Gelfond, Alexandr Osipovich | Encyclopedia.com

(*b*. St. Petersburg [now Leningrad], Russia, 24 October 1906; *d* Moscow, U.S.S.R., 7 November 1968)

mathematics

Gelfond was the son of Osip Isaacovich Gelfond, a physician who also did work in philosophy. From 1924 to 1927 he studied in the division of mathematics of the department of physics and mathematics at Moscow University; later he took a postgraduate course (1927-1930) under A. J. Khintchine and V. V. Stepanov. In 1929–1930 Gelfond taught mathematics at Moscow Technological College, and from 1931 until his death he was at Moscow University, where for a number of years he held the chair of analysis. He later held the chair of the theory of numbers, to which was subsequently added the history of mathematics. From 1933 he also worked in the Soviet Academy of Sciences Mathematical Institute. He became professor of mathematics in 1931 and doctor of mathematics and physics in 1935; he was elected corresponding member of the Academy of Sciences of the U.S.S.R. in 1939 and corresponding member of the International Academy of the History of Science in 1968.

Most important in Gelfond's scientific work were the analytical theory of numbers and the theory of interpolation and approximation of functions of a complex variable. Studies in both fields were closely related; he used and improved methods of the theory of functions in working on the problems of the theory of transcendental numbers.

In 1748 Euler had expressed the idea that logarithms of rational numbers with rational bases are either rational or transcendental. Generalizing that statement, among the famous twenty-three problems that Hilbert posed in 1900, was the hypothesis of the rationality or transcendence of logarithms of algebraic numbers with algebraic bases; i.e., he presumed the transcendence of a^b, where a is any algebraic number not 0 or 1 and b is any irrational algebraic number. For thirty years no approach to solution of this, the seventh of Hilbert's problems, could be found. In 1929 Gelfond established profound connections between the growth and other properties of an entire analytic function and the arithmetic nature of its values when the values of argument belonged to a given algebraic field. This enabled him to find, proceeding from the expansion of the exponential function a^z, a being an algebraic number not 0 or 1, into the interpolting series of Newton.

where $x_0, x_1, x_2 \ldots$ are integers of an algebraic field a solution of the problem in a particular case: the number a^b, where and D is a positive integer that is not a perfect square, is transcendental.

In 1930 R. O. Kuzmin extented Gelfond's method to real , and in 1932 C. L. Siegel applied it the study of the transcendence of the periods of elliptic functions. Soon after, Gelfond consoliated his method with new ingenious ideas and, introducing linear forms of exponential function into consideration, confirmed in 1934 Hilbert's hypothesis in its entirety. His methods and results led to the most important contributions to the theory of transcendental numbers since Hermite's demonstration of the transcendence of e (1873) and K. L. F. Lindemann's of π (1882).

Applying his method to functions of p-adic variables, Gelfond made a number of new discoveries. Among them is the theorem that if $\alpha\ \beta\ \gamma$ are real algebraic numbers and at least one of them is not an algebraic unit, with γ being not equal to 2^n (n is a rational integer), the equation $\alpha^x+\beta^y=\gamma^z$ can possess only a finite number of solutions in rational integers x, y, z (1940). The further development of the method enabled Gelfond to solve a number of problems of mutual algebraic independence of numbers and to construct new classes of transcendental numbers. A considerable part of his discoveries in the theory of transcendental numbers is described in his monograph *Transtsendentnye i algebraicheskie chisla* (1952). Gelfond also wrote on other problems of the theory of numbers, including the diophantine approximations, and elementary methods of analytic theory.

In the theory of functions of a complex variable, Gelfond conducted numerous studies on problems of convergence of interpolation processes depending upon the density of a set of basic points of interpolation and upon the properties of the function to be approximated; on necessary and sufficient conditions for the determination of an entire analytical function on its given values or some other element; and on corresponding methods for the construction of functions. These studies were to a great extent summed up in *Ischislenie konechnykh raznostey* (1952).

Gelfond also promoted the history of mathematics, brilliantly characterizing Euler's work and his investigations in the theory of numbers. For many years he was the chairman of the scientific council that refereed theses on the history of physics and mathematics for the Soviet Academy of Sciences Institute of the History of Science and Technology.

Gelfond, creator of a large scientific school in the Soviet Union. profoundly influenced the advance of the theory of transceudentals and the theory of interpolation and approximation of functions of a complex variable.

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