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(*b*. Vitry-le-François, France, 26 May 1667; *d*. London, England, 27 November 1754)

probability.

De Moivre was one of the many gifted Protestants who emigrated from France to England following the revocation of the Edict of Nantes in 1685. His formal education was French, but his contributions were made within the Royal Society of London. His father, a provincial surgeon of modest means, assured him of a competent but undistinguished classical education. It began at the tolerant Catholic village school and continued at the Protestant Academy at Sedan. After the latter was suppressed for its profession of faith, De Moivre had to study at Saumur. It is said that he read mathematics on the side, almost in secret, and that Christiaan Huygens' work on the mathematics of games of chance, *De ratiociniis in ludo aleae* (Leiden, 1657), formed part of this clandestine study. He received no thorough instruction in mathematics until he went to Paris in 1684 to read the later books of Euclid and other texts under the supervision of Jacques Ozanam.

His Protestant biographers say that De Moivre, like so many of his coreligionists, was imprisoned during the religious tumult of 1685 and not released until 1688. Other, nearly contemporary sources report him in England by 1686. There he took up his lifelong, unprofitable occupation as a tutor in mathematics. On arrival in London, De Moivre knew many of the classic texts, but a chance encounter with Newton's *Principia* showed him how much he had to learn. He mastered the book quickly; later he told how he cut out the huge pages and read them while walking from pupil to pupil. Edmond Halley, then assistant secretary of the Royal Society, was sufficiently impressed to take him up after meeting him in 1692; it was he who communicated De Moivre's first paper, on Newton's doctrine of fluxions, to the Royal Society in 1695 and saw to his election by 1697. (In 1735 De Moivre was elected fellow of the Berlin Academy of Sciences, but not until 1754 did the Paris Academy follow suit.)

Once Halley had made him known, De Moivre's talents became esteemed. He was able to dedicate his first book, *The Doctrine of Chances*, to Newton; and the aging Newton would, it is said, turn students away with "Go to Mr. De Moivre; he knows these things better than I do." He was admired in the verse of Alexander Pope ("Essay on Man" II, 104) and was appointed to the grand commission of 1710, by means of which the Royal Society sought to settle the Leibniz-Newton dispute over the origin of the calculus. Yet throughout his life De Moivre had to eke out a living as tutor, author, and expert on practical applications of probability in gambling and annuities. Despite his powerful friends he found little patronage. He canvassed support in England and even begged Johann I Bernoulli to get Leibniz to intercede on his behalf for a chair of mathematics at Cambridge, but to no avail. He was left complaining of the waste of his time spent walking between the homes of his pupils. At the age of eighty-seven De Moivre succumbed to lethargy. He was sleeping twenty hours a day, and it became a joke that he slept a quarter of an hour more every day and would die when he slept the whole day through.

De Moivre's masterpiece is *The Doctrine of Chances*. A Latin version appeared as "De mensura sortis" in *Philosophical Transactions of the Royal Society* (1711). Successively expanded versions under the English title were published in 1718, 1738, and 1756. The only systematic treatises on probability printed before 1711 were Huygons' *De ratiociniis in ludo aleae* and Pierre Rémond de Montmort's *Essay d'analyse sur les jeux de hazard* (Paris, 1708). Problems which had been posed in these two books prompted De Moivre's earliest work and, incidentally, caused a feud between Montmort and De Moivre on the subject of originality and priority.

The most memorable of De Moivre's discoveries emerged only slowly. This is his approximation to the binomial probability distribution, which, as the normal or Gaussian distribution, became the most fruitful single instrument of discovery used in probability theory and statistics for the next two centuries. In De Moivre's own time his discovery enormously clarified the concept of probability. At least since the fifteenth century there had been substantial work on games of chance that recognized the existence of stable frequencies in nature. But in the classic work of Huygens and even in that of Montmort, the reader was usually given, in the context of a game or lottery, a set of events of equal probability—a set of what were often called "chances"—and he was asked to derive further probabilities or expectations from this fundamental set. No one had a clear mathematical formulation of how "chances" and stable frequencies are related. Jakob I Bernoulli provided a first answer in part IV of his *Ars conjectandi* (Basel, 1713), where he proved what is now called the weak law of large numbers; De Moivre's approximation to the binomial distribution was conceived as an attempt to improve on Bernoulli.

In some experiment, let the ratio of favorable to unfavorable "chances" be p. In n repeated trials of the experiment, let m be the number of successes. Consider any interval around p, bounded by two limits. Bernoulli proved that the probability that m/n should lie between these limits increases with increasing n and approaches l as n grows without bound. But although he could establish the fact of convergence, Bernoulli could not tell at what rate the probability converges. He did obtain some idea of this rate by computing numerical examples for particular values of n and p, but he was unable to state the principles that underlie his discovery. That was left for De Moivre.

De Moivre's solution was published as a Latin pamphlet dated 13 November 1733. Introducing his translation of, and comments on, this work at the end of the last edition of *The Doctrine of Chances*, he took "the liberty to say, that this is the hardest Problem that can be proposed on the Subject of Chance" (p. 242). In this problem the probability of getting exactly m successes in n trials is expressed by the m th term in the expansion of $(a + b)^n$—that is, $a^m b^{n-m}$, where a is the given ratio of chances and $b = 1 - a$. Hence the probability of obtaining a proportion of successes lying between the two limits is a problem in "approximating the Sum of the Terms of the Binomial $(a + b)^n$ expanded into a Series" (p. 243).

Working first with the binomial expansion of $(1 + 1)^n$, De Moivre obtained what is now recognized as n! approximated by Stirling's formula—that is, $cn^{n+1/2} e^{-n}$. He knew the constant c only as the limiting sum of an infinite series: "I desisted in proceeding farther till my worthy and learned Friend Mr. James Stirling, who had applied after me to that inquiry," discovered that (p. 244). Hence what is now called Stirling's formula is at least as much the work of De Moivre as of Stirling.

With his approximation of *n!* De Moivre was able, for example, to sum the terms of the binomial from any point up to the central term. This summation is equivalent to the modern normal approximation and is, indeed, the first occurrence of the normal probability integral. He even appears to have perceived, although he did not name, the parameter now called the standard deviation σ. It was left for Laplace and Gauss to construct the equation of the normal curve in its form

but De Moivre obtained, in a series of examples, expressions that are logically equivalent to this. He understood the rate of the convergence that Bernoulli had discovered and saw that the "error"—that is, the likely difference of the observed frequency from the true ratio of "chances"—decreases in inverse proportion to the square of the number of trials.

De Moivre's approximation is a theorem in probability theory: given the initial law about the distribution of chances, he could approximate the probability that observed frequencies should lie within any two assigned limits. Unlike some later workers, he did not imagine that his result would solve the converse statistical problem—namely, given the observed frequencies, to approximate the probability that the initial law about the ratio of chances lies within any two limits. But he did think his theorem bore on statistics. After summarizing his theorem, he reasoned:

Conversely, if from numberless Observations we find the Ratio of the Events to converge to a determinate quantity, as to the Ratio of P to Q; then we conclude that this Ratio expresses the determinate Law according to which the Event is to happen. For let that Law be expressed not by the ratio P : Q, but by some other, as R : S; then would the Ratio of the Events converge to this last, and not to the former: which contradicts our *Hypothesis* [p. 251].

Nowhere in *The Doctrine of Chances* is this converse reasoning put to any serious mathematical use, yet its conceptual value is great. For De Moivre, it seemed to resolve the philosophical paradox of finding regularities within events postulated to be random. As he expressed it in the third edition, "altho' Chance produces Irregularities, still the Odds will be infinitely great, that in process of Time, those Irregularities will bear no proportion to the recurreney of that Order which naturally results from ORIGINAL DESIGN" (p. 251).

All the mathematical problems treated by De Moivre before setting out his approximation to the binomial distribution are closely related to earlier work by Huygens and Montmort. They include the first intimation of another approximation to the binomial distribution, now usually named for Poisson. In the normal approximation, the given ratio of chances is constant at p; and as n increases, so does up. In the Poisson approximation, np is constant, so that as n grows, p tends to zero. It is useful in studying the probabilities of rather infrequent events. Although De Moivre worked out a particular case of the Poisson approximation, he does not appear to have guessed its subsequent uses in probability theory.

Also included in *The Doctrine of Chances* are great advances in problems concerning the duration of play; a clearer formulation of combinatorial problems about chances; the use of difference equations and their solutions using recurring series; and, as illustrated by the work on the normal approximation, the use of generating functions, which, by the time of Laplace, came to play a fundamental role in probability mathematics.

Although no statistics are found in *The Doctrine of Chances*, De Moivre did have a great interest in the analysis of mortality statistics and the foundation of the theory of annuities. Perhaps this originated from his friendship with Halley, who in 1693 had written on annuities for the Royal Society, partly in protest at the inane life annuities still being sold by the British government, in which the age of the annuitant was not considered relevant. Halley had very meager mortality data from which to work; but his article, together with the earlier "political arithmetic" of John Graunt and William Petty, prompted the keeping of more accurate and more relevant records. By 1724, when De Moivre published the first edition of *Annuities on Lives*, he could base his computations on many more facts. Even so, he found it convenient to base most of his computations on Halley's data, derived from only five years of observation in the city of Breslau; he claimed that other results confirmed the substantial accuracy of those data. In his tables De Moivre found it convenient to suppose that the death rate is uniform after the age of twelve. He did not pretend that the rate is absolutely uniform, as a matter of objective fact, but argued for uniformity partly because of its mathematical simplicity and partly because the mortality records were still so erratically collected that precise curve fitting was unwarranted.

De Moivre's contribution to annuities lies not in his evaluation of the demographic facts then known but in his derivation of formulas for annuities based on a postulated law of mortality and constant rates of interest on money. Here one finds the treatment of joint annuities on several lives, the inheritance of annuities, problems about the fair division of the costs of a tontine, and other contracts in which both age and interest on capital are relevant. This mathematics became a standard part of all subsequent commercial applications in England. Yet the authorship of this work was a matter of controversy. De Moivre's first edition appeared in 1725; in 1742 Thomas Simpson published *The Doctrine of Annuities and Reversions Deduced From General and Evident Principles*. De Moivre republished in the next year, bitter at what, with some justice, he claimed to be the plagiarization of his work. Since the sale of his books was a real part of his small income, money must have played as great a part as pride in this dispute.

Throughout his life De Moivre published occasional papers on other branches of mathematics. Most of them offered solutions to fairly ephemeral problems in Newton's calculus; in his youth some of this work led him into yet another imbroglio about authorship, involving some minor figures from Scotland, especially George Cheyne. In these lesser works, however, there is one trigonometric equation the discovery of which is sufficiently undisputed that it is still often called De Moivre's theorem:

$$(\cos \phi + i \sin \phi)^n = \cos n\phi + \mathrm{i} \sin n\phi$$

This result was first stated in 1722 but had been anticipated by a related formula in 1707. It entails or suggests a great many valuable identities and thus became one of the most useful steps in the early development of complex number theory.

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