Eisenstein, Ferdinand Gotthold Max | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
16-20 minutes

(*b*. Berlin, Germany, 16 April 1823; d. Berlin, 11 October 1852)

Mathematics.

Eisenstein's father, Johann Konstantin Eisenstein, and his mother, the former Helene Pollack, had converted from Judaism to Protestantism before Gotthold was born. His father, who had served eight years in the Prussian army, tried his hand at various commercial enterprises. Including manufacturing, but without financial success. Not until late in life did he begin to make a decent livelihood. Eisenstein's five brothers and sisters, born after him, died in childhood, nearly all of meningitis, which he also contracted. His interest in mathematics, awakened and encouraged by a family acquaintance, began when he was about six. "As a boy of six I could understand the proof of a mathematical theorem more readily than that meat had to be cut with one's knife, not one's fork" ("Curriculum vitae," p. 150). Early, too, Eisenstein showed musical inclinations that continued throughout his life and that found expression in playing the piano and composing.

Evening while he was in elementary school, his persistently poor health prompted his parents to send him for a time to board in the country. From about 1833 to 1837 he was a resident student at the Cauer academy in Charlottenburg (near Berlin), where the quasi-military discipline was little to his taste. The effects upon him of its Spartan pedagogical methods were manifested in frequent, often feverish illnesses and depression. From September 1837 to July 1842 he attended the Friedrich Wilhelm Gymnasium and then, as a senior, the Friedrich Werder Gymnasium in Berlin. In addition, he went to hear Dirichlet and others lecture at the university.

What attracted me so strongly and exclusively to mathematics, apart from its actual content, was especially the specific nature of the mental operation by which mathematical things are dealt with. This way of deducing and discovering new truths from old ones, and the extraordinary clarity and self-evidence of the theorems, the ingeniousness of the ideas... had an irresistible fascination for me.... Starting from the individual theorems, I soon grew accustomed to pierce more deeply into their relationships and to grasp whole theories as a single entity. That is how I conceived the idea of mathematical beauty.... And there is such a thing as a mathematical sense or instinct that enables one to see immediately whether an investigation will bear fruit, and to direct one's thoughts and efforts accordingly ["Curriculum vitae," pp. 156–157].

Eisenstein had the good fortune to find in the meteorologist Heinrich W. Dove and the mathematician Karl Schellbach teachers who understood and encouraged him. What he learned in class and at lectures led him to deeper, independent study of the works of Euler, Lagrange, and Gauss, although it was the last who influenced him most. In the summer of 1842, before completing school, he accompanied his mother to England to join his father, who had gone there two years earlier in search of a better livelihood. In neither England, Wales, nor Ireland could the family gain a firm footing. Eisenstein used the time to steep himself in Gauss's *Disquisitiones arithmeticae* and started on his own to study forms of the third degree and the theory of elliptic functions. In Dublin in early 1843 he made the acquaintance of W. R. Hamilton, who gave him a copy of his work "On the Argument of Abel, Respecting the Impossibility of Expressing a Root of Any General Equation Above the Fourth Degree," to be presented to the Berlin Academy.

By around mid-June 1843 Eisenstein and his mother were back in Berlin. His parents were now living apart, and from then until his death Eisenstein stayed with his mother only briefly from time to time. In August 1843 he applied to the Friedrich Wilhelm Gymnasium in Berlin for permission, as a nonstudent, to take their final examinations (a prerequisite for admission to regular university study). In the brief autobiography appended to his application he mentioned (at age twenty) the "hypochondria that has been plaguing me for two years." On 22 September 1843 Eisenstein passed his final secondary school examination, and Schellbach wrote of him in his report: "His knowledge of mathematics goes far beyond the scope of the secondary-school curriculum. His talent and zeal lead one to expect that some day he will make an important contribution to the development and expansion of science" (a remarkable opinion, compared with the wrong ones put forth by other teachers, Galois for example).

Immediately after passing his examinations, Eisenstein enrolled at the University of Berlin. In January 1844 he delivered to the Berlin Academy the copy of Hamilton's study that he had received in Dublin, using the occasion to submit a treatise of his own on cubic forms with two variables. A. L. Crelle, whom the Academy had commissioned to evaluate Eisenstein's work and make appropriate reply to him on its behalf, accepted the treatise for publication in his *Journal für die reine und angewandte Mathematik*, thus again demonstrating Crelle's keen eye for mathematical genius, which had earlier spotted Abel, Jacobi,

Steiner, and, later, Weierstrass. At the same time, Crelle introduced the young author to [Alexander von Humboldt](), who immediately took an interest in him. Time and again Humboldt requested financial support for Eisenstein from the Prussian ministry of education, the king, and the Berlin Academy, and often helped him out of his own pocket. Eisenstein had no feeling of economic security, since these official grants were awarded only for short periods and always had to be reapplied for, with the approved extensions often arriving late and the sums involved being quite modest and certainly not owed to the recipient. His constant dependence on gifts and charity weighed heavily on him, yet he had found in Humboldt a tireless mentor and protector, the like of which few young talents are ever blessed with. And Humboldt made it clear that he valued Eisenstein not only as a promising young scholar but also as a human being, and with tact and sensitivity he tried (albeit in vain) to divert and cheer him.

The twenty-seventh and twenty-eighth volumes of Crelle's *Journal*, published in 1844, contained twenty-five contributions by Eisenstein. These testimonials to his almost unbelievable, explosively dynamic productivity rocketed him to fame throughout the mathematical world. They dealt primarily with quadratic and cubic forms, the reciprocity theorem for cubic residues, fundamental theorems for quadratic and biquadratic residues, cyclotomy and forms of the third degree, plus some notes on elliptic and Abelian transcendentals. Gauss, to whom he had sent some of his writings, praised them very highly and looked forward with pleasure to an announced visit. In June 1844, carrying a glowing letter of recommendation from Humboldt, Eisenstein went off to see Gauss. He stayed in Göttingen fourteen days. In the course of the visit he won the high respect of the "prince of mathematicians," whom he had revered all his life. The sojourn in Göttingen was important to Eisenstein for another reason: he became friends with Moritz A. Stern—the only lasting friendship he ever made. While the two were in continual correspondence on scientific matters, even Stern proved unable to dispel the melancholy that increasingly held Eisenstein in its grip. Even the sensational recognition that came to him while he was still only a third-semester student failed to brighten Eisenstein's spirits more than fleetingly. In February 1845, at the instance of Ernst E. Kummer, who was acting on a suggestion from Jacobi (possibly inspired by Humboldt), Eisenstein was awarded an honorary doctorate in philosophy by the School of Philosophy of the University of Breslau.

The year 1846 found Eisenstein suddenly involved in an unpleasant priority dispute with Jacobi, who accused him of plagiarism and of misrepresenting known results. Writing to Stern on 20 April 1846, Eisenstein explained that "the whole trouble is that, when I learned of his work on cyclotomy, I did not immediately and publicly acknowledge him as the originator, while I frequently have done this in the case of Gauss. That I omitted to do so in this instance is merely the fault of my naïve innocence." Jacobi charged him with scientific frivolity and appropriating as his own the ideas impaited to him by others, and he maintained that Eisenstein had no original achievements to his credit but had merely cleverly proved certain theorems stated by others and carried out ideas conceived by others. This was in curious contrast with Jacobi's attitude in 1845, when he had recommended Eisenstein for the honorary doctorate.

In 1846–1847 Eisenstein published various writings, mainly on the theory of elliptic functions. Humboldt, who had tried in vain in 1846 to draw the attention of Crown Prince Maximilian of Bavaria to Eisenstein, early in 1847 recommended him for a professorship at Heidelberg—even before he had earned his teaching credentials at the University of Berlin—but again without success. During the summer semester of 1847 Riemann was among those who attended Eisenstein's lecture on elliptic functions. In September 1847 a great honor came to Eisenstein: Gauss wrote the preface to a volume of his collected treatises. No longer extant, unfortunately, is the letter from Gauss to Eisenstein in which, the latter reported to Riemann, Gauss set down the essentials of his proof of the biquadratic reciprocity law with the aid of cyclotomy.

Early in 1848 Eisenstein had attended meetings of certain democratically oriented clubs, although he took no active part in the pre-March political ferment. During the street battles on 19 March, however, he was forcibly removed from a house from which shots had been fired and was taken with other prisoners to the Citadel at Spandau, suffering severe mistreatment en route. Although he was released the next day, the experience gravely affected his health. Moreover, when word spread that he was a "republican", financial support for him dwindled, and it took Humboldt's most strenuous efforts to keep it from drying up altogether. Eisenstein's situation visibly worsened. Alienated from his family and without close friends or any real contact with other Berlin mathematicians, he vegetated. Only occasionally did he feel able to deliver his lectures as *Privatdozent*, from his bed, if he managed to lecture at all. Yet all this time he was publishing one treatise after another in Crelle's *Journal*, especially on the quadratic partition of prime numbers, on reciprocity laws, and on the theory of forms. In August 1851, on Gauss's recommendation, both Eisenstein and Kummer were elected corresponding members of the Göttingen Society, and in March 1852 Dirichlet managed his election to membership in the Berlin Academy. In late July of that year Eisenstein suffered a severe hemorhage. Funds raised by Humboldt so that Eisenstein could spend a year convalescing in Sicily came too late: on 11 October he died of pulmonary tuberculosis. Despite all the public recognition, he ended his days in forlorn solitude. The eighty-three-year-old Humboldt accompanied the graveside.

Eisenstein soon became the subject of legend, and the early literature about him is full of errors. Only latter-day research has illumined the tragic course of his life. For instance, no evidence at all has been found of the dissolute existence that he was frequently rumored to have led. His lectures were usually attended by more than half of Berlin's mathematics students, which was the more remarkable since Dirichlet, Jacobi, and Steiner were then teaching at Berlin. Eisenstein was ever at pains, as he himself emphasized, to bring home to his listeners the most recent research results.

His treatises were written at a time when only Gauss, Cauchy, and Dirichlet had any conception of what a completely rigorous mathematical proof was. Even a man like Jacobi often admitted that his own work sometimes lacked the necessary rigor and self-evidence of methods and proofs. Thus it is not surprising that, as Leo Koenigsberger tells us, Eisenstein's "Study of the

Infinite Double Products, of Which Elliptic Functions Are Composed as Quotients" should have been criticized by Weierstrass, who, in representing his own functions in terms of infinite products, was not picking up the t9orch from his forerunner, Eisenstein, but was drawing directly upon Gauss. Weierstrass correctly rated Riemann over Eisenstein, who was unable to grasp Riemann's general ideas about functions of complex variables. While Klein did concede that the simplest elliptic functions are defined by Eisenstein's everywhere absolutely convergent series, he called Eisenstein a "walking formula who starts out with a calculation and then finds in it the roots of all his knowledge" Unjustly Klein attributed to him a persecution complex and megalomania. Eisenstein's oft-quoted statement to the effect that through his contributions to the theory of forms (including his finding the simplest covariant for the binary cubic form) he hoped "to become a second Newton" (letter to Humboldt, July 1847) is nothing more than a bad joke.

The development that led to the reciprocity law of *nth*-power residues will be permanently associated with Eisenstein's work on cubic and biquadratic reciprocity laws. The Eisenstein series have become an integral part of the theory of modular forms and modular functions. They and the Eisenstein irreducibility law (along with the Eisenstein polynomial and the Eisenstein equation) continue to bear his name and to assure him a position about halfway between that contemptuous assessment by Klein and the verdict of Gauss (expressed, of course, in a letter intended for display), who held Eisenstein's talents to be such as "nature bestows on only a few in each century" (letter to Humboldt, 14 April 1846).

BIBLIOGRAPHY

I. Original Works. Nearly all of Eisenstein's scientific writings were published in the *Journal für die reine und angewandte Mathematik*, specifically in vols. **27** (1844) to **44** (1852); see the bibliography by Kurt-R. Biermann in *Journal für die reine und angewandte Mathematik*, **214/215** (1964), 29–30. Selected *Mathematische Abhandlungen besonders aus dem Gebiete der höhern Mathematik und der elliptischen Functionen* (Berlin, 1847) were published with a preface by Gauss; repr., with intro. by Kurt-R. Biermann (Hildesheim, 1967). An autobiography, "Curriculum vitae des Gotth. Ferdinand Eisenstein," ed. and with intro. by F. Rudio, was published in *Zeitschrift für Mathematik und physik*, **40** (1895), supp., 143–168. The letters from Eisenstein to M. A. Stern were published by A. Hurwitz and F. Rudio in Zeitschrift für Mathematik und Physik, **40** (1895), supp., 169–203. A report by Eisenstein on his imprisonment is found in Adalbert Roerdansz, *Ein Freiheits Martyrium. Gefangene Berliner auf dem Transport nach Spandau am Morgen des 19. März* 1848(Berlin, 1848), pp. 130–135. A bibliography of Eisenstein's writings is given by Kurt-R. Biermann in *Istoriko-mathematicheskie issledovaniya*, **12** (1959), 493–502.

Historical records are available primarily at the following institutions: Archiv der Deutschen Akademie der Wissen schaften zu Berlin; Archiv der Humboldt-Universitat zu Berlin; Deutsche Staatsbibliothek, Göottingen; Archiv der Akademie der Wissenschaften, Göottingen; and Deutsches Zentralarchiv, Historische Abteilung II, Merseburg. See also the survey by Kurt-R. Biermann in *Journal für die reine und angewandte Mathematik*, **214/215** (1964), 28

II. Secondary Literature. See the bibliography by Kurt-R. Biermann in *Journal für die reine und angewandte Mathematik*, **214/215** (1964), 28–29. Only the literature devoted directly to Eisenstein will be cited here. See Wilhelm Ahrens, "Gotthold Eisenstein," in *Deutsche allgemeine Zeitung*, no. 177 (17 April 1923); Moritz Cantor, "Eisenstein," in *Allgemeine deutsche Biographie*, V (1877), 774, which contains errors; J. Loewenberg, "A. V. Humboldt und G. Eisenstein," in *Allgemeine Zeitung des Judenthums*, **55** (1891), 246–248; and Julius Schuster, "A. v. Humboldt und F.G. Eisenstein," in Janus, **26** (1922), 99.

See also the following works by Kurt-R. Biermann: "A. V. Humboldt als Protektor G. Eisensteins und dessen Wahl in die Berliner Akademie," in Forschungen und Fortschritte, **32** (1958), 78–81; "Zur Geschichte der Ehrenpromotion G. Eisensteins," *ibid.*,332–335; "Die Briefe A. v. Humboldts an F.G. M. Eisenstein," in Alexander uon Humboldt, gedendschrift (Berlin, 1959), 117–159; "A. L. Crelles Verhaltnis zu G. Eisentein," in *Monatsbericht der Deutschen Akademie der Wissenschaften zu Berlin*, **1** (1959), 67–72; "Eisenstein," in *Neue deutsche Biographie*, IV (1959), 420–421; "Einige neue Ergebnisse der Eisenstein-Forschung," in *Zeitschrift für Geschichte der Naturwissenschaften, Technik und Medizin*, **1** no. **2** (1961), 1–12; and "G. Eisenstein, Die wichtigsten Daten seines Lebens und Wirkens," in *Journal für die reine und angewandte Mathematik*, **214/215** (1964), 19–30.

Kurt-R. Biermann