

# Gottlob Frege | Encyclopedia.com

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(*b.* Wismar, Germany, 8 November 1848; *d.* Bad Kleinen, Germany, 26 July 1925),

*mathematics, logic, foundations of mathematics.* For the original article on Frege see *DSB*, vol. 5.

Although a mathematician, [Gottlob Frege](#) is regarded as one of the founding fathers of modern (analytical) philosophy. With his *Begriffsschrift* (concept script) of 1879 he created modern mathematical logic. He used it as a linguistic tool for a program of founding mathematical concepts exclusively on logical concepts (logicism). Frege was involved in controversies with representatives of the algebraic tradition in logic concerning the power of the different systems of [symbolic logic](#), and with [David Hilbert](#) on the nature of mathematical axiom systems.

**Frege in Jena** . Frege spent most of his academic life at the University of Jena, except for five semesters of studies in Göttingen (1871–1873). He took courses in mathematics, physics, chemistry, and philosophy. Among his most important teachers in Jena were Karl Snell, who followed [Jakob Friedrich Fries](#) as chair of mathematics and physics, and Ernst Carl Abbe, at that time privatdozent for mathematics. Abbe became Frege's mentor. He encouraged Frege to transfer to Göttingen in order to complete his university studies and later supported him in his career.

Back in Jena, Frege applied for the post of a privatdozent for mathematics, submitting as *Habilitationsschrift* "Methods of Calculation Based on an Extension of the Concept of Quality," which contributed to the theory of functional equations, in particular iteration theory. Abbe initiated Frege's promotion to außerordentlicher Professor (roughly equivalent to associate professor or reader) of mathematics in 1879. This early promotion was possible because Frege had published his first monograph, *Begriffsschrift*, in January of that year. In 1866 Abbe had become a scientific consultant for improving the construction of microscopes built by the [Carl Zeiss](#) optics industry. In 1875 he became an associate and limited partner of Zeiss. Abbe set up the [Carl Zeiss](#) Foundation (1889) by first establishing a fund for scientific purposes (Ministerialfond für wissenschaftliche Zwecke) in 1886, supporting teaching and research in mathematics and sciences at the University of Jena. Abbe's foundation also made an improvement in Frege's remuneration possible, and later financially supported his promotion to *ordentlicher Honorarprofessor* (a payroll professorship in honor of the person) in 1896.

In 1907 Frege was awarded the prestigious title of *Hofrat*. His growing reputation is indicated by [Ludwig Wittgenstein](#)'s visit in 1911 (further visits took place in 1912 and 1913). In summer semester 1913 [Rudolf Carnap](#) attended Frege's course *Begriffsschrift II*, and another course, *Logic in Mathematics*, in 1914. In 1918 Frege retired after having been on sick leave for a year. He moved to Bad Kleinen, a resort near Wismar. On the initiative of Heinrich Scholz, Frege's *Nachlass* (literary estate) was transferred in 1935 to Münster, where it was purportedly destroyed during a bomb raid on Münster on 25 March 1945.

**Logic** . Frege's publication of the *Begriffsschrift* is regarded in the early twenty-first century as "the single most important event in the development of modern logic" (Thiel and Beaney, p. 26). In this work, Frege created the first strict logical calculus in the modern sense, based on precise definitions of expressions and deduction rules arriving for the first time at an axiomatic development of classical quantification theory. Frege replaced the traditional analysis of elementary statements into subject and predicate with an analysis of a proposition into function and argument, which could be used to express the generality of a statement (and with this also existence statements) by bound variables and quantifiers.

It can be assumed that Frege took over the term *Begriffsschrift* from Friedrich Adolf Trendelenburg's characterization of [Gottfried Wilhelm Leibniz](#)'s general characteristics (1854). The term had, however, already been used by Wilhelm von Humboldt in a treatise (1824) on the letter script and its influence on the construction of language.

In a lengthy review of the *Begriffsschrift* (1880), Ernst Schröder accused Frege of ignoring [George Boole](#)'s algebra of logic, first presented in *The Mathematical Analysis of Logic* (1847). Frege answered in articles (published only posthumously) by comparing Boole's calculatory logic with his own, where he determined quantification theory as the main point of deviation (Frege, 1880–1881, 1882). It is indeed historically true that the Booleans had no quantification theory at that time, but this cannot be regarded as an essential difference between these variations of [symbolic logic](#) on a systematic level, because in 1883 the U.S. logician Charles S. Peirce and his student Oscar Howard Mitchell developed an almost equivalent quantification theory within the algebra of logic. The essential difference between the algebra of logic and Frege's mathematical logic can thus be seen in different interpretations of the judgment. Another essential difference can be seen in the fact that Frege aimed at giving a logical structure of judgeable contents, which implied an inherent semantics. The Booleans, in contrast, were interested in logical structures themselves, which could be applied in different domains. Their systems allowed various interpretations. This required a supplementary external semantics.

**Logicism** . Frege's work was above all devoted to investigations on the nature of number. It was, thus, essentially philosophical. There is evidence that he was influenced by the philosophy of his contemporaries, especially by neo-Kantian approaches. These influences found their way into Frege's philosophy of mathematics with its metaphysical qualities.

Contrary to [Immanuel Kant](#), who regarded mathematical (arithmetical and geometrical) propositions as examples for synthetic a priori propositions, that is, propositions that are not empirical, but enlarging knowledge, Frege wanted to prove that arithmetic could completely be founded on logic, that is, that each arithmetical concept, in particular the concept of number, could be derived from logical concepts. Arithmetic was, thus, analytical.

The logicist program is only sketched in the *Begriffsschrift*, where Frege gave purely logical definitions of equinumerosity and the successor relation. In his next book, the *Grundlagen der Arithmetik* (1884). Frege formulated the classical logicistic definition of number, according to which the number  $n$  is defined as the extension of the concept "equinumerous to the concept  $F_n$ " with  $F_n$  standing for a concept with exactly  $n$  objects falling under it. The series of  $F_n$ s starts with  $F_0 = \neg x = x$ .  $F_{n+1}$  can be reconstructed recursively from  $F_n$ . The number 0 is defined as the extension of the concept "equinumerous to the concept 'different from itself,'" and the number 1 as the extension of the concept "equinumerous to the concept 'equals 0.'" The purely logical foundation of mathematics should not only disprove the Kantian paradigm, but also refute empiricist approaches to mathematics such as the one advocated by [John Stuart Mill](#), and with this psychological interpretations of numbers as mental constructions. Frege pointedly expressed this criticism of psychologism in his harsh review of [Edmund Husserl](#)'s *Philosophie der Arithmetik* in 1894, as a result of which Husserl was brought to revise the foundational program of phenomenology and convert to antipsychologism.

Frege elaborated the logicistic program in the two volumes of the *Grundgesetze der Arithmetik* (1893/1903). In this last of his monographs Frege also presented the mature version of his ontology, developed earlier in the three papers "Function and Concept" (1891), "On *Sinn und Bedeutung*" (1892), and "Concept and Object" (1892), all three currently regarded as classic texts of analytical philosophy. There he further elaborated his earlier distinction between concept and object. In particular he introduced in the *Grundgesetze* value-ranges considered as a special kind of objects. The identity criterion is given in Basic Law V, according to which the value-ranges of two functions are identical if the functions coincide in their values for every argument, with this giving the modern abstraction schema. In terms of concepts the law says that whatever falls under the concept  $F$  falls under the concept  $G$  and vice versa, if and only if the concepts  $F$  and  $G$  have the same extension.

Frege's conception of logicism failed, as Frege himself diagnosed, because of Basic Law V. Frege suggested an ad hoc solution forbidding that the extension of a concept may fall under the concept itself. This solution was proved to be insufficient by Stanisław Leśniewski (1939, unpublished) and Willard Van Orman Quine in 1955, but it indicates that the logical form of Basic Law V may be innocent of the emergence of the paradox and that the formation rules for function names may be too liberal in allowing impredicative function names.

In his latest publications Frege gave up logicism. He abandoned the talk of extensions of concepts and value-ranges, and the idea of numbers as logical objects, although he still held that they are objects of some other kind, based on the source of "geometrische Erkenntnisquelle," that is, pure intuition.

Frege's logicist program was later revived by the proponents of Frege-arithmetic and neologicism. In some of these directions Basic Law V is replaced by Hume's principle, according to which two concepts  $F$  and  $G$  have the same number if and only if they are equinumerous, that is, if there is a one-to-one correspondence between the  $F$ 's and  $G$ 's.

**The Nature of Axiomatics** . After the failure of his logicistic program, Frege focused his research on geometry as the foundational discipline of mathematics. He kept the traditional understanding of geometry as an intuitive discipline, thereby opposing [David Hilbert](#)'s new formalistic approach to geometry that came along with a new kind of axiomatics. Frege's opposition had its prehistory in his criticism of the older arithmetical formalism presented in a paper "Über formale Theorien der Arithmetik" (1885), taken up again in papers against his Jena colleague in mathematics, Carl Johannes Thomae (1906/1908). In these papers Frege opposes the understanding of arithmetic as a purely formal game with calculations bare of any contents. The older formalism regards arithmetic as a game like chess. It starts from certain initial formulas, then derives new formulas using a fixed set of transformation rules. But, neither the initial formulas nor the transition rules are justified, so the derived formulas are not justified either. Therefore, Frege concludes, these approaches could not provide any contribution to the foundations of arithmetic.

Hilbert overcame the traditional conception of axiomatics according to the model of Euclid's *Elements* by giving an example. In his *Grundlagen der Geometrie* of 1899 he gave an axiomatic presentation of Euclidean geometry. Hilbert's system proceeds from "thought things" in the Kantian sense, products of the human mind, but empty concepts because of lacking any element of (empirical) intuition. The geometrical concepts were not directly defined, but implicitly gained as concepts obeying the features set by some group of axioms, and justified by proving the independence of the axioms from one another, the completeness of the system, and its consistency. The formalistic approach aims at a theory of structures. This is pointedly expressed in Hilbert's letter to Frege of 29 December 1899, in which Hilbert claimed that every theory is only a half-timbering or schema of concepts and implications with arbitrary basic elements. If instead of the system of points some system love, law, chimney sweep is thought, and if all axioms are regarded as relations between these elements, then all theorems, for example Pythagoras's theorem, would be valid for them.

In a letter sent two days earlier, Frege had correctly criticized Hilbert's use of implicit definitions arguing that he had blurred the differences between axioms and definitions. It became clear that Frege stuck to the traditional (Aristotelian) understanding of axioms in geometry, calling axioms sentences that are true but not proved, because they have emerged from a source of knowledge completely different from the logical, a source that can be called spatial intuition. From the truth of the axioms follows that they do not contradict each other, so no consistency of proof was needed. Hilbert answered that if the arbitrarily set axioms do not contradict each other with all their implications, then they are true and the defined objects exist. For Hilbert consistency (logical possibility) is, thus, the criterion of truth and existence.

Hilbert rejected Frege's suggestion to publish the exchange of letters, so Frege took up his criticism in a series of papers published in 1903 and 1906 on the foundations of geometry, where he argued for his antiformalistic position. He demanded that after having defined the concept "point," it should be possible to determine whether a certain object, for example, his pocket watch, was a point or not.

The two controversies mentioned show that Frege followed that traditional understanding of philosophy as all-embracing fundamental discipline that formed, along with logic, logical ontology, and epistemology the foundation for mathematics and sciences. He did not share the pragmatic attitude of some influential contemporaries in mathematics and sciences (like Hilbert) to keep philosophy away from their mathematical and scientific practice by simply fading out philosophical problems. Nevertheless, Frege opened the way for directions like philosophy of science, which aimed at bridging the gap between philosophy on the one hand and mathematics and science on the other, and which became successful in the twentieth century. His influence on Bertrand Russell's logicism, codified in *Principia Mathematica*, Russell's joint work with [Alfred North Whitehead](#) (1910/13), is well known. Through Carnap, Frege gained influence on logic and foundational research in the neopositivist movement of the Vienna Circle, which constituted the context of Kurt Gödel's shaping of modern logic and foundational studies.

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