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(b. Auxerre, France, 21 March 1768; d. Paris, France, 16 May 1830)

mathematics, mathematical physics.

Fourier lost both his father (Joseph, a tailor in Auxerre) and his mother (Edmée) by his ninth year and was placed by the archbishop in the town’s military school, where he discovered his passion for mathematics. He wanted to join either the artillery or the engineers, which were branches of the army then generally available to all classes of society; but for some reason he was turned down, and so he was sent to a Benedictine school at St. Benoît-sur-Loire in the hope that he could later pursue his special interests at its seminary in Paris. The French Revolution interfered with these plans, however, and without regret he returned in 1789 to Auxerre and a teaching position in his old school.

During the Revolution, Fourier was prominent in local affairs, and his courageous defense of the victims of the Terror led to his arrest in 1794. A personal appeal to Robespierre was unsuccessful; but he was released after Robespierre’s execution on 28 July 1794 and went as a student to the ill-fated École Normale, which opened and closed within that year. He can have spent only a short time there, but nevertheless he made a strong impression; and when the école Polytechnique started in 1795 he was appointed administrateur de police, or assistant lecturer, to support the teaching of Lagrange and Monge. There he fell victim of the reaction to the previous regime and was, ironically, arrested as a supporter of Robespierre (who had declined his earlier appeal). But his colleagues at the École successfully sought his release, and in 1798 Monge selected him to join Napoleon’s Egyptian campaign. He became secretary of the newly formed Institut d’Égypte, conducted negotiations between Napoleon and Sitty-Nefiçah (the wife of the chief bey, Murad), and held other diplomatic posts as well as pursuing research. He does not appear to have been appointed governor of southern Egypt, however, as has often been reported.

After his return to France in 1801, Fourier wished to resume his work at the École Polytechnique; but Napoleon had spotted his administrative genius and appointed him prefect of the department of Isère, centered at Grenoble and extending to what was then the Italian border. Here his many administrative achievements included the reconciliation of thirty-seven different communities to the drainage of a huge area of marshland near Bourgoin to make valuable farming land, and the planning and partial construction of a road from Grenoble to Turin (now route N91-strada 23), which was then the quickest route between Turin and Lyons. In 1808 Napoleon conferred a barony on him.

While in Egypt, Fourier suggested that a record be made of the work of the Institut d’Égypte, and on his return to France he was consulted on its organization and deputed to write the “Preface historique” on the ancient civilization and its glorious resurrection. This he completed in 1809, but some of its historical details caused controversy. Napoleon supported it, and it appeared in the Description de l’Égypte.

Fourier was still at Grenoble in 1814 when Napoleon fell. By geographical accident the town was directly on the route of the party escorting Napoleon from Paris to the south and thence to Elba; to avoid an embarrassing meeting with his former chief Fourier negotiated feverishly for a detour in the route of the cortege. But no such detour was conceivable on Napoleon’s return and march on Paris in 1815, and so Fourier compromised, fulfilling his duties as prefect by ordering the preparation of the defenses—which he knew to be useless—and then leaving the town for Lyons by one gate as Napoleon entered by another. He did return, however, and the two friends met at Bourgoin. Fourier need have had no fears, for Napoleon made him a count and appointed him prefect of the neighboring department of the Rhône, centered at Lyons. But before the end of Napoleon’s Hundred Days, Fourier had resigned his new title and prefecture in protest against the severity of the regime and had come to Paris to try to take up research full time (it had previously been only a spare-time activity).

This was the low point of Fourier’s life—he had nobility, only a small pension, and a low political reputation. But a former student at the École Polytechnique and companion in Egypt, Chabrol de Volvic, was now prefect of the department of the Seine and appointed him director of its Bureau of Statistics, a post without onerous duties but with a salary sufficient for his needs.

In 1816 Fourier was elected to the reconstituted Académie des Sciences, but Louis XVIII could not forgive his having accepted the prefecture of the Rhône from Napoleon, and the nomination was refused. Diplomatic negotiation eventually cleared up the situation, and his renomination in 1817 was not opposed. He also had some trouble with the second edition of the Description
de l’Égypte (for now his references to Napoleon needed rethinking) but in general his reputation was rising fast. He was left in a position of strength after the decline of the Société d’Arcueil, led by Laplace in the physical sciences, and gained the favor and support of the aging Laplace himself in spite of the continued enmity of Poisson. In 1822 he was elected to the powerful position of secrétaire perpétuel of the Académie des Sciences, and in 1827—after further protests—to the Académie Française. He was also elected a foreign member of the Royal Society.

Throughout his career, Fourier won the loyalty of younger friends by his unselfish support and encouragement; and in his later years he helped many mathematicians and scientists, including Oersted, Dirichlet, Abel, and Sturm. An unfortunate incident occurred in 1830 when he lost the second paper on the resolution of equations sent to the Academy by Evariste Galois; but this would appear to be due more to the disorganization of his papers than—as Galois believed—to deliberate suppression.

The Egyptian period of his life had one final consequence in his last years. While there he had caught some illness, possibly myxedema, which necessitated his increasing confinement to his own heated quarters. He lived at 15 rue pavée St. André des Arts (now 15 rue Ségur) until 1829, then at 19 rue d’Enfer (now the site of 73 Boulevard St. Michel) until his death. On 4 May 1830 he was struck down while descending some stairs, and he allowed the symptoms to become worse until he died at the Église St. Jacques de Haut Pas, and he was buried in the eighteenth division of the cemetery of Pere Lachaise.

Various memorials have been made to Fourier. A bust by Pierre-Alphonse Fessard was subscribed in 1831 but was destroyed during World War II. A similar fate overtook the bronze statue by Faillot erected in Auxerre in 1849; the Nazis melted it down for armaments. During the night before its destruction, however, the mayor rescued two of its bas-reliefs, which were mounted on the walls of the town hall after the war. A medallion was founded in the town in 1952, and since 1950 the Annales de l’Institut Fourier have been published by the University of Grenoble. In 1968 the bicentenary of his birth was celebrated and the secondary school in Auxerre was renamed the Lycee Fourier.

Heat Diffusion and Partial Differential Equations. Fourier’s achievements lie in the study of the diffusion of heat and in the mathematical techniques he introduced to further that study. His interest in the problem may have begun when he was in Egypt, but the substantial work was done at Grenoble. In 1807 he presented a long paper to the Academy on heat diffusion between disjoint masses and in special continuous bodies (rectangle, annulus, sphere, cylinder, prism), based on the diffusion equation

\[ \frac{\partial u}{\partial t} = \alpha \nabla^2 u \]

(in three variables). Of the examiners, Laplace, Monge, and Lacroix were in favor of accepting his work, but Lagrange was strongly opposed to it—due, to some extent, to the Fourier series

required to express the initial temperature distribution in certain of these bodies, which contradicted Lagrange’s own denigration of trigonometric series in his treatment of the vibrating string problem in the 1750’s. The paper was therefore never published.

A prize problem on heat diffusion was proposed in 1810, however, and Fourier sent in the revised version of his 1807 paper, together with a new analysis on heat diffusion in infinite bodies. In these cases the periodicity of the Fourier series made it incapable of expressing the initial conditions, and Fourier substituted the Fourier integral theorem, which he wrote in forms such as

The last sections of the paper dealt with more physical aspects of heat, such as the intensity of radiation, and these became more important in Fourier’s thought during his later years. Fourier’s paper won the competition, but the jury—probably at the insistence of Lagrange—made criticisms on grounds of “rigor and generality,” which Fourier considered an unjustified reproach. He expanded the mathematical parts of the paper into his book Théorie analytique de la chaleur. An extended treatment of the physical aspects was first planned for further chapters of this book and then for a separate book, Théorie physique de la chaleur, but it was never achieved.

The history of Fourier’s main work in mathematics and mathematical physics has long been confused by an exclusive concentration on only two results, Fourier series and Fourier integrals, and by the application of anachronistic standards of rigor in judgments on their derivation. Fourier’s achievement is better understood if we see it as twofold: treating first the formulation of the physical problem as boundary-value problems in linear partial differential equations, which (together with his work on units and dimensions) achieved the extension of rational mechanics to fields outside those defined in Newton’s Principia; and second, the powerful mathematical tools he invented for the solution of the equations, which yielded a long series of descendants and raised problems in mathematical analysis that motivated much of the leading work in that field for the rest of the century and beyond.

Fortunately for the historian, Fourier reproduced, nearly intact, almost all his successful results in the several versions of his basic work. A comparison of these, in correlation with unpublished sources, biographical information, and separate papers, enables a firm reconstruction of the sequence of his researches. Moreover, from this sequence and from his style of presentation, we can identify several crucial points at which he failed to solve problems as well as the reasons for his failure.

Fourier’s first work in the rational mechanics of heat used a model of heat being transferred by a shuttle mechanism between discrete particles. The physical theory was a simple method of mixtures, and the mathematics was of the 1750’s. Of the two
Fourier’s successful establishment of the equation of heat flow was probably indebted to early work of J. B. Biot on the steady temperatures in a metal bar, wherein Biot distinguished between internal conduction and external radiation. Biot’s analysis was crippled by a faulty physical model for conduction which yielded an “inhomogeneous” equation $\frac{\partial^2 \psi}{\partial x^2} - kv \frac{\partial \psi}{\partial x} = 0$; and Fourier was able to concoct a physical model which resolved the difficulty. The full time-dependent equations for one and two dimensions of the type (1) then came easily.

Fourier’s masterstroke was in the choice of configuration for a problem in which to apply the equation. The semi-infinite strip, uniformly hot at one end and uniformly cold along the sides, combines the utmost simplicity with physical meaning, in the tradition of rational mechanics deriving from the Bernoullis and Euler. The steady-state case is simply Laplace’s equation in Cartesian coordinates. Fourier probably tried complex-variable methods (a solution along these lines, probably retrospective, is in the Théorie) but then used separation of variables to yield a series solution and thus the boundary-condition equation

$$\frac{\partial^2 \psi}{\partial x^2} - kv \frac{\partial \psi}{\partial x} = 0$$

The earliest records of this first successful investigation show its exploratory character and Fourier’s excitement with his achievement. Also, in this work there are traces of the influence of Monge in the notation, in the representation of the solution as a surface, and in the separate expression of boundary values in determining the solution of a differential equation. Thenceforth, Fourier proceeded into new territory with assuredness. The three-dimension case caused some difficulties, which were resolved by splitting the original equation into two, one for interior conduction and the other relating radiation to the temperature gradient at the surface. Applied to the sphere, in spherical coordinates, this gave a nonharmonic trigonometric expansion, where the eigenvalues are the roots of a transcendental equation. Fourier used his knowledge of the theory of equations (see below) to argue for the reality of all roots, but the question caused him trouble for many years. The problem of heat conduction in a cylinder gave rise to a further generalization. Fourier’s solution was in what are called Bessel functions—derived several years before Friedrich Bessel—by techniques made fully general in the theory created by Fourier’s later associates, J. Charles François Sturm and Joseph Liouville.

The study of diffusion along an infinite line, involving the development of the Fourier integral theorem, probably depended on the idea of Laplace of expressing the solution of the heat equation as an integral transform of an arbitrary function representing the initial temperature distribution. Fourier derived the cosine and sine transforms separately for configurations symmetrical and antisymmetrical about the origin, by the extension of the finite-interval expansion. Only gradually did he come to appreciate the generality of the odd-and-even decomposition of a given function.

Fourier’s last burst of creative work in this field came in 1817 and 1818, when he achieved an effective insight into the relation between integral-transform solutions and operational calculus. There was at that time a three-cornered race with Poisson and Cauchy, who had started using such techniques by 1815. In a crushing counterblow to a criticism by Poisson, Fourier exhibited integral-transform solutions of several equations which had long defied analysis, and gave the lead to a systematic theory. This was later achieved by Cauchy, en route to the calculus of residues.

As a mathematician, Fourier had as much concern for practical problems of rigor as anyone in his day except Cauchy and N. H. Abel, but he could not conceive of the theory of limiting processes as a meaningful exercise in its own right. The famous referees’ criticisms of the 1811 prize essay, concerning its defects of rigor and generality, have long been misinterpreted. Much of the motivation for them was political; Poisson and Biot, outclassed rivals in the theory of heat diffusion, tried for years to denigrate Fourier’s achievements. The criticism of rigor was probably based on Poisson’s point that the eigenvalues in the sphere problem were not proven to be all real; and complex roots would yield a physically impossible solution. (Poisson himself solved the problem for Fourier years later.) The supposed lack of generality in Fourier’s series solution (2) was
Fourier’s sensibility was that of rational mechanics. He had a superb mastery of analytical technique and notation (is his invention, for example); and this power, guided by his physical intuition, brought him success. Before him, the equations used in the leading problems in rational mechanics were usually nonlinear, and they were solved by ad hoc approximation methods. Similarly, the field of differential equations was a jungle without pathways. Fourier created and explained a coherent method whereby the different components of an equation and its series solution were neatly identified, with the different aspects of the physical situation being analyzed. He also had a uniquely sure instinct for interpreting the asymptotic properties of the solutions of his equations for their physical meaning. So powerful was his approach that a full century passed before nonlinear differential equations regained prominence in mathematical physics.

For Fourier, every mathematical statement (although not all intermediate stages in a formal argument) had to have a physical meaning, both in exhibiting real motions and in being capable, in principle, of measurement. He always interpreted his solutions so as to obtain limiting cases which could be tested against experiment, and he performed such experiments at the earliest opportunity. He rejected the prevailing Laplacian orthodoxy of analyzing physical phenomena through the assumption of imperceptible molecules connected by local Newtonian forces; because of his approach to physical theory, together with his enmity to Poisson, he was adopted as philosophical patron by Auguste Comte in the development and popularization of philosophie positive.

Although the physical models of his earliest drafts were very sketchy, by the time of the 1807 paper he had fully incorporated physical constants into his theory of heat. The concern for physical meaning enabled him to see the potential in his formal technique for checking the coherence of the clumps of physical constants appearing in the exponentials of the Fourier integral solutions. From this came the full theory of units and dimensions (partly anticipated by Lazare Carnot), the first effective advance since Galilei in the theory of the mathematical representation of physical quantities. A comparison with the confused struggles of contemporaries such as Biot with the same problem illuminates Fourier’s achievement.

Although Fourier studied the physical theory of heat for many years, his contributions, based primarily on the phenomena of radiation, did not long survive. His concern for applying his theory produced an analysis of the action of the thermometer, of the heating of rooms, and, most important, the first scientific estimate of a lower bound for the age of the earth. It is puzzling that in spite of his faith in the importance of heat as a primary agent in the universe, Fourier seems to have had no interest in the problem of the motive power of heat; and so, along with nearly all his contemporaries, he remained in ignorance of the essay on that topic by Lazare Carnot’s son, Sadi.

On the side of real-variable analysis, the problems suggested especially by Fourier series lead directly through Dirichlet, Riemann, Stokes, and Heine to Cantor, Lebesgue, F. Riesz, and Ernst Fischer. Such deep results are not the chance products of algebraic doodling. None of Fourier’s predecessors or contemporaries did—or could—exploit trigonometric expansions of arbitrary functions to their full effectiveness, nor could they recognize and accept their implications for the foundations of pure and applied analysis. Such achievements required a great master craftsman of mathematics, endowed with a lively imagination and holding a conscious philosophy of mathematics appropriate for his work. For Fourier, this was expressed in his aphorism, “Profound study of nature is the most fertile source of mathematical discoveries.”

Theory of Equations. In contrast with his famous work on heat diffusion, Fourier’s interest in the theory of equations is remarkably little known. Yet it has a much longer personal history, for it began in his sixteenth year when he discovered a new proof of Descartes’s rule of signs and was just as much in progress at the time of his death. This rule may be stated as follows:

Let \( f(x) = x^n + a_1x^{n-1} + \ldots + a_m x + a_0 \).

Then there will be a sequence of signs to the coefficients of \( f(x) \). If we call a pair of adjacent signs of the same type (i.e., ++ or --) a preservation and a pair of the opposite type a variation, then the number of positive (or negative) roots of \( f(x) \) is at most the number of variations (or preservations) of sign in the sequence. Fourier’s proof was based on multiplying \( f(x) \) by \((x+p)\), thus creating a new Polynomial which contained one more sign in its sequence and one more positive (or negative) root, according as \( p \) was less (or greater) than zero, and showing that the number of preservations (or Variations) in the new sequence was not increased relative to the old sequence. Hence the number of variations (or preservations) is increased by at least one, and the theorem follows. The details of the proof may be seen in any textbook dealing with the rule, for Fourier’s youthful achievement quickly became the standard proof, even if its authorship appears to be virtually unknown.

Fourier generalized Descartes’s rule to estimate the number of real roots \( f(x) \) within a given interval \([a, b]\), by taking the signs of the terms in the sequence

\[ f^m(x), f^{m-1}(x), \ldots, f'(x), f(x), f(x). \]

When \( x = -\infty \), the series will be made up totally of the variations

\[ + - + - \ldots. \]
while at \( x = + \infty \) it is entirely preserved:

\[ + + + + \ldots \]

Fourier showed that as \( x \) passes from \(- \infty\) to \(+ \infty\) the variations are lost by the crossing of a real (possibly multiple) root, or the skirting of a pair of complex conjugate roots, and that the number of real roots within \([a, b]\) is at most the difference between the number of variations in the sequence when \( x = a \) and the number when \( x = b \). This theorem received an important extension in Fourier’s own lifetime by Sturm, who showed in 1829 that the number of real roots is exactly the difference in the number of variations formulated above for the sequence of functions

\[ f_0(x), f_{m-1}(x) \ldots f_2(x), f'(x), f(x) \]

where \( f_2(x) \ldots \) are defined algorithmically from \( f(x) \) and \( f(x) \). This is the famous Sturm’s theorem.

Fourier appears to have proved his own theorem while in his teens and he sent a paper to the Academy in 1789. However, it disappeared in the turmoil of the year in Paris, and the pressure of administrative and other scientific work delayed publication of the results until the late 1810’s. Then he became involved in a priority row with Ferdinand Budan de Boislaurent, a Part-time mathematician who had previously published similar but inferior results. At the time of his death, Fourier was trying to prepare these and many other results for a book to be called Analyse des équations déterminées; he had almost finished only the first two of its seven livres. His friend Navier edited it for publication in 1831, inserting an introduction to establish from attested documents (including the 1789 paper) Fourier’s priority on results which had by then become famous. Perhaps Fourier was aware that he would not live to finish the work, for he wrote a synopsis of the complete book which also appeared in the edition. The synopsis indicated his wide interests in the subject, of which the most important not yet mentioned were various means of distinguishing between real and imaginary roots, refinements to the Newton-Raphson method of approximating to the root of an equation, extensions to Daniel Bernoulli’s rule for the limiting value of the ratio of successive terms of a recurrent series, and the method of solution and applications of linear inequalities. Fourier’s remarkable understanding of the last subject makes him the great anticipator of linear programming.

Fourier’s other mathematical interests included a general search for problems in dynamics and mechanics, shown by a published paper on the principle of virtual work. In his later years his directorship of the Bureau of Statistics brought him in touch with the problems of probability and errors, and he wrote important papers on estimating the errors of measurement from a large number of observations, published in the Bureau’s reports for 1826 and 1829.

**BIBLIOGRAPHY**

I. Original Works. Fourier’s most famous work is *Théorie analytique de la chaleur* (Paris, 1822; repr. Breslau, 1883). An English trans. was prepared by A. Freeman (Cambridge, 1878; repr. New York, 1955). A 2nd French ed. appeared in 1888 as vol. 1 of the *Oeuvres* of Fourier, Gaston Darboux, ed. Vol. II, containing the majority of the rest of Fourier’s published works, appeared in 1890. The list of works given by Darboux in the intro. to this vol. shows that his principal omission was the *Analyse des équations déterminées* (Paris, 1831). Two German trans. of this book have been made: by C. H. Schnuse (Brunswick, 1836; notes added in 1846), and by A. Loewy, Ostwald’s Klassiker, no.127 (Leipzig, 1902). The other main omissions from Darboux’s ed. were the first 79 articles of the 1811 prize paper on heat diffusion, in *Mémoires de l’Académie des sciences*, 4 (1819–1820), 185–555, which were largely in common with sees. of the book, and a joint paper with H. C. Oersted on thermoelectric effects, in *Annales de Chimie et de physique*, 12 (1823), 375–389. Darboux’s list also omitted the papers read by Fourier at the Institute d’Égypte, which are listed in Cousin’s obit. of Fourier cited below and partially in Navier’s introduction to the *Analyse*. As secrétaire perpetuel of the Académie des Sciences, Fourier wrote the *Analyse des travaux* (1823–1827) et éloges on Delambre, Herschel, Breguet, Charles, and Laplace. The references are in Darboux’s list.

The main source of unpublished MSS is the twenty-nine vols. in the Bibliotheque Nationale (MSS fonds franc. 22501–22529), totaling about 5,200 sheets. Of these, 22501 is a set of miscellaneous studies and letters; 22502–22516 deal with the theory of equations, including topics only summarized in the *Analyse*; 22517–22522 cover extended work in mechanics, dynamics, and errors of measurement, etc.; and 22523–22529 are concerned with various aspects of heat diffusion and its associated mathematics and experimental work, including in 22525, fols. 107–149, the “first draft” of his 1807 paper on heat diffusion. This paper itself was discovered by Darboux in the library of the Ecole Nationale des Ponts et Chaussees (MS 267, now numbered 1851), which also contains several cahiers of lecture notes, totaling 386 pp., given by Fourier at the Ecole Polytechnique in 1795–1796 (MS 668, and 18520). Another set of lecture notes, partly in common with this set, is in a vol. of 559 pp. in the Bibliotheque de l’Institut de France (MS 2044) where an early four-page article on Descartes’ rule (MS 2038) may also be found. There is a scattering of letters in connection with his secretariaship of the Academy and his prefectures in various public MS collections in France and in the archives of various learned institutions.

With regard to his extrascientific writings, his notebook of the Egyptian campaign may be read in *Bibliotheque Egyptologique*, VI (Paris, 1904), 165–214. The various versions of the “Préface historique” are best compared on pp. 88–172 of the book by J. J. Champollion-Figeac cited below. Fourier wrote an article in the *Description* on the government of Egypt, and another on the astronomical monuments of the country (including a discussion of the Zodiacs). Details are given in the bibliographical account and collation of the *Description de l’Égypte* (London, 1838). He also contributed notes to a trans. by a. de Grandsagne
of Pliny’s *Natural History* and, although they are unsigned, it is clear that he annotated Pliny’s discussion of Egypt, in *Histoire naturelle* 20 vols. (Paris, 1829–1840), IV, 190–209. He also wrote the articles “Rallier des Ourmes,” “Viete,” and “Jean Wallis” in the Biographie Universelle, 52 vols. (Paris, 1811–1829).


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