parallel to VB, and $A'B'$ is parllel to VC. Now draw a line through A' parallel to BC, through B' parallel to AC, and through C' parallel to AB. These lines will be concurrent.

Note that each diagram is the reciprocal of the other, and that forces applied at any node of one are parallel to and proportional to the sides of the other. Note also that three forces in equilibrium in one figure, and therefore represented by a triangle, have as their images in the reciprocal figure three concurrent lines. It is this property that makes reciprocal figures so useful.

Again, the clarity and elegance of Cremona's presentation helped to disseminate and popularize the theorem and its consequences. Moreover, he collated results obtained by others and made them more readily available. Thus, his *Graphical Statics* contains not only signed lines and signed angles, which are fairly well known, but also the concept of signed and weighted areas. The development of the concepts of centroids of figures is elegant and clear.

The third area to which Cremona contributed was that of projective geometry; in fact, this discipline prevades all his work. Birational transformations arose from the concept of a curve and its projection onto another plane, and graphic statics makes extensive use of projective techniques. It is true that Cremona made no startling discoveries in this area; but he did derive many properties of projectively related figures, and he did present the subject to his classes in a manner calculated to clarify and bring out relationships most simply.

Cremona made use of Euclidean geometry when he thought it most effective, and it may be said that this introduced extraneous factors into projective geometry. It must be remembered, however, that his training and temperament favored the use of intuitive rather than strictly postulational methods.

As an organizer and popularizer of those areas of mathematics in which he did his work, Cremona has had few peers. His works may still be read with profit and enjoyment.

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