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(*b*. York, Pennsylvania, 13 January 1876; *d*. Princeton, New Jersey, 28 October 1965)

mathematics.

Eisenhart was the second son of Charles Augustus Eisenhart and the former Emma Pfahler. His father was a dentist, a founder Emma Pfahler. His father was a dentist, founder of the Edison Electric Light and York Telephone companies, and secretary of the Sunday school of St. Paul' Lutheran Church. Eisenhart was taught by his mother before he entered school and completed grade school in three years. He then attended York High School until, in his junior year, he was encouraged by the principal to withdraw and devote his time to the independent study of Latin and Greek for early admission to Gettysburg College, which he attended from 1892 to 1896. Being the only upper-division mathematics student, during the last two years of college Eisenhart studied mathematics through independent guided reading.

After teaching for a year at the preparatory school of the college, he began graduate study at Johns Hopkins University in 1897 and obtained the Ph.D. in 1900 with a thesis whose topic, "Infinitesimal Deformations of Surfaces," he had chosen himself. He was introduced to differential geometry through a lecture by Thomas Craig and studied the subject through the treatises of Gaston Darboux. According to his own testimony, the experience of independent study led Eisenhart to propose the four-course plan of study adopted at Princeton in 1923, which provides for independent study and the preparation of a thesis. Eisenhart's scientific career was spent at Princeton; he retired in 1945.

In 1908 Eisenhart married Anna Maria Dandridge Mitchell of Charles Town, West Virginia; she died in 1913. In 1918 he married Katharine Riely Schmidt of York, Pennsylvania. He had one son, Churchill, by his first marriage and two daughters, Anna and Katharine, by his second.

Eisenhart's work in differential geometry covers two distinct periods and fields. The first period, to about 1920, was devoted mainly to the theory of deformations of surfaces and systems of surfaces.

Modern differential geometry was founded by Gaston Darboux as a field of applications of partial differential equations. His methods were taken up by Luigi Bianchi, who created an extensive theory of the deformations of surfaces of constant negative curvature. In another direction, Claude Guichard showed between 1897 and 1899 how the partial differential equations of the deformations of triply orthogonal systems of surfaces can be interpreted in terms of the systems of lines connecting a point and its image point. These discoveries made the theory of deformations of surfaces one of the focal points of geometric research in Europe at the turn of the century. Although there were quite a number of able mathematicians working in America in the field of geometry at that time, Eisenhart was the only one to turn to the topic of deformations. His main contribution to the topic of deformations. His main contribution to the theory was a unifying principle: The deformation of a surface defines the congruence (two-parameter family) of lines connecting a point and its image (following Guichard). In general, a congruence contains two families of developable surfaces (a developable surface is formed by the tangent to a space curve). Eisenhart recognized that in all known cases, the intersections of these surfaces with the given surface and its image form a net of curves with special properties. This allows not only a unified treatment of many different subjects and a replacement of tricks by methods, but also leads to many new results that round off the theory. Eisenhart gave a coherent account of the theory in *Transformations of Surfaces* (1923). The book also contains most of Eisenhart's previous results either in the text or in the exercises, with references. Some aspects of the theory were taken up later in the projective setting by Eduard Čech and his students. All these investigations deal with small neighborhoods for which existence theorems for solutions of differential equations are available.

Of the few papers not dealing with deformations dating from this period, a noteworthy one is "Surfaces Whose First and Second Forms Are Respectively the Second and First Forms of Another Surface" (1901), one of the first differential geometric characterizations of the sphere, a topic started by Heinrich Liebmann in 1899. Eisenhart proved that the unit sphere is the only surface whose first and second fundamental forms are, respectively, the second and first fundamental forms of another surface.

Einstein's general theory of relativity (1916) made Riemannian geometry the center of geometric research. The analytic tools that turned Riemannian geometry from an idea into an effective instrument were Ricci's covariant differential calculus and the related notion of Levi-Civita's parallelism. These tools had been thoroughly explored in Luigi Bianchi's *Lezioni di geometria differenziale*. As a consequence, the attention of geometers immediately turned to the generalization of Riemannian geometry. Most of Eisenhart's work after 1921 was in this direction. The colloquium lectures *Non-Riemannian Geometry* (1927) contain his account of the main results obtained by him and his students and collaborators. An almost complete coverage of Eisenhart's results, with very good references, is given in Schouten's *Ricci Calculus*. Three directions of generalization of Riemannian geometry were developed in the years after 1920. They are connected with the names of Élie Cartan, Hermann Weyl, and Eisenhart. Cartan considered geometries that induce a geometry of a transitive transformation group in any tangent space. Weyl gave an axiomatic approach to the maps of tangent spaces by parallelism along any smooth curve. Eisenhart's approach, inspired by Oswald Veblen's work on the foundations of projective geometry and started in cooperation with Veblen, is the only one to deal directly with the given space. In Riemannian geometry, the measure of length is prescribed and the geodesic lines are determined as the shortest connections between nearby points. In Eisenhart's approach, the geodesics are given as the solution of a prescribed system of second-order differential equations and the non-Riemannian geometries are obtained by asking that there should exist a Levi-Civita parallelism for which the tangents are covariant constant.

While Cartan's and Weyl's generalizations have become the foundations of the fiber space theory of differentiable manifolds, Eisenhart's theory does not fit the framework of these topological theories. The reason is that the geometric objects intrinsically derived from the "paths" of the geometry, the projective parameters of Tracy Y. Thomas, have a more complicated transformation law than the generalized Christoffel symbols of Cartan and Weyl. However, there are a number of modern developments, such as the theory of Finsler spaces and the general theory of the geometric object, that fit Eisenhart's framework but not that of the algebraic-topological approach. As far as metric geometry is concerned, the most fruitful approach seems to be to give the geodesics directly as point sets and to throw out all differential equations and analytical apparatus. On the other hand, for nonmetric geometries Eisenhart proved (in "Spaces With Corresponding Paths"[1922])that for every one of his geometries there exists a unique geometry with the same paths and for which the mapping of tangent spaces induced by the flow of tangent vectors with unit speed along the paths is volume-preserving. For the latter geometry, which would appear to give a natural setting for topological dynamics, the Cartan, Weyl, and Eisenhart approaches are equivalent.

A Number of interesting avenues of development of Riemannian geometry were opened by Eisenhart. The papers "Fields of Parallel Vectors in the Geometry of Paths"(1922) and "Fields of Parallel Vectors in a Riemannian Geometry" (1925) started the topic of recurrent fields and harmonic spaces (for a report with later references, see T. J. Willmore, *An Introduction to Differential Geometry*, ch. 7, sec. 13). The so-called Eisenhart's theorem appears in "Symmetric Tensors of the Second Order Whose First Covariant Derivatives Are Zero" (1923): If a Riemannian geometry admits a second-order, symmetric, covariant constant tensor other than the metric, the space behaves locally like the product of two lowerdimensional spaces. Together with a theorem of Georges de Rham to the effect that a simply connected, locally product Riemannian space is in fact a Cartesian product of two spaces, the theorem is an important tool in global differential geometry. An extension of the theorem is given in "Parallel Vectors in Riemannian Space" (1938).

The basic equations for the vectors of a group of motions in a Riemannian space had been given by Killing in 1892. Eisenhart developed a very powerful analytical apparatus for these questions; the results are summarized in *Riemannian Geometry* (1926; ch. 6) and *Continuous Groups of Transformations* (1933). The later developments are summarized in Kentaro *Yano's Group of Transformations in Generalized Spaces*(1949).

Eisenhart's interest in mathematical instruction found its expression in a number of influential text-books—such as *Differential Geometry of Curves and Surfaces* (1909), *Riemannian Geometry*(1926),*Continuous Groups of Transformations* (1933), *Coordinate Geometry* (1939), *An Introduction to Differential Geometry With Use of the Tensor Calculus* (1940)—some in fields that until then had been dependent upon European monographs devoid of exercises and other student aids. His interest in history resulted in several papers; "Lives of Princeton Mathematicians" (1931), "Plan for a University of Discoverers" (1947), "Walter Minto and the Earl of Buchan" (1950), and the preface to "Historic Philadelphia" (1953).

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