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(*b*.Auch, Gers, France, 5 January 1884; *d*. Paris, France, 21 January 1974)

mathematical analysis.

Denjoy was the son of Jean Denjoy, a wine merchant in Perpignan and of a woman surnamed Jayez, who was from Catalonia. After secondary education at Auch and Montpellier, in 1902 he entered the École Normale Supérieure in Paris, where he studied under Émile Borel, Paul Painlevé and Charles Picard, and graduated first in his class. In 1905 he received a Fondation Thiers fellowship for a three-year period. Denjoy completed his dissertation ("Sur les produits canoniques d'ordre infini") in 1909 and was named *maître de conférences* at Montpellier University, where he taught until 1914. Poor eyesight kept him from military service during World War I. In 1917 he received a professorship at Utrecht, and in 1922 he accepted a position at the University of Paris, where he remained until his retirement in 1955.

On 15 June 1923 Denjoy married Thérèse-Marie Chevresson: they had three sons.

Denjoy led a quiet life, working most of the day at home. During the summers he enjoyed cycling on forest trails or walking along rivers and lakes. His death resulted from a fall in his home.

Elected to the Académic des Sciences in Paris on 15 June 1942. Denjoy was also a member of several other academies, including the Academy of Sciences of Amsterdam, the Société des Sciences et Lettres of Warsaw, and the Société Royale des Sciences of Liège. He was vice president of the International Mathematical Union in 1954. Although he did not write joint papers, he maintained contact with most of the great mathematical analysts, especially those from Russia, Poland, the Netherlands, and Germany.

Denjoy's weak voice and bad eyesight did not make him a notable lecturer, but in private talks he was very entertaining. His written work displayed his gift for brilliant metaphors to convey mathematical discoveries. He coined many illuminating mathematical terms, such as *clairsemé, gerbe, résiduel, plénitude*, and *épaisseur*.

Denjoy was an atheist, but tolerant of others' religious views; he was very interested in philosophical, psychological, and social issues. Throughout his life he wrote about them in his (unpublished) diary. In 1964 he published some of his thoughts in *Hommes, formes et le nombre*, which deals mainly with mathematical discoveries and concepts and with men of science. He liked neither the Bourbaki approach to mathematics nor its style, and at the time of his death he was planning a sharp account of his criticisms.

His activity in the Radical-Socialist Party, at the time headed by Édouard Herriot, led to Denjoy's election to the Montpellier town council in 1912 and to the Gers county council in 1920, and he served on the latter until 1940. From 1949 to 1950, just before Francois Duvalier became dictator, Denjoy held a cultural diplomatic position in Haiti. In 1960 he joined the Comité de l'Union Rationaliste, which aims to spread the spirit of science and the experimental method, and to fight dogmatism and fanaticism.

Denjoy was the youngest of the prestigious quartet of French mathematicians—the others were Émile Borel, René Baire, and Henri Lebesgue—that devised the theory of functions of real variables at a time when analytic functions *f(z)* were more commonly studied.

By the time Denjoy completed his dissertation in 1909, Borel had introduced a theory of countably additive measure, divergent series, scales of growth and zero measure, and monogenic functions. Baire had initiated a new approach to real functions by introducing semicontinuity sets of the first category and the transfinite scale of (Baire) functions. Lebesgue had firmly established the notion of measure and his theory of integration. With thoseingredients Denjoy realized a synthesis combining topological and metric tools: topology to reduce problems to basics, and metric notions for the final blow.

Denjoy had a strong classical background in complex function theory, differential equations, and continued fractions that permeated all his work. His dissertation, for instance, presented results concerning the series $\Sigma A_n/(z-a_n)$, canonical Weierstrass products for integral functions, asymptotic values of integral functions of finite order, and boundary behavior of conformal representation. Al though some of these results are now considered among his best contributions, he wrote in 1934 that he considered his achievements to be (1) the integration of derivatives, (2) the computation of the coefficients of any converging

trigonometric series for which the sum is given, (3) his theorem on quasianalytic functions, and (4) differential equations on a torus.

Posterity has not always confirmed this hierarchy. For instance, the fourth achievement, which completely clarifies the "last Poincaré theorem," has grown into a vast field involving dynamical systems. His theorem concerning quasi-analytical functions (related to Borel monogeneity) and the Denjoy conjecture were the source of many subsequent studies (for example, Benoit Mandelbrot's). The first two are sometimes considered more feats of intellectual strength than sources of practical applications. It remains true, however, that they both had profound implications and that they were needed. It is likely that nobody but Denjoy could have achieved the computation of the coefficients, the results of which were first published (as were most of Denjoy's theorems) as notes in the *Comptes rendus* of the Academy of Sciences (1921). When confronted with growing skepticism, he published the complete proofs in five volumes (1941–1949). They contain much more than was required for the proofs and are an explosion of beautiful theorems and examples. Denjoy's *Lecons sur le calcul des coefficients d'une série trigonométrique* was for a long time recommended reading for research students at Moscow University. It contains, in addition to the Denjoy integral, a wealth of tangential properties of continuous functions that have inspired considerable research by Andrew Bruckner. Some of the properties were generalized by Frédéric Roger and Gustave Choquet.

BIBLIOGRAPHY

The best guide to Denjoy's works is "Arnaud Denjoy, évocation de l'homme et de l'oeuvre," in *Astérisqiue* nos. 28–29. See also his *Leçons sur le calcul des coeffecients d'une série trigonométrique*, 5 vols. (Paris, 1941–1949); *L'énumération transfinie*, 5 vols. (Paris, 1946–1954); *Mémoire sur la deérivation et son calcul inverse* (Paris, 1954); *Articles et mémoires*. 2 vols. (Paris. 1955); *Un demi-siècle de notes*. 2 vols. (Paris, 1957); and *Hommes, formes et le nombre* (Paris, 1964).

Gustave Choquet