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(*b*. Lisbon, Portugal, 1744; *d*. Lisbon, 1 January 1787)

mathematics.

The son of Lorenzo da Cunha, a painter, and his wife, Jacinta Ignes, da Cunha learned grammar, rhetoric, and logic at the Lisbon school of the Congregation of the Oratory, he also studied mathematics and physics on his own. At the age of nineteen he volunteered as a lieutenant in the artillery and spent nearly ten years at Valença do Mi-nho. At this time Portugal was experiencing ami feudal and anticlerical reforms, which were carried out by the marquis of Pombal, minister of King Joseph I. Da Cunha belonged to a group of free-thinking intellectuals who supported Pombal and disseminated ideas of the Enlightenment, and he became known as a progressive thinker, talented poet, and author of an original memoir on ballistics. In 1773 Pombal appointed da Cunha as a geometry professor at the Faculty of Mathematics of Coimbra University. Da Cunha's university career was short. In 1777, after the death of Joseph I, reactionaries returned to power and Pombal was dismissed, then exiled. In the same year da Cunha was arrested and imprisoned by the Inquisition. Charged with supporting heretical doctrines, in October 1778 he was sentenced by the General Council of the Inquisition in Lisbon to three years in prison. Freed at the beginning of 1781, da Cunha, under the protection of a high official, obtained a mathematical professorship at the College of Sao Lucas and resumed his scientific research. His health had been weakened in jail, however, and he died before his forty-fourth birthday.

Da Cunha's main scientific work was *Principios marhemdticos,* published serially beginning in 1782 and as a complete book in 1790. Intended to be a textbook, this work is a concise encyclopedia of mathematics in twenty-one parts that embrace all basic branches of the science, from geometry and arithmetic to the solution of differential equations and problems in the calculus of variations.

Excessive conciseness was the pedagogical deficiency of this exposition, which contained many fresh and fruitful ideas. The most striking feature was da Cunha's tendency to rigorous exposition of mathematics in general and of the calculus in particular. Needless to say, not all of his attempts in this direction were successful.

In book IX of *Principios* da Cunha presented a new theory of the exponential function that anticipated the methods of the modern theory of analytic functions and that was based on the use of solely convergent series (a very uncommon restriction at that time). The convergence of series in question was tested by comparing the given series with a convergent geometric series with each term greater than the series. Let a be a (positive) number which series is seen to converge for all values of c. Then the exponential function a sup(x)to the base a is defined for all values of x as the sum of the series to that end da Cunha demonstrated that every positive number a may be represented by the series From this definition the laws of exponents were derived. The binomial theorem-the power-series expansion of the function $(1+x)^n$- was obtained in a very ingenious way: da Cunha represented both 1+x and $(1+x)^n$ in the form of the exponential expansions, a device also used in the modern theory of complex functions.

In book XV, devoted to the elements of the calculus, the fundamental concepts were those of infinitely great and infinitely small variables, the concept of limit was not explicitly used. Following Leiniz notations, da Cunha profited to some extent from Newton's terminology, for instance, for the differential he adopted the symbol d but called it, as did some other mathematicians, "fluxion," a word that Newton used to designate the velocity of change of the variable or fluent. The definition of "differential" given by da Cunha was remarkable. During the eighteenth century the differential of a function, $y = f(x)$, was generally understood to be, and was defined as, its infinitely small increment, $dy = \Delta y$; but in practice dy was calculated as a part of the increment linear with respect to Δx, a distinction that was one of the sources of paradoxes and endless discussions. The definition proposed by da Cunha legitimated the procedures of differential calculus and was equivalent to one introduced in the nineteenth century following the works of Cauchy: If the increment $\Delta y=f(x+\Delta x)-f(x)$ can be represented as $\Delta y=A\Delta x + \varepsilon\Delta x$, where a does not depend on Δx and ε approaches zero together with Δx, then $A\Delta x$ is called the differential of function y,$dy=A\Delta x$. In this way da Cunha deduced some formulas of the differential calculus.

Da Cunha was one of the main precursor of the reform of the foundations of infinitesimal calculus, initiated in the first decades of the nineteenth century. Neither the Portuguese nor the French edition of his *Prinicipios* had a wide circulation, however, and they did not greatly influence the development of mathematics. Da Cunha's manuscripts on the problems of mathematics and its foundations are briefly mentioned in the preface to the French edition of *Principios,* but their subsequent fate is unknown.

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