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(*b*. Borgo San-Sepolcro [now Sansepolcro], Italy, in the 1410s, *d*. Borgo San-Sepolcro, 12 October 1492),

mathematics, perspective, painting.

For the original article on Piero della Francesca see *DSB*, vol. 5.

Piero is one of the few persons who figures prominently both in the history of mathematics and art. While he has received much attention as a painter, his contributions to mathematics are not so well known and will be the main topic of this addendum (for Piero and mathematics in general, see the 2005 book by J.V. Field and its references; for Piero and perspective, see also the 2007 book by Kirsti Andersen and its references).

Although many scholars have worked on Piero during the decades since the original *DSB* article and thrown much light on his work, basic knowledge about him is still lacking. Thus, his exact year of birth is unknown, just as it is unclear how and where he learned mathematics and painting. Likewise, scholars have no dates for when he composed his three books on mathematics. Of these, his *Trattato d'abaco* (referred to as *Trattato*) is supposed to be the earliest. It belongs to the so-called abacus books, which, despite their name, presented not the abacus but mathematics on an elementary level for future merchants, bank clerks, artisans, and artists. The books dealt primarily with arithmetic, but often included some algebra and practical geometry as well—which is also true of Piero's *Trattato*. In addition, Piero treated some more advanced geometrical objects, such as the regular polyhedra. He returned to these solids in the last of his books *Libellus de quinque corporibus regularibus* (referred to as *Libellus*), in which he also included five semi-regular, also called Archimedean, polyhedra (Field, 1997). These solids are called Archimedean because Pappus of Alexandria in his *Collection* stated that Archimedes had found thirteen convex polyhedra that have regular polygons of more than one kind as faces. It is unlikely that Piero knew about Pappus's claim, and he may actually himself have rediscovered some of the Archimedean polyhedra. In *Trattato* he described two of these, one—later called a cuboctahedron—having eight equilateral triangles and six squares as faces, obtained by cutting off the corners of a cube through the midpoints of the edges. The second was a truncated tetrahedron that Piero constructed by cutting off the vertices of a tetrahedron through points situated one third of the edge length from the corners. In *Libellus* he came back to the truncated tetrahedron and then added the truncated versions of the remaining four regular polyhedra.

As to the rest of the contents of *Trattato* and *Libellus*, Piero benefited a great deal from earlier treatises—the material for which mainly dates back to Euclid, alKhwarizmi, and Leonardo da Pisa (also called Fibonacci) (on Piero and Euclid, see Folkerts, 1996; and Piero and the algebra tradition, see Giusti, 1993).

While able to draw on a tradition for the themes of the *Trattato* and *Libellus*, Piero was pioneering new ground when he composed his third book *De prospectiva pingendi* (On the perspective of painting, referred to as *De prospectiva*), which is the first work devoted entirely to perspective. Most of the book consists of descriptions of various constructions in a style that tends to become tedious, but is useful because it guides the reader through the entire construction. Piero not only wanted to explain the *how* of perspective constructions, but the *why* as well. In other words, he aimed to provide a scientific foundation for his subject. This was a task that he had to start from scratch, and he did get started. Taken stepwise, Piero's mathematical reasoning was sound, but he did not always provide all the arguments that were needed for his conclusions. For instance, in his "proof" of theorem 24 in the first book, he proved with great accuracy that some triangles are similar, but overlooked the fact that the similarity in itself did not prove his original statement.

De prospectiva is divided into three parts; in the first Piero discussed the problem of constructing perspective images of figures situated in a horizontal plane applying a total of three different methods. In the second part, Piero threw some three-dimensional figures into perspective. The third part of *De prospectiva* is devoted to a method that applies to both plane and solid figures. This method is based on making a plan and an elevation of a configuration consisting of the eye point, the picture plane, and an object to be drawn in perspective. The method then provides what corresponds to the horizontal and vertical coordinates of the perspective images of special selected points in the object. There are no traces of Piero's predecessors having applied this method, but still he takes it for granted that his readers knew the technique of constructing plans and elevations of objects. The essential novelty was that Piero applied this technique to perspective, and he may well have been the first to do so. He himself stated that he found this perspective method less abstract and at the same time more powerful than the constructions presented in the two first parts of the book. In his own words, it was "easy to demonstrate and understand" and advantageous to apply "to more difficult solids" (Piero, 1974, p. 129). And he did present impressive examples, among them how to construct the perspective image of a tilted cube in which none of the edges are horizontal or parallel to the picture plane, and a trompe l'oeil giving the impression that a bowl pops up from a table when seen from the intended eye point. What made the example with the tilted cube difficult was to construct its plan and elevation; for this Piero applied a plan, an elevation, and a third projection perpendicular to the two others to describe a rotation of a cube having its sides parallel to the plans of reference.

De prospectiva did in general not receive the appreciation it deserved. Presumably, its mathematical arguments were too difficult for practitioners to follow and its long and detailed descriptions of how to perform constructions deterred mathematicians from studying it carefully.

Piero was admired by Luca Pacioli and Giorgio Vasari, but by the end of the sixteenth century he seems to have been forgotten both as a mathematician and as a painter. Thus, it took four hundred years after his death before his name appeared as the author of a printed book. Although his work was not mentioned often, Piero was not completely without influence. Presumably through Pacioli's praise of *De prospectiva*, Daniele Barbaro became aware of the work and copied longer passages from it almost verbatim without revealing his source. Before Barbaro, he himself had behaved similarly by including an Italian version of *Libellus* as a third section of his own *De divina proportione*(1509) with out referring to Piero. Through these publications, and presumably also through manuscripts that applied Piero's material, some of his ideas became part of textbooks. For instance, it is striking that in his books on geometry and proportions, the German painter and mathematician Albrecht Dürer applied methods that are similar to Piero's, including the technique of involving a plan and an elevation to describe movements of bodies in space.

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Kirsti Andersen