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In this section, Descartes's mathematics is discussed separately. The physics is discussed in two subsections: Optics and Mechanics.

Mathematics. The mathematics that served as model and touchstone for Descartes's philosophy was in large part Descartes's own creation and reflected in turn many of his philosophical tenets.[1] Its historical foundations lie in the classical analytical texts of Pappus (*Mathematical Collection*) and Diophantus (*Arithmetica*) and in the cossist algebra exemplified by the works of Peter Rothe and Christoph Clavius. Descartes apparently received the stimulus to study these works from Isaac Beeckman; his earliest recorded thoughts on mathematics are found in the correspondence with Beeckman that followed their meeting in 1618. Descartes's command of cossist algebra (evident throughout his papers of the early 1620's) was perhaps strengthened by his acquaintance during the winter of 1619–1620 with Johann Faulhaber, a leading German cossist in Ulm.[2] Descartes's treatise *De solidorum elementis*, which contains a statement of "Euler's Theorem" for polyhedra ($V + F = E + 2$), was quite likely also a result of their discussions. Whatever the early influences on Descartes's mathematics, it nonetheless followed a relatively independent line of development during the decade preceding the publication of his magnum opus, the *Géométrie* of 1637.[3]

During this decade Descartes sought to realize two programmatic goals. The first stemmed from a belief, first expressed by Petrus Ramus,[4] that cossist algebra represented a "vulgar" form of the analytical method employed by the great Greek mathematicians. As Descartes wrote in his *Rules for the Direction of the Mind* (*ca*. 1628):

... some traces of this true mathematics [of the ancient Greeks] seem to me to appear still in Pappus and Diophantus.... Finally, there have been some most ingenious men who have tried in this century to revive the same [true mathematics]; for it seems to be nothing other than that art which they call by the barbarous name of "algebra," if only it could be so disentangled from the multiple numbers and inexplicable figures that overwhelm it that it no longer would lack the clarity and simplicity that we suppose should obtain in a true mathematics.[5]

Descartes expressed his second programmatic goal in a letter to Beeckman in 1619; at the time it appeared to him to be unattainable by one man alone. He envisaged "an entirely new science,"

... by which all questions can be resolved that can be proposed for any sort of quantity, either continuous or discrete. Yet each problem will be solved according to its own nature, as, for example, in arithmetic some questions are resolved by rational numbers, others only by irrational numbers, and others finally can be imagined but not solved. So also I hope to show for continuous quantities that some problems can be solved by straight lines and circles alone; others only by other curved lines, which, however, result from a single motion and can therefore be drawn with new forms of compasses, which are no less exact and geometrical, I think, than the common ones used to draw circles; and finally others that can be solved only by curved lines generated by diverse motions not subordinated to one another, which curves are certainly only imaginary (e.g., the rather well-known quadratrix). I cannot imagine anything that could not be solved by such lines at least, though I hope to show which questions can be solved in this or that way and not any other, so that almost nothing will remain to be found in geometry[6].

Descartes sought, then, from the beginning of his research a symbolic algebra of pure quantity by which problems of any sort could be analyzed and classified in terms of the constructive techniques required for their most efficient solution. He took a large step toward his goal in the *Rules* and achieved it finally in the *Géométrie*.

Descartes began his task of "purifying" algebra by separating its patterns of reasoning from the particular subject matter to which it might be applied. Whereas cossist algebra was basically a technique for solving numerical problems and its symbols therefore denoted numbers, Descartes conceived of his "true mathematics" as the science of magnitude, or quantity, per se. He replaced the old cossist symbols with letters of the alphabet, using at first (in the *Rules*) the capital letters to denote known quantities and the lowercase letters to denote unknowns, and later (in the *Géométrie*) shifting to the $a,b,c; x,y,z$ notation still in use today. In a more radical step, he then removed the last vestiges of verbal expression (and the conceptualization that accompanied it) by replacing the words "square," "cube," etc., by numericalsuperscripts. These superscripts, he argued (in rule XVI), resolved the serious conceptual difficulty posed by the dimensional connotations of the words they replaced. For the square of a magnitude did not differ from it in kind, as a geometrical square differs from a line; rather, the square, the cube, and all powers differed from the base quantity only in the number of "relations" separating them respectively from a common unit quantity. That is, since

$1{:}x = x{:}x^2 = x^2{:}x^3 = \ldots$

(and, by Euclid V, ratios obtain only among homogeneous quantities), x^3 was linked to the unit magnitude by three "relations," while x was linked by only one. The numerical superscript expressed the number of "relations."

While all numbers are homogeneous, the application of algebra to geometry (Descartes's main goal in the *Géométrie*) required the definition of the six basic algebraic operations (addition, subtraction, multiplication, division, raising to a power, and extracting a root) for the realm of geometry in such a way as to preserve the homogeneity of the products. Although the Greek mathematicians had established the correspondence between addition and the geometrical operation of laying line lengths end to end in the same straight line, they had been unable to conceive of multiplication in any way other than that of constructing a rectangle out of multiplier and multiplicand, with the result that the product differed in kind from the elements multiplied. Descartes's concept of "relation" provided his answer to the problem: one chooses a unit length to which all other lengths are referred (if it is not given by the data of the problem, it may be chosen arbitrarily). Then, since $1{:}a = a{:}ab$, the product of two lines a and b is constructed by drawing a triangle with sides 1 and a; in a similar triangle, of which the side corresponding to I is b, the other side will be ab, a line length. Division and the remaining operations are defined analogously. As Descartes emphasized, these operations do not make arithmetic of geometry, but rather make possible an algebra of geometrical line segments.

The above argument opens Descartes's *Géométrie*and lays the foundation of the new analytic geometry contained therein, to wit, that given a line x and a polynomial $P(x)$ with rational coefficients it is possible to construct another line y such that $y = P(x)$. Algebra thereby becomes for Descartes the symbolic method for realizing the second goal of his "true mathematics," the analysis and classification of problems. The famous "Problem of Pappus," called to Descartes's attention by Jacob Golius in 1631, provides the focus for Descartes's exposition of his new method. The problem states in brief: given n coplanar lines, to find the locus of a point such that, if it is connected to each given line by a line drawn at a fixed angle, the product of $n/2$ of the connecting lines bears a given ratio to the product of the remaining $n/2$ (for even n; for odd; n, the product of $(n + 1)/2$ lines bears a given ratio to k times the product of the remaining $(n - 1)/2$, where k is a given line segment). In carrying out the detailed solution for the case $n = 4$, Descartes also achieves the classification of the solutions for other n.

Implicit in Descartes's solution is the analytic geometry that today bears his name. Taking lines *AB, AD, EF, GH* as the four given lines, Descartes assumes that point *C* lies on the required locus and draws the connecting lines *CB, CD, CF, CH*. To apply algebraic analysis, he then takes the length *AB*, measured from the fixed point *A*, as his first unknown, x, and length *BC* as the second unknown, y. He thus imagines the locus to be traced by the endpoint C of a movable ordinate *BC* maintaining a fixed angle to line *AB* (the axis) and varying in length as a function[7] of the length *AB*. Throughout the *Géométrie*, Descartes chooses his axial system to fit the problem; nowhere does the now standard—and misnamed— "Cartesian coordinate system" appear.

The goal of the algebraic derivation that follows this basic construction is to show that every other connecting line may be expressed by a combination of the two basic unknowns in the form $\alpha x + \beta y + \gamma$, where α, β, γ derive from the data. From this last result it follows that for a given number n of fixed lines the power of x in the equation that expresses the ratio of multiplied connecting lines will not exceed $n/2$ (n even) or $(n - 1)/2$ (n odd); it will often not even be that large. Hence, for successive assumed values of y, the construction of points on the locus requires the solution of a determinate equation in x of degree $n/2$, or $(n - 1)/2$; e.g., for five or fewer lines, one need only be able to solve a quadratic equation, which in turn requires only circle and straightedge for its constructive solution.

Thus Descartes's classification of the various cases of Pappus' problem follows the order of difficulty of solving determinate equations of increasing degree.[8] Solution of such equations carries with it the possibility of constructing any point (and hence all points) of the locus sought. The direct solvability of algebraic equations becomes in book II Descartes's criterion for distinguishing between "geometrical" and "nongeometrical" curves; for the latter (today termed "transcendental curves") by their nature allow the direct construction of only certain of their points. For the construction of the loci that satisfy Pappus' problem for $n \leq 5$, i.e., the conic sections, Descartes relies on the construction theorems of Apollonius' *Conics* and contents himself with showing how the indeterminate equations of the loci contain the necessary parameters.

Descartes goes on to show in book II that the equation of a curve also suffices to determine its geometrical properties, of which the most important is the normal to any point on the curve. His method of normals—from which a method of tangents follows directly—takes as unknown the point of intersection of the desired normal and the axis. Considering a family of circles drawn about that point, Descartes derives an equation $P(x) = 0$, the roots of which are the abscissas of the intersection points of any circle and the curve. The normal is the radius of that circle which has a single intersection point, and Descartes finds that circle on the basis of the theorem that, if $P(x) = 0$ has a repeated root at $x = a$, then $P(x) = (x - a)^2 R(x)$, where $R(a) \neq 0$. Here a is the abscissa of the given point on the curve, and the solution follows from equating the coefficients of like powers of x on either side of the last equation. Descartes's method is formally equivalent to Fermat's method of maxima and minima and, along with the latter, constituted one of the early foundations of the later differential calculus.

The central importance of determinate equations and their solution leads directly to book III of the *Géométrie* with its purely algebraic theory of equations. Entirely novel and original, Descartes's theory begins by writing every equation in the form $P(x) = 0$, where $P(x)$ is an algebraic polynomial with real coefficients.[9] From the assertion, derived inductively, that every such equation may also be expressed in the form

$(x - a)\ (x - b) \ldots (x - s) = 0$

where *a, b*,..., *s* are the roots of the equation, Descartes states and offers an intuitive proof of the fundamental theorem of algebra (first stated by Albert Girard in 1629) that an *n*th degree equation has exactly *n* roots. The proof rests simply on the principle that every root must appear in one of the binomial factors of $P(x)$ and that it requires *n* such factors to achieve x^n as the highest power of *x* in that polynomial. Descartes is therefore prepared to recognize not only negative roots (he gives as a corollary the law of signs for the number of negative roots) but also "imaginary" solutions to complete the necessary number.[10] In a series of examples, he then shows how to alter the signs of the roots of an equation, to increase them (additively or multiplicatively), or to decrease them. Having derived from the factored form of an equation its elementary symmetric functions,[11] Descartes uses them to eliminate the term containing x^{n-1} in the equation. This step paves the way for the general solution of the cubic and quartic equations (material dating back to Descartes's earliest studies)and leads to a general discussion of the solution of equations, in which the first method outlined is that of testing the various factors of the constant term, and then other means, including approximate solution, are discussed.

The *Géométrie* represented the sum of mathematical knowledge to which Descartes was willing to commit himself in print. The same philosophical concepts that led to the brilliant new method of geometry also prevented him from appreciating the innovative achievements of his contemporaries. His demand for strict a priori deduction caused him to reject Fermat's use of counterfactual assumptions in the latter's method of maxima and minima and rule of tangents.[12] His demand for absolute intuitive clarity in concepts excluded the infinitesimal from his mathematics. His renewed insistence on Aristotle's rigid distinction between "straight" and "curved" led him to reject from the outset any attempt to rectify curved lines.

Despite these hindrances to adventurous speculation, Descartes did discuss in his correspondence some problems that lay outside the realm of his *Géométrie*. In 1638, for example, he discussed with Mersenne, in connection with the law of falling bodies, the curve now expressed by the polar equation $\varrho = a\lambda^{\vartheta}$ (logarithmic spiral)[13] and undertook the determination of the normal to, and quadrature of, the cycloid. Also in 1638 he took up a problem posed by Florimond Debeaune: (in modern terms) to construct a curve satisfying the differential equation $a(dy/dx) = x - y$. Descartes appreciated Debeaune's quadrature of the curve and was himself able to determine the asymptote $y = x - a$ common to the family, but he did not succeed in finding one of the curves itself[14]

By 1638, however, Descartes had largely completed his career in mathematics. The writing of the *Meditations* (1641), its defense against the critics, and the composition of the magisterial *Principia philosophiae* (1644) left little time to pursue further the mathematical studies begun in 1618.

Optics. In addition to presenting his new method of algebraic geometry, Descartes's *Géométrie* also served in book II to provide rigorous mathematical demonstrations for sections of his *Dioptrique* published at the same time. The mathematical derivations pertain to his theory of lenses and offer, through four "ovals," solutions to a generalized form of the anaclastic problem.[15] The theory of lenses, a topic that had engaged Descartes since reading Kepler's *Dioptrica* in 1619, took its form and direction in turn from Descartes's solution to the more basic problem of a mathematical derivation of the laws of reflection and refraction, with which the *Dioptrique* opens.

Background to these derivations was Descartes's theory of light, an integral part of his overall system of cosmology.[16] For Descartes light was not motion (which takes time) but rather a "tendency to motion," an impulsive force transmitted rectilinearly and instantaneously by the fine particles that fill the interstices between the visible macrobodies of the universe. His model for light itself was the blind man's cane, which instantaneously transmits impulses from the objects it meets and enables the man to "see." To derive the laws of reflection and refraction, however, Descartes required another model more amenable to mathematical description. Arguing that "tendency to motion" could be analyzed in terms of actual motion, he chose the model of a tennis ball striking a flat surface. For the law of reflection the surface was assumed to be perfectly rigid and immobile. He then applied two fundamental principles of his theory of collision: first, that a body in motion will continue to move in the same direction at the same speed unless acted upon by contact with another body; second, that a body can lose some or all of its motion only by transmitting it directly to another. Descartes measured motion by the product of the magnitude of the body and the speed at which it travels. He made a distinction, however, between the speed of a body and its "determination" to move in a certain direction.[17] By this distinction, it might come about that a body impacting with another would lose none of its speed (if the other body remained unmoved) but would

receive another determination. Moreover, although Descartes treated speed as a scalar quantity, determination was (operationally at least) always a vector, which could be resolved into components.[18] When one body collided with another, only those components of their determinations that directly opposed one another were subject to alteration.

Imagine, then, says Descartes, a tennis ball leaving the racket at point *A* and traveling uniformly along line *AB* to meet the surface *CE* at *B*. Resolve its determination into two components, one (*AC*) perpendicular to the surface and one (*AH*) parallel to it. Since, when the ball strikes the surface, it imparts none of its motion to the surface (which is immobile), it will continue to move at the same speed and hence after a period of time equal to that required to traverse *AB* will be somewhere on a circle of radius *AB* about *B*. But, since the surface is impenetrable, the ball cannot pass through it (say to *D*) but must bounce off it, with a resultant change in determination. Only the vertical component of that determination is subject to change, however; the horizontal component remains unaffected. Moreover, since the body has lost none of its motion, the length *HF* of that component after collision will equal the length *AH* before. Hence, at the same time the ball reaches the circle it must also be at a distance $HF = AH$ from the normal HB, i.e., somewhere on line FE. Clearly, then, it must be at F, and consideration of similar triangles shows that the angle of incidence ABH is equal to the angle of reflection HBF.

For the law of refraction, Descartes altered the nature of the surface met by the ball; he now imagined it to pass through the surface, but to lose some of its motion (i.e., speed) in doing so. Let the speed before collision be to that after as $p:q$. Since both speeds are uniform, the time required for the ball to reach the circle again will be to that required to traverse AB as $p:q$. To find the precise point at which it meets the circle, Descartes again considered its determination, or rather the horizontal component unaffected by the collision. Since the ball takes longer to reach the circle, the length of that component after collision will be greater than before, to wit, in the ratio *of* $p:q$, Hence, if $FH: AH = p:q$, then the ball must lie on both the circle and line FE. Let I be the common point.

The derivation so far rests on the assumption that the ball's motion is decreased in breaking through the surface. Here again Descartes had to alter his model to fit his theory of light, for that theory implies that light passes more easily through the denser medium. For the model of the tennis ball, this means that, if the medium below the surface is denser than that above, the ball receives added speed at impact, as if it were struck again by the racket. As a result, it will by the same argument as given above be deflected toward the normal as classical experiments with airwater interfaces said it should.

In either case, the ratio $p:q$ of the speeds before and after impact depended, according to Descartes, on the relative density of the media and would therefore be constant for any two given media. Hence, since

$$FH:AH = BE:BC = p:q$$

it follows that

which is the law of refraction.

The vagueness surrounding Descartes's concept of "determination" and its relation to speed makes his derivations difficult to follow. In addition, the assumption in the second that all refraction takes place at the surface lends an ad hoc aura to the proof, which makes it difficult to believe that the derivation represented Descartes's path to the law of refraction (the law of reflection was well known). Shortly after Descartes's death, prominent scientists, including Christian Huygens, accused him of having plagiarized the law itself from Willebrord Snell and then having patched together his proof of it. There is, however, clear evidence that Descartes had the law by 1626, long before Golius uncovered Snell's unpublished memoir.[19] In 1626 Descartes had Claude Mydorge grind a hyperbolic lens that represented an anaclastic derived by Descartes from the sine law of refraction. Where Descartes got the law, or how he got it, remains a mystery; in the absence of further evidence, one must rest content with the derivations in the *Dioptrique*.

Following those derivations, Descartes devotes the remainder of the *Dioptrique* to an optical analysis of the human eye, moving from the explanation of various distortions of vision to the lenses designed to correct them, or, in the case of the telescope, to increase the power of the normal eye. The laws of reflection and refraction reappear, however, in the third of the *Essais* of 1637, the *Météores*. There Descartes presents a mathematical explanation of both the primary and secondary rainbow in terms of the refraction and internal reflection of the sun's rays in a spherical raindrop.[20] Quantitatively, he succeeded in deriving the angle at which each rainbow is seen with respect to the angle of the sun's elevation. His attempted explanation of the rainbow's colors, however, rested on a general theory of colors that could at the time only be qualitative. Returning to the model of the tennis ball, Descartes explained color in terms of a rotatory motion of the ball, the speed of rotation varying with the color. Upon refraction, as through a prism, those speeds would be altered, leading to a change in colors.

Mechanics. Descartes's contribution to mechanics lay less in solutions to particular problems than in the stimulus that the detailed articulation of his mechanistic cosmology provided for men like Huygens.[21] Concerned with the universe on a grand scale, he had little but criticism for Galileo's efforts at resolving more mundane questions. In particular, Descartes rejected much of Galileo's work, e.g., the laws of free fall and the law of the pendulum, because Galileo considered the phenomena in a vacuum, a vacuum that Descartes's cosmology excluded from the world. For Descartes, the ideal world corresponded to the real one. Mechanical phenomena took place in a plenum and had to be explained in terms of the direct interaction of the bodies that constituted it, whence the central role of his theory of impact.[22]

Two of the basic principles underlying that theory have been mentioned above. The first, the law of inertia, followed from Descartes's concept of motion as a state coequal with rest; change of state required a cause (i.e., the action of another moving body) and in the absence of that cause the state remained constant. That motion continued in a straight line followed from the privileged status of the straight line in Descartes's geometrical universe. The second law, the conservation of the "quantity of motion" in any closed interaction, followed from the immutability of God and his creation. Since bodies acted on each other by transmission of their motion, the "quantity of motion" (the product of magnitude and speed) served also as Descartes's measure of force or action and led to a third principle that vitiated Descartes's theory of impact. Since as much action was required for motion as for rest, a smaller body moving however fast could never possess sufficient action to move a larger body at rest. As a result of this principle, to which Descartes adhered in the face of both criticism and experience, only the first of the seven laws of impact (of perfectly elastic bodies meeting in the same straight line) is correct. It concerns the impact of two equal bodies approaching each other at equal speeds and is intuitively obvious.

Descartes's concept of force as motive action blocked successful quantitative treatment of the mechanical problems he attacked. His definition of the center of oscillation as the point at which the forces of the particles of the swinging body are balanced out led to quite meager results, and his attempt to explain centrifugal force as the tendency of a body to maintain its determination remained purely qualitative. In all three areas—impact, oscillation, and centrifugal force—it was left to Huygens to push through to a solution, often by discarding Descartes's staunchly defended principles.

Descartes met with more success in the realm of statics. His *Explication des engins par l'aide desquels on peut avec une petite force lever un fardeau fort pesant*, written as a letter to Constantin Huygens in 1637, presents an analysis of the five simple machines on the principle that the force required to lift a pounds vertically through b feet will also lift na pounds b/n feet. And a memoir dating from 1618 contains a clear statement of the hydrostatic paradox, later made public by Blaise Pascal.[23]

NOTES

1. Cf. Gaston Milhaud, *Descartes savant* (Paris, 1921), and Jules Vuillemin, *Mathématiques et métaphysique chez Descartes*(Paris, 1960).

2. Cf. Milhaud, pp. 84–87.

3. Defending his originality against critics, Descartes repeatedly denied having read the algebraic works of François Viète or Thomas Harriot prior to the publication of his own *Géométrie*. The pattern of development of his ideas, especially during the late 1620's, lends credence to this denial.

4. In his *Scholarum mathematicarum libri unus et triginta* (Paris,1569; 3rd. ed., Frankfurt am Main, 1627), bk. I (p. 35 of the 3rd ed.). Descartes quite likely knew of Ramus through Beeckman, who had studied mathematics with Rudolph Snell, a leading Dutch Ramist.

5.*Regulae ad directionem ingenii*, in *Oeuvres de Descartes*, Adam and Tannery, eds., X (Paris, 1908). rule IV, 376–377.

6. Descartes to Beeckman (26 Mar. 1619), *Oeuvres*, X, 156–158. By "imaginary" curve, Descartes seems to mean a curve that can be described verbally but not accurately constructed by geometrical means.

7. Both the term and the concept it denotes are certainly anachronistic. Descartes speaks of the indeterminate equation that links x and y as the "relation [*rapport*] that all the points of a curve have to all those of a straight line" (*Géométrie*, p. 341). Strangely, Descartes makes no special mention of one of the most novel aspects of his method, to wit, the establishment of a correspondence between geometrical loci and indeterminate algebraic equations in two unknowns. He does discuss the correspondence further in bk. II, 334–335, but again in a way that belies its novelty. The correspondence between determinate equations and point constructions (i.e., section problems) had been standard for some time.

8. For problems of lower degree, Descartes maintains the classification of Pappus. Plane problems are those that can be constructed with circle and straightedge, and solid problems those that require the aid of the three conic sections. Where, however, Pappus grouped all remaining curves into a class he termed linear, Descartes divides these into distinct classes of order. To do so, he employs in bk. I a construction device that generates the conic sections from a referent triangle and then a new family of higher order from the conic sections, and so on.

9. Two aspects of the symbolism employed here require comment. First, Descartes deals for the most part with specific examples of polynomials, which he always writes in the form $x^n + a_1x^{n-1} + ... + a_n = 0$; the symbolism $P(x)$ was unknown to him. Second, instead of the equal sign, =, he used the symbol ∞, most probably the inverted ligature of the first two letters of the verb *aequare* ("to equal").

10. One important by-product of this structural analysis of equations is a new and more refined concept of number. See Jakob Klein, *Greek Mathematical Thought and the Origins of Algebra* (Cambridge, Mass., 1968).

11. Here again a totally anachronistic term is employed in the interest of brevity.

12. Ironically, Descartes's method of determining the normal to a curve (bk. II, 342 ff.) made implicit use of precisely the same reasoning as Fermat's. This may have become clear to Descartes toward the end of a bitter controversy between the two men over their methods in the spring of 1638.

13. Cf. Vuillemin, pp. 35–55.

14.*Ibid*., pp. 11–25; Joseph E. Hofmann, *Geschichte der Mathematik*, II (Berlin, 1957), 13.

15. The anaclastic is a refracting surface that directs parallel rays to a single focus; Descartes had generalized the problem to include surfaces that refract rays emanating from a single point and direct them to another point. Cf. Milhaud, pp. 117–118.

16. The full title of the work Descartes suppressed in 1636 as a result of the condemnation of Galileo was *Le monde, ou Traitéde la lumière*. It contained the basic elements of Descartes's cosmology, later published in the *Principia philosophiae* (1644). For a detailed analysis of Descartes's work in optics, see A. I. Sabra, *Theories of Light From Descartes to Newton* (London,1967), chs. 1–4.

17. "One must note only that the power, whatever it may be, that causes the motion of this ball to continue is different from that which determines it to move more toward one direction than toward another," *Dioptrique* (Leiden, 1637), p. 94.

18. Cf. Descartes to Mydorge (1 Mar. 1638), "determination cannot be without some speed, although the same speed can have different determinations, and the same determination can be combined with various speeds" (quoted by Sabra, p. 120). A result of this qualification is that Descartes in his proofs treats speed operationally as a vector.

19. See the summary of this issue in Sabra, pp. 100 ff.

20. Cf. Carl B. Boyer, *The Rainbow: From Myth to Mathematics* (New York, 1959).

21. For a survey of Descartes's work on mechanics, which includes the passages pertinent to the subjects discussed below, see René Dugas, *La mécanique au XVII[e] siècle* (Neuchâtel, 1954), ch. 7.

22. Presented in full in the *Principia philosophiae*, pt. II, pars.24–54.

23. Cf. Milhaud, pp. 34–36.

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II. Secondary Literature. In addition to the works cited in the notes, see also J. F. Scott, *The Scientific Work of René Descartes* (London, 1952); Carl B. Boyer, *A History of Analytic Geometry* (New York, 1956); Alexandre Koyré *Études galiléennes* (Paris, 1939); E. J. Dijksterhuis, *The Mechanization of the World Picture* (Oxford, 1961). See also the various histories of seventeenth-century science or mathematics for additional discussions of Descartes's work.

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