Fagnano Dei Toschi, Giulio Carlo | Encyclopedia.com

(*b*. Sinigaglia, Italy, 6 December 1682; *d*. Sinigaglia, 26 September 1766)

mathematics.

Fagnano, the son of Francesco Fagnano and Camilla Bartolini, was born into a noble family that had included Pope Honorius II and had been established in his native town for nearly 350 years. In 1723 he was appinted *goonfaloniere* of Sinigaglia; while he held this office he was subjected to calumny by envious fellow citizens. He was the father of many children, among them Giovanni Francesco, a distinguished mathematicaian.

Fagnano began to study mathematics after reading the first volume of Malebranche's *Recherche de laverité;* and although he was self-educated, he soon made such progress that he became famous both in Italy and abroad. In 1721 Louis XV conferred upon him the title of count; in 1745 Pope Benedict XIV made him a marquis of Sant' Onofrio. He belonged to the Royal Society of London and the Berlin Academy of Sciences, and at his death he had been nominated for membership in the Paris Academy of Sciences. Fagnano maintained correspondence with many contemporary mathematicians, especially Grandi, Riccati, Leseur, and Jacquier; he was praised by Euler and Fontenelle, the permanent secretary of the French Academy. Lagrange, at the age of twenty, turned to him for help in publishing his first work.

Fagnano's works were published at intervals in the *Giornale dei letterati* and in the Raccolta Calogera. These were later collected and with other, unpublished works included in *Produzioni matematiche*.

In algebra Fagnano suggested new methods for the solution of equations of the second, third, and fourth degrees. He also organized ina rational manner the knowledge that scientists had of imaginary numbers, establishing for them a special algorithm that was far better that Bombelli's primitive one. In this field he established the well-known formula

This is reminiscent of Euler's celebrated formula

$e^{\pi i} = -1$,

Which unites the four most important numbers in mathematics.

In geometry Fagnano formulated a beneral theory of geometric proportions that is more noteworthy than the countless writings, published previously, that were intended to illustrate book V of Euclid's *Elements*. Much more important, however, is his work on the triangle, for which he may well be considered the founder of the geometry of the triangle. Some of the problems solved are as follows:

To find in the plane of a triangle, *ABC*, a point, *P*, that will reduce to the minimum the sum $PA + PB + PC$ or the sum $PA^2 + PB^2 + PC^2$.

To find in the plane of a quadrangle, *ABCD*, a point that will render minimum the sum $PA + PB + PC + PD$.

Two of Fagnano's major findings are (1) that the sum of the squares of the distances of the center of gravity of a triangle from the vertices equals one-third the sum of the squares of the sides and (2) that given a triangle, *ABC*, for every point *P* of *BC* we may construct an inscribed triangle, with its vertex at *P*, of minimum perimeter. He also solved the problem Proposed by Simon Lhuilier: Draw through a given point the straight line on which two given straight lines cut off the minimum segment. This leads analytically to a third-degree equation.

The most important results achieved by Fagnano, however, were in analytical geometry and in integral calculus. He rectified the ellipse $x^2 + 2y^2 = a^2$, which has as its major axis the mean proportion between the minor axis and its double. The equation

$(x^2 + y^2)^2 - 2a^2(x^2 - y^2) = 0$

represents a fourt-degree curve that, owing to its shape, is called “lemniscate” a term derived from the Greek *lemniscata*. This was first studied by Jakob I Bernoulli (1694), but it was made famous by Fagnano’s research. He established its rectification and demonstrated that each of its arcs may be divided with ruler and compass into *n* equal parts when *n* is of the from 2. 2^m, 3. $2^{m,}$ 5. 2^m

He gave the name “elliptical integrals” to integrals of the from

in which p *(x)* is a polynomial, of the third or fourth degree. Euler found a basic result in their theory, known as the theorem of addition, that includes the results first found by Fagnano in the lemniscate arcs. For this reason some have considered Fagnano in the lemniscate arcs. For this rerason some have considered Fagnano’s work the foreunner of the theory of elliptic functions—a claim undoubtedly put forward by Legendre.

Disputes, encouraged by Fagnano’s uncle Giovanni, arose over who deserved the credit of priority in these studies—Fagnano or Nikolaus I Bernoulli. On the advice of this friends Riccati, Fagnano soon put an end to these arguments.

Fagnano also found the area of the lemniscate, thus demonstrating that Tschirnhausen’ opinion was erroneous and that it was impossible to square the area composed of several leaves.

In 1714 Fagnano proposed the following problem: Given a biquadratic parabola of the from $y = x^4$, and given a portion of it, determine another portion of the same curve in such a manner that the difference between the portions is rectifiable. Since no one replied, Fagnano himself published the solution, thus extending the method to infinite species of rectifiable parabolas. Fagnano also studied the problem of squaring hyperbolic spaces.

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II. Secondary Literature. On Fagnano or his work, see Luigi Bianchi, *Lezioni sulla teoria delle funzioni di variable complessa e delle funzioni ellittiche* (Pisa, 1901), p. 250; Gino Loria, *Curve piane speciali algebriche e trascendenti. Teoria e storia* (Milan, 1930), I, 257 ff., and *Storia delle matematiche*, 2nd ed. (Milan, 1950), pp. 664–666; and A. Natucci, “Anton Maria Legendre inventore della teoria delle funzioni ellittiche” in *Archimade*, **4** no. 6 (1952), 261.

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