Ferrari, Ludovico | Encyclopedia.com

(*b*. Bologna, Italy, 2 February 1522; *d*. Bologna, October 1565)

algebra.

Little is known of Ferrari's life. His father, Alessandro, was the son of a Milanese refugee who had settled in Bologna. Following his father's death Ferrari went to live with his uncle Vincenzo. In November 1536 he was sent to Milan by his uncle to join the household of Girolamo Cardano, replacing his uncle's son Luca, who was already in Cardano's service. Although he had not received a formal education, Ferrari was exceptionally intelligent. Cardano therefore instructed him in Latin, Greek, and mathematics and employed him as amanuensis. In Cardano's autobiography, written many years later, Ferrari is described as having "excelled as a youth all my pupils by the high degree of his learning" (*De vita propria liber* [1643], p. 156).

In 1540 Ferrari was appointed public lecturer in mathematics in Milan, and shortly afterward he defeated Zuanne da Coi, a mathematician of Brescia, at a public disputation. He also collaborated with Cardano in researches on the cubic and quartic equations, the results of which were published in the *Ars magna* (1545). The publication of this book was the cause of the celebrated feud between Ferrari and Niccold Tartaglia of Brescia, author of *Quesiti et inventioni diverse* (1546). In the wake of the resulting public disputation, Ferrari received offers of employment from many persons of importance, including Emperor Charles V, who wanted a tutor for his son, and Ercole Gonzaga, cardinal of Mantua. He accepted Gonzaga's offer and, at the request of the cardinal's brother, Ferrante, then governor of Milan, he carried out a survey of that province. After this he was in the cardinal's service for some eight years. On his retirement because of ill health Ferrari went to Bologna to live with his sister. From September 1564 until his death in October 1565, he held the post of *lector ad mathematicam* at the University of Bologna.

When Ferrari went to live with Cardano, the latter was earning his livelihood by teaching mathematics. Although Cardano was a qualified physician, he had not yet been accepted by the College of Physicians and was then preparing his first works on medicine and mathematics for publication. It is likely that Ferrari was introduced to mathematics through Cardano's *Practica arithmetice* (1539). While this work was in preparation, news reached Cardano that a method of solving the cubic equation of the form $x^3 + ax = b$, where a and b are positive, was known to Niccolò Tartaglia of Brescia. Until then Cardano had accepted Luca Pacioli's statement in the *Summa de arithmetica, geometria, proportioni et proportionalita* (1494) that the cubic equation could not be solved algebraically. On learning that Tartaglia had solved the equation in the course of a disputation with Antonio Maria Fiore in 1535, Cardano probably tried to find the solution himself, but without success. In 1539, before his book was published, he asked Tartaglia for the solution, offering to include it in his forthcoming book under Tartaglia's name. Tartaglia refused, on the ground that he wished to publish his discovery himself. But when he visited Cardano in Milan in March 1539, he gave him the solution on the solemn promise that it would be kept secret. In 1542, however, Cardano learned that the cubic equation had been solved several years before Tartaglia by Scipione Ferro, *lector ad mathematicam* at the University of Bologna from 1496 to 1526. During a visit to Bologna, Cardano and Ferrari were shown Ferro's work, in manuscript, by his pupil and successor Annibale dalla Nave. After this Cardano did not feel obliged to keep his promise.

Having learned the method of solving one type of cubic equation, Cardano and Ferrari were encouraged to extend their researches to other types of cubics and to the quartic. Ferrari found geometrical demonstrations for Cardano's formulas for solving $x^3 + ax = bx^2 + c$ and $x^3 + ax^2 = b$; he also solved the quartic of the form $x^4 + ax^2 + b = cx$ where a, b, c, are positive. The results were embodied in Cardano's *Ars magna* (1545). In it he attributed the discovery of the method of solving the equation $x^3 + ax = b$ to Scipione Ferro and its rediscovery to Tartaglia. That this apparent breach of secrecy angered Tartaglia is evident from book IX of his *Quesiti et inventioni diverse* (1546), where he recounted the circumstances in which he had made his discovery and Cardano's attempts to obtain the solution from him. He also gave a verbatim account of the conversation at their meeting in Milan, along with his comments.

Ferrari, loyal to his master and impetuous by nature, reacted quickly. In February 1547 he wrote to Tartaglia, protesting that the latter had unjustly and falsely made statements prejudicial to Cardano. Having criticized the mathematical content of Tartaglia's work and accused him of repetition and plagiarism, Ferrari challenged him to a public disputation in geometry, arithmetic, and related disciplines. Scholarly disputations, common in those days, were often the means of testing the professional ability of the participants. Since both Ferrari and Tartaglia were engaged in the public teaching of mathematics, a disputation was a serious matter. In his reply Tartaglia, while insisting that Cardano had not kept his promise, said that he had used injurious words in order to provoke Cardano to write to him. He asked Ferrari to leave Cardano to fight his own battles; otherwise, Ferrari should admit that he was writing at Cardano's instigation. Saying that he would accept the challenge if

Cardano at least countersigned Ferrari's letter, Tartaglia went on to raise objections to the conditions of the proposed disputation—the subjects, the location, the amount of caution money to be deposited, and the judges.

Twelve letters were exchanged, full of charges and insults, each party trying to justify his position. Tartaglia maintained that Cardano had broken his promise and that Ferrari was writing at Cardano's instance. Ferrari asserted that the solution of the cubic equation was known to both Scipione Ferro and Antonio Maria Fiore long before Tartaglia had discovered it and that it was magnanimous of Cardano to mention Tartaglia in the *Ars magna*. He also denied that he was writing on Cardano's behalf. In the course of this correspondence each party issued a series of thirty-one problems for the other to solve. Tartaglia sent his problems in a letter dated 21 April 1547. The problems were no more difficult than those found in Pacioli's *Summa*. On 24 May 1547 Ferrari replied with thirty-one problems of his own but did not send the solutions to those set by Tartaglia. In his reply (July 1547) Tartaglia sent the solutions to twenty-six of Ferrari's problems, leaving out those which led to cubic equations; a month later he gave his reasons for not solving these five problems. In a letter dated October 1547 Ferrari replied, criticizing Tartaglia's solutions and giving his solutions to the problems set by the latter. Tartaglia, replying in June 1548, said he had not received Ferrari's letter until January and that he was willing to go to Milan to take part in the disputation. In July 1548 both parties confirmed their acceptance.

There is no record of what happened at the meeting except for scattered references in Tartaglia's *General trattato di numeri, et misure* (1556–1560). The parties met on 10 August 1548 in the church of Santa Maria del Giardino dei Minori Osservanti in the presence of a distinguished gathering that included Ferrante Gonzaga, governor of Milan, who had been named judge. Tartaglia says that he was not given a chance to state his case properly. Arguments over a problem of Ferrari's that Tartaglia had been unable to resolve lasted until suppertime, and everyone was obliged to leave. Tartaglia departed the next day for Brescia, and Ferrari was probably declared the winner.

Ferrari's method of solving the quartic equation $x^4 + ax^2 + b = cx$ was set out by Cardano in the *Ars magna*. It consists of reducing the equation to a cubic. The discovery was made in the course of solving a problem given to Cardano by Zuanne da Coi: "Divide 10 into three proportional parts so that the product of the first and second is 6." If the mean is x, it follows that $x^4 + 6x^2 + 36 = 60x$, or $(x^2 + 6)^2 = 60x + 6x^2$. This last equation can be put in the form

$$(x^2 + 6 + y)^2 = 6x^2 + 60x + y^2 + 12y + 2yx^2$$

or

$$(x^2 + 6 + y)^2 = (2y + 6)x^2 + 60x + (y^2 + 12y),$$

were y is a new unknown. If y is chosen so that the right-hand side of the equation is a perfect square, then y satisfies the condition

$$60^2 = 4(2y + 6)(y^2 + 12y),$$

which can be reduced to the cubic equation

$$y^3 + 15y^2 + 36y = 450.$$

That Ferrari's method of solution is applicable to all cases of the quartic equation was shown by Rafael Bombelli in his *Algebra* (1572).

BIBLIOGRAPHY

I. Original Works. The letters exchanged by Ferrari and Tartaglia were printed, and copies were sent to several persons of influence in Italy. (A complete set of these letters is in the Department of Printed Books of the British Museum.) They have been published by Enrico Giordani in *I sei cartelli di matematica disfida, primamente intorno alla generale risoluzione delle equazioni cubiche, di Lodovico Ferrari, coi sei contro-cartelli in risposta di Nicolò Tartaglia, comprendenti le soluzioni de' quesiti dall'una e dall'altra parse proposti* (Milan, 1876). Ferrari's work on the cubic and quartic equations is described in Cardano's *Artis magnae, sine de regulis algebraicis* (Nuremberg, 1545).

II. Secondary Literature. Cardano wrote a short biography of Ferrari, "Vita Ludovici Ferrarii Bononiensis," in his *Opera omnia* (Lyons, 1663), IX, 568–569. References to Ferrari in Cardano's other works are cited in J. H. Morley, *Life of Girolamo Cardano of Milan, Physician* (London, 1854), I, 148–149, 187. The history of mathematics in sixteenth-century Italy is outlined in Ettore Bortolotti, *Storia della matematica nella Università di Bologna* (Bologna, 1947), pp. 35–80. Arnaldo Masotti, "Sui cartelli di matematica disfida scambiati fra Lodovico Ferrari e Niccolò Tartaglia," in *Rendiconti dell'Istituto lombardo di scienze e lettere*, **94** (1960), 31–41, cites the important secondary literature on Ferrari.

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