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(*b*. Cyrene [now Shahhat, Libya], *ca*. 276 b.c.; *d*. Alexandia, *ca*. 195 b.c.)

geography, mathematics.

Eratosthenes, son of Aglaos, was born in Cyrene but spent most of his working life in Alexandria, where he was head of the library attached to the famous Museum from *ca*. 235 until his death. At some period during his early manhood he went to Athens for the ancient equivalent of a university education, and there he associated with the Peripatetic Ariston of Chios, Arcesilaus and Apelles of the Academy, and Bion the Cynic (Strabo, *Geography*, 15). When he was about thirty, he was invited to Alexandria by King Ptolemy III (Euergetes I), possibly at the instigation of Eratosthenes' fellow countryman Callimachus, who had already been given a post in the library by Ptolemy II (Philadelphus). On the death of the first chief librarian, Zenodotus, *ca*. 235, Eratosthenes was appointed to the post, Callimachus having died *ca*. 240 (*Suda Lexicon, s.v*., calls Eratosthenes a pupil of Callimachus). At some time during his stay in Alexandria he became tutor to Euergetes' son and remained in favor with the royal court until his death. (See the anecdote related in Athenaeus, *Deipnosophistai*, VII, 276a, concerning Eratosthenes and Queen Arsinoe III.)

The above represents the most probable account of Eratosthenes' life according to the consensus of scholarly opinion, but the exact dates of the stages of his career are disputed and certainty is unattainable. In particular, Knaack puts the date of his birth back to *ca*. 284, and Jacoby (*Fragmente der griechischen Historiker*, IIB [1930], 704) even as far as 296 (suggesting that in the *Suda Lexicon, s.v*., ϱκς′ is a copyist's error for ϱκα′, which then refers to the 121st olympiad, i.e., 296–293, not the 126th, i.e., 276–273), while the date of his death becomes either about 203 (Knaack) or 214 (Jacoby), both scholars accepting the testimony of our sources that Eratosthenes died at eighty (the *Suda Lexicon*) or eighty-one (Censorinus, *De die natali*, p. 15) or eighty-two (Pseudo-Lucian, *Μακϱόβιοι*, p. 27). The reason for supposing that he must have been born earlier than 276 is that Strabo calls him γνώϱιμος of Zeno of Citium (the founder of Stoicism), a word that often means "pupil" in such a context; but Zeno died in 262, and Eratosthenes could hardly have studied under him at the tender age of fourteen. To this it may be answered that γνώϱιμος can also mean simply "acquainted with," and that the date of Zeno's death may be as late as 256 (see Diogenes Laertius, VII, 6: 28). There is also considerable doubt about the order of succession of the early librarians at Alexandria. A papyrus fragment (*Oxyrhynchus papyri*, X, 1241, col. 2) lists them as Zenodotus (whose name is presumed to have occurred at the damaged end of the previous column), Apollonius Rhodius, Eratosthenes, Aristophanes of Byzantium, Aristarchus of Samothrace, and another Apollonius; but there are several mistakes and chronological difficulties in this list (*cf*. Grenfell and Hunt, *ad loc*.), and it is by no means certain that Apollonius Rhodius succeeded Zenodotus directly—the *Suda Lexican* (*s.v*. "Apollonius") has him succeeding Erastosthenes, although this may arise from confusion with the later Apollonius (if he is correctly placed).

Eratosthenes was one of the foremost scholars of his time and produced works (of which only fragments remain) on geography, mathematics, philosophy, chronology, literary criticism, and grammar as well as writing poetry. According to the *Suda Lexicon*, he was described as Πένταθλος ("All-Rounder"). "another Plato," and "Beta"—the last possibly because, working in so many fields (and polymathy was greatly admired by the Alexandrians), he just failed to achieve the highest rank in each (see Strabo's remark that Eratosthenes was a mathematician among geographers and a geographer among mathematicians: *Geography*, **94** ; *cf*. 15), or perhaps simply because he was the second chief librarian. His most enduring work was in geography (particularly notable is his measurement of the circumference of the earth), but he himself seems to have taken most pride, as regards his scientific work, in his solution to the famous problem of doubling the cube, to celebrate which he composed an epigram disparaging previous solutions and dedicated to Euergetes and his son; the authenticity of this poem has been questioned (by Hiller and by Powell), but on inadequate grounds. As a mathematician, Eratosthenes ranked high enough in the estimation of the great Archimedes to have one of the latter's treatises, the *Method*, dedicated to him and to be the recipient of a difficult problem in indeterminate analysis, known as the "Cattle Problem," for communication to the mathematicians of Alexandria. In philosophy, Eratosthenes was an eclectic and, according to Strabo (*Geography*, 15), somewhat of a dilettante. He was the first Greek writer to make a serious study of chronological questions and established the system of dating by olympiads, while as an authority on Old Comedy he is constantly cited in the scholia to Aristophanes' plays.

Eratosthenes' *Geography* (Γεωγϱαφικά) was in three books, as we learn from Strabo, who quotes from it frequently and is, in fact, the chief source of our knowledge of it. It long remained a prime authority on geographical matters; Julius Caesar evidently consulted it, since in his description of the Germans he mentions that Eratosthenes knew of the Hercynian Forest (*De bello Gallico*, VI, 24), and Strabo (writing around the turn of the Christian era) admits that for the southeastern quarter of the inhabited world (*oikoumene*) he has no better authority than Eratosthenes (*Geography*, 723). The work was the first scientific attempt to put geographical studies on a sound mathematical basis, and its author may be said to have been the founder of mathematical geography. It was concerned with the terrestrial globe as a whole, its division into zones, changes in its surface, the position of the *oikoumene* as then known, and the actual mapping of it, with numerous estimates of distances along a few roughly defined parallels and meridians; but it also contained a certain amount of material descriptive of peoples and places.

Strabo, who disliked the mathematical side of the subject and much preferred purely descriptive geography (see *Geographical Fragments of Hipparchus*, pp. 36, 162, 164, 171, 191), several times complains that Eratosthenes put too much emphasis on mathematical topics such as the above (*Geography*, 48–49, 62, 65). Hipparchus (second century B.C.), on the other hand, criticizes his predecessor for not making sufficient use of astronomical data in fixing the reference lines of his map and not treating the subject in a mathematical enough manner. (Hipparchus wrote a work in three books, *Against the Geography of Eratosthenes*, of which we have substantial fragments quoted by Strabo, often inextricably mingled with citations from Eratosthenes himself—see *Geographical Fragments of Hipparchus*.) One of Eratosthenes' main purposes was to correct the traditional Ionian map, which had a round oikoumene with Delphi at the center, wholly surrounded by a circular ocean (as envisaged, e.g., by Anaximander and Hecataeus and already ridiculed by Herodotus, *History*, IV, 36, 2), and to sketch a better one (Strabo, *Geography*, 68), making use of all the data at his command—which, as head of the largest library in antiquity, must have been considerable (*ibid*., 69).

Eratosthenes used as his base line a parallel running from Gibraltar through the middle of the Mediterranean and Rhodes, to the Taurus Mountains (Toros Dağlari, in Turkey), which were extended due east to include the Elburz range (south of the Caspian), the Hindu Kush, and the Himalayas, which formed the northern boundary of India (such a line, approximately bisecting the known world, had already been suggested in the previous century by Dicaearchus, a pupil of Aristotle—see *Geographical Fragments of Hipparchus*, p. 30). Intersecting this main parallel at right angles was a meridian line taken as passing through Meroë, Syene (modern Aswan, on the Tropic of Cancer), Alexandria, Rhodes, and the mouth of the Borysthenes (modern Dnieper—*ibid*., pp. 146–147). Wherever Eratosthenes found in his sources data (such as distances in stades, similarities in fauna, flora, climate, or astronomical phenomena, lengths of the longest days, etc., recorded at different places) that he could correlate with one or both of the above base lines, he was enabled to sketch in other parallels. In addition, he divided at least the southeastern quarter of the *oikoumene* (we have no information about his treatment of the remainder) into rough geometrical figures shaped like parallelograms, which he called "seals", (σφραγῖδες) forming the first "seal" out of India and working westward (*ibid*., pp. 128–129).

Naturally, the data at his disposal, mainly travelers' estimates of days' voyages and marches, which are notoriously unreliable—the only scientific data available were the gnomon measurements of Philo, prefect of Ptolemy, at Meroë (Strabo, *Geography*, 77), of Eratosthenes himself at Alexandria, and of Pytheas at Marseilles (*ibid*., 63), together with some sun heights recorded by the latter (*Geographical Fragments of Hipparchus*, p. 180)—were of dubious accuracy, and any mapping done on the basis of them was bound to be largely guesswork. Hipparchus has no difficulty in showing that the figures and distances given by Eratosthenes are mathematically inconsistent with each other, and he therefore rejects them, together with some of the sensible alterations proposed by Eratosthenes for the traditional map, thus demonstrating that inspired guesswork sometimes gives better results than scientific caution (*ibid*., pp. 34–35).

It is uncertain whether the measurement of the earth's circumference was first published in the *Geography* or in a separate treatise; if the latter, it would at any rate have been mentioned in the larger work. The method is described in detail by Cleomedes (*De motu circulars*, I, 10), the only ancient source to give it. Assuming that Syene was on the Tropic of Cancer (because there, at midday on the summer solstice, the gnomon—i.e., a vertical pointer set upright on a horizontal base—cast no shadow and a well, especially dug for this purpose [according to Pliny, *Natural History*, **II,** 73] was illuminated to its bottom by the sun's rays), and that this town and Alexandria were on the same meridian, Eratosthenes made a measurement of the shadow cast at Alexandria at midday on the solstice by a pointer fixed in the center of a hemispherical bowl, known as a "scaphe" (σκάφη—presumably he used this form of gnomon because the shadow of a thin stylus would be better defined than that of a large pillar or post) and estimated that the shadow amounted to 1/25 of the hemisphere, and thus 1/50 of the whole circle. Since the rays of the sun can be regarded as striking any point on the earth's surface in parallel lines, and the lines produced through the vertical gnomons at each place meet at the center of the earth, the angle of the shadow at Alexandria (*ABC* in Figure 1) is equal to the alternate angle (*BCD*) subtended by the arc *BD*, which is the distance along the meridian between Alexandria and Syene, estimated by Eratosthenes at 5,000 stades; and since it is 1/50 of the whole circle, the total circumference must be 250,000 stades. This

is the figure reported by Cleomedes. Hipparchus accepts a figure of 252,000 stades as Eratosthenes' measurement (Strabo, *Geography*, 132, corroborated by Pliny, *Natural History*, II, 247, whose further statement that Hipparchus added 26,000 stades to Eratosthenes' figure is incorrect—see *Geographical Fragments of Hipparchus*, p. 153), and it seems fairly certain that Eratosthenes himself added the extra 2,000 in order to obtain a number readily divisible by 60; he divided the circle into sixtieths only (Strabo, *Geography*, 113–114), the familiar division into 360° being unknown to him and first introduced into Greek science by Hipparchus (*Geographycal Fragments of Hipparchus*, pp. 148–149; D. R. Dicks, "Solstices, Equinoxes, and the Pre-Socratics," in *Journal of Hellenic Studies*, **86** [1966], 27–28).

The method is sound in theory, as Hipparchus recognized, but its accuracy depends on the precision with which the basic data could be determined. The figure of 1/50 of the circle (equivalent to 7°12′) for the difference in latitude is very near the truth, but Syene (lat. 24°4′N.) is not directly on the tropic (which in Eratosthenes' time was 23°44′N.), Alexandria is not on the same meridian (lying some 3° to the west), and the direct distance between the two places is about 4,530 stades, not 5,000. Probably Eratosthenes himself was aware that this last figure was doubtful (without trigonometrical methods, which he certainly did not know, it would have been impossible to measure the distance accurately), and so felt at liberty to increase his final result by 2,000. Nonetheless, the whole measurement was a very creditable achievement and one that was not bettered until modern times. On the most probable value of the stade Eratosthenes used (on this vexed question, see *Geographical Fragments of Hipparchus*, pp. 42–46), 252,000 stades are equivalent to about 29,000 English miles, which may be compared with the modern figure for the earth" circumference of a little less than 25,000 miles.

He obtained a value for the obliquity of the ecliptic

equivalent to 23; 51, 20° a figure accepted as accurate by both Hipparchus and Ptolemy. Apparently he estimated the arc between the greatest and least meridian altitudes of the sun (at summer and winter solstices) to be 11/83 of a great circle. This value, which is twice that for the obliquity of the ecliptic, is 47; 42, 39° +. How he discovered this curious ratio (if he did) is not clear (*ibid*., fr. 41 and comment, pp. 167–168), and whether this measurement was fully described in the *Geography* or elsewhere cannot be determined—Strabo does not mention it, and he was undoubtedly writing with a copy of the *Geography* before him. What certainly would have found a place in this work was Eratosthenes' division of the terrestrial globe into zones. Of these he envisaged five (see Figure 2); a frigid zone around each pole, with a radius of 6/60 each, or 25,200 stades on the meridian circle (in his division of the circle into sixtieths, each sixtieth = 252,000 ÷ 60 = 4,200 stades), a temperate zone between each frigid zone and the tropics, with a radius of 5/60 or 21,000 stades, and a torrid zone comprising the two areas from the equator to each tropic, with a radius of 4/60 or 16,800 stades each (4/60 is equivalent to 24° an approximate figure for the obliquity of the ecliptic known probably from the time of Eudoxus and used occasionally even by Hipparchus, e.g., *Commentarii in Arati et Eudoxi Phaenomena*, I, 10, 2)—making a total of 126,000 stades from pole to pole, i.e., half the whole circumference (see Geminus, *Isagoge*, XVI, 6f.; V, 45 f.; Strabo, *Geography*, 113–114; *cf*. 112). The frigid zones were arbitrarily defined by the "arctic" and "antarctic" circles of an observer on the main parallel of latitude (roughly 36° N.), i.e., the circles marking the limits of the circumpolar stars that never rise or set and the stars that are never visible at that latitude (see *Geographical Fragments of Hipparchus*, pp. 165–166). Within this framework the *oikoumene*, according to Eratosthenes, has a "breadth" (north-south, as always in Greek geography) of 38,000 stades from the Cinnamon country (south of Meroe) to Thule, and a "length" (east-west) of 77,800 stades from the further side of India to beyond the Straits of Gibraltar (Strabo, *Geography*, 62–63, 64).

Although it is clear from the *Geography* that Eratosthenes was familiar with the concept of the celestial sphere, he does not seem to have done any original work in astronomy apart from the above measurements made in a geographical context; his name is not connected with any purely astronomical observation (figures for the distance and size of the sun attributed to him by Eusebiuse of Caesarea, *Praeparatio evangelica*, XV, 53, and Macrobius, *In somnium Scipionis*, I, 20, 9 are worthless, coming from these sources), he does not appear among the authorities cited by Ptolemy in the *Phaseis* for data relating to the parapegmata or astronomical calendars (see *Geographical Fragments of Hipparchus*, pp. 111–112), and only one astronomical title is attributed to him (and that wrongly): the fragmentary *Catasterismoi* (Robert, ed. [Berlin, 1878]; see Maass, "Analecta Eratosthenica," in *Philogische Untersuchungen*, **6** [1883], 3–55), which tells how various mythical personages were placed among the stars and gave their names to the different constellations, descriptions of which are given. It is possible that an inferior second-century compilation of the same nature, called *Poetica astronomica* (Bunte, ed. [Leipzig, 1875]) and going under the name of the Augustan scholar Hyginus, is based partly on a work of Eratosthenes, who is cited some twenty times (as against, e.g., ten times for Aratus), but this is hardly serious astronomy (see Rose, *Handbook of Latin Literature*, 3rd ed. [1954], p. 447).

In mathematics, Eratosthenes' chief work seems to have been the *Platonicus*, of which we have a few extracts given by Theon of Smyrna, who wrote in the second century (*Expositio rerurn mathematicarum ad legendum Platonem utilium*, Hiller, ed. [Leipzig, 1878], pp. 2, 127, 129, 168). In this work, Eratosthenes apparently discussed from a mathematical and philosophical point of view such topics as proportion and progression (essential tools in Greek mathematics) and, arising from this, the theory of musical scales (Ptolemy, *Harmonica*, II, 14, Düring, ed. [Göteborg, 1930], pp. 70 f.; see Düring's ed. of Porphyry's commentary on this [1932], p. 91). Also in this work he gave his solution of the famous Delian problem of doubling the cube and described a piece of apparatus by which a solution could be obtained by mechanical means; the description is preserved for us by Eutocius, a sixth-century commentator on the works of Archimedes, and includes Eratosthenes' epigram (mentioned above) commemorating his achievement (*Eutocii commentarii in libros de sphaera et cylindro*, II, I, in *Archimedes opera omnia*, Heiberg ed., III, 88 f.; epigram, p. 96); Pappus also describes the apparatus and the method (*Collectio*, III, F. Hultsch, ed. [Berlin, 1876], 22–23, 56–58). Eutocius gives his information in the form of a letter from Eratosthenes to King Ptolemy Euergetes; the "letter" is almost certainly not genuine, but there is no reason to doubt that the contents represent the matter of Eratosthenes' solution (perhaps at least partly in his own words) or that the epigram is his.

The history of the problem of doubling the cube and the various solutions proposed are fully discussed by Health (*History of Greek Mathematics*, I, 244–270). Briefly, the problem resolves itself into finding two mean proportionals in continued proportion between two given straight lines: if a and b are the two given straight lines and we find x and y such that $a{:}x = x{:}y = y{:}b$, then $y = x^2/a = ab/x$; eliminating y, we have $x^3 = a^2 b$, and in the case where b is twice a, $x^3 = 2a^3$, and thus the cube is doubled. Eratosthenes' mechanical solution envisaged a framework of two parallel rulers with longitudinal grooves along which could be slid three rectangular (or, according to Pappus, *loc. cit*., triangular) plates (marked with their diagonals parallel—see Figure 3) moving independently of each other and able to overlap; if one of the plates remains fixed and the other two are moved so that they overlap as in Figure 4, it can easily be shown that points A, B, C, D lie on a straight line in such a way that AE, BF, CG, DH are in continued proportion, and BF and CG are the required mean proportionals between the given straight lines AE and DH.

In arithmetic Eratosthenes invented a method called the "Sieve" (*κόσκινον*) for finding prime numbers (Nicomachus, *Introduction arithmetica*, I, 13, 2–4). According to this, one writes down consecutively the odd numbers, starting with 3 and continuing as long as desired; then, counting from 3, one passes over two numbers and strikes out the third (a multiple of 3 and hence not prime) and continues to do this until the end—thus 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35, etc. The same process is gone through with 5, but this time passing over four numbers and striking out the fifth (a multiple of 5)—3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35, etc. The process may be repeated with consecutive odd numbers as many times as one likes, on each occasion, if *n* is the odd number, *n*-1 numbers being passed over and the next struck out; the remaining numbers will all be prime. Pappus (late third century) also attributes to Eratosthenes a work *On Means* (Περί μεσοτήτων), the contents of which are a matter of conjecture but which was important enough to form part of what Health calls the *Treasury of Analysis* (ἀναλνόμενος τόπος), comprising works by Euclid, Apollonius, Aristaeus, and Eratosthenes (Pappus, *Collectio*, Hultsch, ed., VII 3, p. 636, 24; see Heath, *History of Greek Mathematics*, II, pp. 105, 399 ff.).

In chronology, Eratosthenes apparently wrote two works, *Chronography* (χρονογραφίαι) and *Olympic Victors* (Ολνμπιονίκαι); both must have entailed considerable original research (he was the first Greek writer we know to have made a scientific study of the dating of events), and the former seems to have been a popularizing work containing a number of anecdotes, several of which are repeated by Plutarch (e.g., "Demosthenes," Loeb ed., IX, 4; Tenbner ed., XXX, 3; "Alexander,' Loeb ed., III, 2; Teubner ed., XXXXI, 2). Eratosthenes' datings remained authoritative throughout antiquity and in many cases cannot be improved upon today—e.g., the fall of Troy, 1184/1183 B. C.; the Dorian migration, 1104/1103; the first olympiad, beginning 777/776; the invasion of Xerxes, 480/479; the outbreak of the Peloponnesian War, 432/431.

In literary criticism Eratosthenes wrote a work in not less than twelve books entitled *On the Old Comedy*, the contents of which ranged over textual criticism, discussion of the authorship of plays from the dates of performances, and the meanings and usages of words; it was highly thought of by ancient scholars, being frequently cited, and its loss is greatly to be regretted. He also seems to have written a separate work on grammar. Finally, as befitted an Alexandrian polymath, he had a not inconsiderable reputation as a poet; his three main poetical works were *Hermes, Erigone*, and *Anterinys or Hesiod* (apparently alternative titles). The first had the same theme at the beginning as the well-known Homeric hymn but went on to draw a picture of the ascent of Hermes to the heavens and to give a vividly imaginative description of the zones of the earth as seen from there (Achilles Tatius,*Isagoge*, p. 153c in Petavius" *Uranologion* [1630]-the lines are reprinted by Hiller and by Powell); this passage was copied by Vergil (*Georgics*, I, 233–239). The *Erigone* was a star legend dealing with the story of Icarius, his daughter Erigone, and her dog, all of whom in this version were translated to the heavens as Boötes, Virgo, and Sirius, the Dog Star. The subject matter of the third poem is unknown. Only a few fragments of Eratosthenes' poetry are extant (the longest, some sixteen lines, being the passage from the *Hermes* mentioned above), and it is impossible to judge its intrinsic merit from these.

BIBLIOGRAPHY

The only published collection of all Eratosthenes' fragments is G. Bernhardy, *Eratosthenica* (Berlin, 1822), which is now greatly out of date. I have been much indebted in the preparation of this article to R. M. Bentham's unpublished Ph.D. thesis (London) entitled "The Fragments of Eratosthenes of Cyrene." It was made available to me through the kindness of his supervisor, Prof. E. H. Warmington (formerly of Birkbeck College, University of London), following the unfortunate death of the author before he submitted his thesis.

See also G. Knaack, "Eratosthenes," in Pauly-Wissowa, VI(1907), cols.358–388;E. H. Bunbury, *History of Ancient Geography*, **I** (London, 1879), ch. 16; E. H. Berger, *Die geographische Fragmente des Eratosthenes* (Leipzig, 1880); A. Thalamas, *La géographe d'Ératosthène* (Versailles, 1921); D. R. Dicks, *The Geographical Fragments of Hipparchus* (London, 1960); E. Hiller, *Eratosthenis carminum reliquiae* (Leipzig, 1872); and J. U. Powell, *Collectanea Alexandrina* (Oxford, 1925).

D. R. Dicks